Integrated Pathway Year 3 Integrated Math III Test Specifications

DCAS Aligned to Common Core State Standards

The following document shows the alignment of the Delaware Comprehensive Assessment System (DCAS) to the Delaware Common Core State Standards (CCSS), as reflected on the computer-adaptive test for Integrated Math III. To respect the intent of the CCSS, all standards are represented in either the traditional or integrated pathway documents. Therefore, all standards should be taught in grades 9 through 11 to ensure continuity of learning. The number for standards not assessed on the DCAS for Algebra II is highlighted in yellow.

The standards that are identified in the Integrated Math III document are specific to this course. Where there are gaps in numbering of the standards, please refer to the Overview of the Pathway document to see the vertical alignment within that CCSS domain.

<table>
<thead>
<tr>
<th>Number and Quantity</th>
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<tbody>
<tr>
<td>9–12.N.RN – The Real Number System Domain</td>
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<tr>
<td>• Extend the properties of exponents to rational exponents.</td>
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<tr>
<td>N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.</td>
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<tr>
<td>N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.</td>
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<tr>
<td>N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</td>
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</table>

| 9-12.N.Q – Quantities Domain |
| • Reason quantitatively and use units to solve problems. |
| • Standard Clarification: Foundation for work with expressions, equations, and functions. |
| N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. |
| N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. |
| N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
## Algebra

### 9-12.A.SSE – Seeing Structure in Expressions Domain

- **Interpret the structure of expressions.**
  - **Standard Clarification:** Polynomial and rational

  - **A.SSE.1** Interpret expressions that represent a quantity in terms of its context.*
  - **A.SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
  - **A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret \( P(1+r)^n \) as the product of \( P \) and a factor not depending on \( P \).*

  - **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see \( x^4 - y^4 \) as \((x^2)^2 - (y^2)^2\), thus recognizing it as a difference of squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\).*

- **Write expressions in equivalent forms to solve problems.**
  - **Standard Clarification:** Quadratic and exponential

  - **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
  - **A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
  - **A.SSE.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - **A.SSE.3.c** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression \( 1.15^t \) can be rewritten as \((1.15^{\frac{1}{12}})^{12t} \approx 1.012^{12t}\) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

  - **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

### 9-12.A.APR – Arithmetic with Polynomials and Rational Expressions Domain

- **Perform arithmetic operations on polynomials.**
  - **Standard Clarification:** Quadratic and beyond

  - **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

  - **A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
### 9.12.A.CED – Creating Equations* Domain

- **Create equations that describe numbers or relationships.**
  - *Standard Clarification: Equations using all available types of expressions, including simple root functions*

  **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

  **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

  **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.***

  **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$.***


- **Understand solving equations as a process of reasoning and explain the reasoning.**
  - *Standard Clarification: Simple radical and rational*

  **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

  **A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

- **Solve equations and inequalities in one variable.**
  - *Standard Clarification: Quadratics with real coefficients*

  **A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

  **A.REI.4** Solve quadratic equations in one variable.

  **A.REI.4.a** Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

  **A.REI.4.b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

- **Represent and solve equations and inequalities graphically**
  - *Standard Clarification: Combine polynomial, rational, radical, absolute value, and exponential functions*

  **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

  **A.REI.11** Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

  **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
## Functions

### 9-12.F.IF – Interpreting Functions Domain

- **Understand the concept of a function and use function notation.**
  
  F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

  F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

- **Interpret functions that arise in applications in terms of the context.**
  
  - Standard Clarification: Include rational, square root and cube root; emphasize selection of appropriate models
  
  F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include:** intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

  F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.*

  F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

- **Analyze functions using different representations.**
  
  - Standard Clarification: Include rational and radical; focus on using key features to guide selection of appropriate type of model function
  
  F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

  F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

  F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

  F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

  F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

  F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

  F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

  F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. **For example, identify percent rate of change in functions such as \( y = (1.02)^t \), \( y = (0.97)^t \), \( y = (1.01)^{12t} \), \( y = (1.2)^{t/10} \), and classify them as representing exponential growth or decay.**

  F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.**
### 9-12.F.BF – Building Functions Domain

- **Build a function that models a relationship between two quantities.**
  - *Standard Clarification: Include all types of functions studied*
    - F.BF.1 Write a function that describes a relationship between two quantities.*
      - F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
      - F.BF.1b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    - F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
      - *Standard Clarification: Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*

- **Build new functions from existing functions.**
  - F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

### 9-12.F.LE – Linear, Quadratic, and Exponential Models Domain

- **Construct and compare linear, quadratic, and exponential models and solve problems.**
  - *Standard Clarification: Logarithms as solutions for exponentials*
    - F.LE.4 For exponential models, express as a logarithm the solution to \( ab^{ct} = d \) where a, c, and d are numbers and the base b is 2, 10, or \( e \); evaluate the logarithm using technology.

### 9-12.F.TF – Trigonometric Functions Domain

- **Extend the domain of trigonometric functions using the unit circle.**
  - F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **Model periodic phenomena with trigonometric functions.**
  - F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
- **Prove and apply trigonometric identities.**
  - F.TF.8 Prove the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \) and use it to find \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \) given \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \) and the quadrant of the angle.
### Geometry

**9-12.G.C – Circles Domain**

- **Understand and apply theorems about circles**
  - **G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
  - **G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

- **Find arc lengths and areas of sectors of circles**
  - *Radian introduced only as unit of measure*
  - **G.C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

**9-12.G.GPE – Expressing Geometric Properties with Equations Domain**

- **Translate between the geometric description and the equation for a conic section**
  - **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

**9-12.G.MG – Modeling with Geometry Domain**

- **Apply geometric concepts in modeling situations**
  - **G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
### Statistics and Probability

#### 9-12.S.ID – Interpreting Categorical and Quantitative Data Domain
- **Summarize, represent, and interpret data on a single count or measurement variable.**
- **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
- **S.ID.6a** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

- **Understand independence and conditional probability and use them to interpret data.**
  - **Standard Clarification:** Link to data from simulations or experiments
- **S.CP.3** Understand the conditional probability of A given B as \( P(A \text{ and } B)/P(B) \), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
- **S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
- **Use the rules of probability to compute probabilities of compound events in a uniform probability model.**
- **S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
- **S.CP.7** Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model.

#### 9-12.S.MD – Using Probability to Make Decisions Domain
- **Use probability to evaluate outcomes of decisions.**
  - **Standard Clarification:** Include more complex situations
- **S.MD.5** (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
- **S.MD.5a** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
- **S.MD.5b** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

*Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (†).*