Delaware’s
Common Core State Standards for High School Mathematics
Assessment Examples

Compiled by:

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Delaware Common Core State Standards for High School Mathematics

Overview

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in fourth credit courses or advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college- and career-ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories including Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability.

Conceptual categories portray a coherent view of high school mathematics. A student's work with functions, for example, crosses a number of traditional course boundaries potentially up through and including calculus. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a standard for mathematical practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

<table>
<thead>
<tr>
<th>Number and Quantity</th>
<th>Algebra</th>
<th>Functions</th>
<th>Geometry</th>
<th>Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The Real Number System (N-RN)</td>
<td>• Seeing Structure in Expressions (A-SSE)</td>
<td>• Interpreting Functions (F-IF)</td>
<td>• Congruence (G-CO)</td>
<td><strong>Modeling</strong></td>
</tr>
<tr>
<td>• Quantities (N-Q)</td>
<td>• Arithmetic with Polynomials and Rational Expressions (A-APR)</td>
<td>• Building Functions (F-BF)</td>
<td>• Similarity, Right Triangles, and Trigonometry (G-SRT)</td>
<td></td>
</tr>
<tr>
<td>• The Complex Number System (N-CN)</td>
<td>• Creating Equations (A-CED)</td>
<td>• Linear, Quadratic, and Exponential Models (F-LE)</td>
<td>• Circles (G-C)</td>
<td></td>
</tr>
<tr>
<td>• Vector and Matrix Quantities (N-VM)</td>
<td>• Reasoning with Equations and Inequalities (A-REI)</td>
<td>• Trigonometric Functions (F-TF)</td>
<td>• Expressing Geometric Properties with Equations (G-GPE)</td>
<td></td>
</tr>
</tbody>
</table>

Statistics and Probability

• Interpreting Categorical and Quantitative Data (S-ID)
• Making Inferences and Justifying Conclusions (S-IC)
• Conditional Probability and the Rules of Probability (S-CP)
• Using Probability to Make Decisions (S-MD)
High School Mathematics – Unpacking the Delaware Common Core State Standards

This document is designed to help understand the Common Core State Standards in providing examples that show a range of format and complexity. It is a work in progress, and it does not represent all aspects of the standards.

What Is the Purpose of This Document?

This document may be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the expectations. This document, along with ongoing professional development, is one of many resources used to understand and teach the Delaware Common Core State Standards.

This document contains descriptions of what each standard means and what a student is expected to know, understand, and be able to do. This is meant to eliminate misinterpretation of the standards.

References

This document contains explanations and examples that were obtained from State Departments of Education for Utah, Arizona, North Carolina, Kansas, and New York with permission.

How Do I Send Feedback?

This document is helpful in understanding the Common Core but is an evolving document where more comments and examples might be necessary. Please feel free to send feedback to the Delaware Department of Education at rfly@doe.k12.de.us, and we will use your input to refine this document.
High School – Number and Quantity Overview

**The Real Number System (N-RN)**
- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers

**Quantities (N-Q)**
- Reason quantitatively and use units to solve problems

**The Complex Number System (N-CN)**
- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

**Vector and Matrix Quantities (N-VM)**
- Represent and model with vector quantities
- Perform operations on vectors
- Perform operations on matrices and use matrices in applications

**Mathematical Practices (MP)**
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Number and Quantity: The Real Number System (N-RN)

Extend the properties of exponents to rational exponents

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>✷ N.RN.1 – Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{2/3})^3 = 5^{1/3 \cdot 3}$ to hold, so $(5^{1/3})^3$ must equal 5. Explanation: Students may explain orally or in written format. Example:</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expression</th>
<th>Numerical Value</th>
<th>Expression</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4^{1/2} =$?</td>
<td>$\sqrt{4} = 2 =$?</td>
<td>$\frac{2}{\sqrt{4}} = \sqrt{4} =$?</td>
<td></td>
</tr>
<tr>
<td>$64^{1/3} =$?</td>
<td>$\sqrt[3]{64} = 4 =$?</td>
<td>$\sqrt[3]{64} = \sqrt[3]{4^3} = 4 =$?</td>
<td></td>
</tr>
<tr>
<td>$8^{2/3} =$?</td>
<td>$\sqrt[3]{8^2} = 4 =$?</td>
<td>$\sqrt[3]{8^2} = \sqrt[3]{2^4} = 4 =$?</td>
<td></td>
</tr>
<tr>
<td>$16^{1/4} =$?</td>
<td>$\sqrt[4]{16} = 2 =$?</td>
<td>$\sqrt[4]{16^1} = 2 =$?</td>
<td></td>
</tr>
<tr>
<td>$25^{-1/2} =$?</td>
<td>$(\sqrt{25})^{-1} = \frac{1}{5} =$?</td>
<td>$(\sqrt{25})^{-1} = \frac{1}{\sqrt{25}} = \frac{1}{5} =$?</td>
<td></td>
</tr>
<tr>
<td>$(2^3)^{1/2} =$?</td>
<td>$\sqrt{2}^3 = ?$</td>
<td>$\sqrt{2^3} = \sqrt{8} = 2\sqrt{2} =$?</td>
<td></td>
</tr>
</tbody>
</table>

1. What did you notice about your answers to the problems in the same rows?
2. Is there some pattern that relates the two expressions in each row to one another? Describe the pattern.
3. Given the expression $(5^{3})^{1/2}$, what expression using a root symbol would yield the same numerical value?
4. Given the expression $\sqrt[3]{54}$, what expression utilizing a fractional exponent would yield the same numerical value?
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷ | N.RN.2 – Rewrite expressions involving radicals and rational exponents using the properties of exponents.  
Example:  
- $\sqrt[3]{5^{2}} = 5^{\frac{2}{3}}$  
- Rewrite using fractional exponents:  
  - $\sqrt[5]{16} = \sqrt[5]{2^{4}} = 2^{\frac{4}{5}}$  
- Rewrite $\frac{\sqrt{x}}{x^{2}}$ in at least three alternate forms.  
  Solution: $x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{\sqrt{x^{3}}} = \frac{1}{x\sqrt{x}}$  
- Rewrite $\sqrt[4]{2-4}$ using only rational exponents.  
- Rewrite $\sqrt[3]{x^{3} + 3x^{2} + 3x + 1}$ in simplest form. |
| 9–10 | N.RN.3 – Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.  
Explanation: Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between multiplication and addition).  
Example:  
- Explain why the number $2\pi$ must be irrational, given that $\pi$ is irrational. Answer: if $2\pi$ were rational, then half of $2\pi$ would also be rational, so $\pi$ would have to be rational as well, but $\pi$ is irrational therefore, $2\pi$ is irrational. |
Number and Quantity: Quantities★ (N-Q)

Reason quantitatively and use units to solve problems

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10 + ★ | N.Q.1 – Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.  
**Explanation:** Include word problems where quantities are given in different units, which must be converted to make sense of the problem.  
**Example:**  
- A problem might have an object moving 12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour:  
  \[
  \frac{12 \text{ ft.}}{1 \text{ sec.}} \cdot \frac{60 \text{ sec.}}{1 \text{ min.}} \cdot \frac{60 \text{ min.}}{1 \text{ hr.}} = \frac{1 \text{ mile}}{5280 \text{ ft.}} = 8.18 \text{ mph}
  \]  
  Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatter plots, and multi-bar graphs. |
| 9–10 + ★ | N.Q.2 – Define appropriate quantities for the purpose of descriptive modeling.  
**Examples:**  
- What type of measurements would one use to determine their income and expenses for one month?  
- How could one express the number of accidents in Delaware? |
| 9–10 ★ | N.Q.3 – Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.  
**Explanation:** The margin of error and tolerance limit varies according to the measure, tool used, and context.  
**Example:**  
- Determining price of gas by estimating to the nearest cent is appropriate because you will not pay in fractions of a cent but the cost of gas is \( \frac{\$3.479}{\text{gallon}} \). |
Number and Quantity: The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
<tbody>
<tr>
<td> N.CN.1 – Know there is a complex number ( i ) such that ( i^2 = -1 ), and every complex number has the form ( a + bi ) with ( a ) and ( b ) real.</td>
<td></td>
</tr>
</tbody>
</table>

**Example:**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
<th>bi Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{-36} )</td>
<td>( \sqrt{-36} = \sqrt{-1} \cdot \sqrt{36} = 6i )</td>
<td>6i</td>
</tr>
<tr>
<td>2. ( 2\sqrt{-49} )</td>
<td>( 2\sqrt{-49} = 2\sqrt{-1} \cdot \sqrt{49} = 2 \cdot 7i = 14i )</td>
<td>14i</td>
</tr>
<tr>
<td>3. ( -3\sqrt{-10} )</td>
<td>( -3\sqrt{-10} = -3\sqrt{-1} \cdot \sqrt{10} = -3 \cdot i \cdot \sqrt{10} = -3i\sqrt{10} )</td>
<td>-3i\sqrt{10}</td>
</tr>
<tr>
<td>4. ( 5\sqrt{-8} )</td>
<td>( 5\sqrt{-8} = 5\sqrt{-1} \cdot \sqrt{8} = 5 \cdot i \cdot 2\sqrt{2} = 10i\sqrt{2} )</td>
<td>10i\sqrt{2}</td>
</tr>
</tbody>
</table>

|  N.CN.2 – Use the relation \( i^2 = -1 \) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. |

**Example:**

- Simplify the following expression. Justify each step using the commutative, associative, and distributive properties.

\[
(3 - 2i)(-7 + 4i)
\]

Solutions may vary—one solution follows:

\[
(3 - 2i)(-7 + 4i)
\]

\[
3(-7 + 4i) - 2i(-7 + 4i) \quad \text{Distributive Property}
\]

\[
-21 + 12i + 14i - 8i^2 \quad \text{Distributive Property}
\]

\[
-21 + (12i + 14i) - 8i^2 \quad \text{Associative Property}
\]

\[
-21 + i(12 + 14) - 8i^2 \quad \text{Distributive Property}
\]

\[
-21 + 26i - 8i^2 \quad \text{Computation}
\]

\[
-21 + 26i - 8(-1) \quad i^2 = -1
\]

\[
-21 + 26i + 8 \quad \text{Computation}
\]

\[
-21 + 8 + 26i \quad \text{Commutative Property}
\]

\[
-13 + 26i \quad \text{Computation}
\]
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>N.CN.3 – Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</td>
</tr>
</tbody>
</table>

**Example:**

- Given \( w = 2 - 5i \) and \( z = 3 + 4i \)
  
  a. Use the conjugate to find the modulus of \( w \).
  
  b. Find the quotient of \( z \) and \( w \).

<table>
<thead>
<tr>
<th>Solution a:</th>
<th>Solution b:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>w</td>
</tr>
<tr>
<td>(</td>
<td>w</td>
</tr>
<tr>
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<td>(</td>
<td>w</td>
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<tr>
<td>(</td>
<td>w</td>
</tr>
</tbody>
</table>
### Represent complex numbers and their operations on the complex plane

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | N.CN.4 – Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.  
*Explanation:* Students will represent complex numbers using rectangular and polar coordinates. |
|       | a + bi = r(cos θ + sin θ) |
|       | ![Diagram of complex numbers on the complex plane](image)|
|       | Examples: |
|       | • Plot the points corresponding to 3 – 2i and 1 + 4i. Add these complex numbers and plot the result. How is this point related to the two others? |
|       | • Write the complex number with modulus (absolute value) 2 and argument π/3 in rectangular form. |
|       | • Find the modulus and argument (0 < θ < 2π) of the number √6 + √−6. |
| +     | N.CN.5 – Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.  
*For example,* (−1 + √3i)³ = 8 because (−1 + √3i) has modulus 2 and argument 120°. |
<p>|       | Examples: |
|       | • Perform the following operations with complex numbers: |
|       | a. (3 − 5i) + (−10 + 12i) |
|       | b. (18 − 3i) − (2 − 6i) |
|       | c. (4 + 18i)(2 − 3i) |
|       | d. (\frac{4 + 10i}{5 + \sqrt{3}i}) |
|       | • Find the reciprocal of the complex number 8 − 2√2i |
|       | • Graph the complex numbers: −2, (\frac{1}{2}), 2³, and (−6i) |
|       | • Calculate their norms (absolute value). |</p>
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | N.CN.6 – Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.  
*Example:*  
- Which of the complex numbers $z_1$, $z_2$, $z_3$, $z_4$, and $z_5$ below has the greatest modulus? |

![Diagram](image)

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷     | N.CN.7 – Solve quadratic equations with real coefficients that have complex solutions.  
*Examples:*  
- Within which number system can $x^2 = -2$ be solved? Explain how you know.  
- Solve $x^2 + 2x + 2 = 0$ over the complex numbers.  
- Find all solutions of $2x^2 + 5 = 2x$ and express them in the form $a + bi$. |

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
</table>
| +     | N.CN.8 – Extend polynomial identities to the complex numbers. *For example, rewrite $x + 4$ as $(x + 2i)(x - 2i)$.  
*Example:*  
- Factor the polynomial $x^3 + 4x^2 + 5x$ completely (a) over the real numbers and (b) over the complex numbers.  
*Solution:*  
  a. $x^3 + 4x^2 + 5x = x(x^2 + 4x + 5)$  
  b. $x^3 + 4x^2 + 5x = x(x + 2 - i)(x + 2 + i)$ |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | N.CN.9 – Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.  
   Examples:  
   - How many zeros does $-2x^2 + 3x - 8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra.  
   - How many complex zeros does the following polynomial have? How do you know?  
     $p(x) = (x^2 - 3)(x^2 + 2)(x - 3)(2x - 1)$ |
### Number and Quantity: Vector and Matrix Quantities (N-VM)

**Represent and model with vector quantities**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | N.VM.1 – Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \( \vec{v} \), \( |\vec{v}| \), \( ||\vec{v}|| \), \( \vec{v} \)).  
*Example:*  
- Find the vector with the given magnitude and in the same direction as \( \vec{u} \).  
  \( \|\vec{v}\| = 9 \) and \( \vec{u} = (2, 5) \) |
| +     | N.VM.2 – Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.  
*Example:*  
- Write a linear combination of the standard unit vectors \( i \) and \( j \), given the initial and terminal points, respectively.  
  \((-2, 4) \) and \((1, 5)\) |
| +     | N.VM.3 – Solve problems involving velocity and other quantities that can be represented by vectors.  
*Examples:*  
- A motorboat traveling from one shore to the other at a rate of 5 m/s east encounters a current flowing at a rate of 3.5 m/s north.  
  - What is the resultant velocity?  
  - If the width of the river is 60 meters wide, then how much time does it take the boat to travel to the opposite shore?  
  - What distance downstream does the boat reach the opposite shore?  
- A ship sails 12 hours at a speed of 15 knots (nautical miles per hour) at a heading of 68° north of east. It then turns to a heading of 75° north of east and travels for 5 hours at 8 knots. Find its position north and east of its starting point. (For this problem, assume the earth is flat.)  
  - The solution may require an explanation, orally or in written form, that includes understanding of velocity and other relevant quantities. |

*Label Legend:* 9-10 = Standards for Grades 9 and 10; ✦ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
### Perform operations on vectors

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>N.VM.4 – Add and subtract vectors.</td>
</tr>
<tr>
<td></td>
<td>a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</td>
</tr>
<tr>
<td></td>
<td>b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</td>
</tr>
<tr>
<td></td>
<td><em>Example:</em></td>
</tr>
<tr>
<td></td>
<td>Addition of vectors is used to determine the resultant of two given vectors. This can be done by lining up the vectors end to end, adding the components, or using the parallelogram rule. Students may use applets to help them visualize operations of vectors given in rectangular or polar form.</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram of vector operations" /></td>
</tr>
<tr>
<td>+</td>
<td>c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of $w$, with the same magnitude as $w$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</td>
</tr>
<tr>
<td></td>
<td><em>Example:</em></td>
</tr>
<tr>
<td></td>
<td>• Given two vectors $u$ and $v$, can the magnitude of the resultant be found by adding the magnitude of each vector? Use an example to illustrate your explanation.</td>
</tr>
<tr>
<td></td>
<td>• If $u = (-2, -8)$ and $v = (2, 8)$ find $u + v$, $u = (-v)$, and $u - v$. Explain the relationship between $u + -v$ and $u - v$ in terms of the vector components.</td>
</tr>
<tr>
<td></td>
<td>• A plane is flying due east at an average speed of 500 miles per hour. There is a crosswind from the south at 60 miles per hour. What is the magnitude and direction of the resultant?</td>
</tr>
</tbody>
</table>
### Label: N.VM.5 – Multiply a vector by a scalar.

#### a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

**Explanation:** The result of multiplying a vector $v$ by a positive scalar $c$ is a vector in the same direction as $v$ with a magnitude of $cv$. If $c$ is negative, then the direction of $v$ is reversed by scalar multiplication. Students will represent scalar multiplication graphically and component-wise. Students may use applets to help them visualize operations of vectors given in rectangular or polar form.

**Example:**
- Given $u = (2, 4)$, write the components and draw the vectors for $u$, $2u$, $\frac{1}{2}u$, and $-u$. How are the vectors related?

#### b. Compute the magnitude of a scalar multiple $cv$ using $||cv|| = |c|v$. Compute the direction of $cv$ knowing that when $|c|v \neq 0$, the direction of $cv$ is either along $v$ (for $c > 0$) or against $v$ (for $c < 0$).

**Example:**
- If $v = (-3, 5)$, find $||cv||$ and $\theta$, where $c = 2$ and $0 \leq \theta < 360^\circ$. Round off $\theta$ to the nearest tenth of a degree.

### Perform operations on matrices and use matrices in applications

#### Label: N.VM.6 – Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

**Example:**
- Write an inventory matrix for the following situation. A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of paint, 12 brushes, and 17 canvases. For class 2, the teacher buys 20 tubes of paint, 14 brushes and 15 canvases. Next year, she has 3 times as many students in each class. What affect does this have on the amount of supplies?

**Solution:**

<table>
<thead>
<tr>
<th>Class</th>
<th>P</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class 1</td>
<td>24</td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>Class 2</td>
<td>20</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>Year 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class 1</td>
<td>72</td>
<td>36</td>
<td>51</td>
</tr>
<tr>
<td>Class 2</td>
<td>60</td>
<td>42</td>
<td>45</td>
</tr>
<tr>
<td>Label</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>----------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>☐ N.VM.7 – Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Example: Students may use graphing calculators and spreadsheets to create and perform operations on matrices.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|  | \[
\begin{bmatrix}
-7 & 19 & 15 \\
-3 & 41 & -63 & 20 \\
2 & 0 & -8 
\end{bmatrix}
\] |
|  | • The following is an inventory matrix for Company A’s jellybean, lollipop, and gum flavors. The price per unit is $0.03 for jellybeans, gum, and lollipops. Determine the gross profit for each flavor and for the entire lot. |
|  | F1 | F2 | F3 | F4 | F5 | F6 | F7 |
|  | C1 | 327 | 818 | 465 | 211 | 127 | 134 | 705 |
|  | C2 | 513 | 222 | 312 | 446 | 645 | 671 | 101 |
|  | C3 | 1878 | 901 | 51 | 156 | 711 | 423 | 344 |
|  | C1 = Jellybeans | F1 = Vanilla |
|  | C2 = Lollipops | F2 = Banana |
|  | C3 = Gum | F3 = Strawberry |
|  | | F4 = Tangerine |
|  | | F5 = Coconut |
|  | | F6 = Mint |
|  | | F7 = Licorice |
| ☐ N.VM.8 – Add, subtract, and multiply matrices of appropriate dimensions. |
|  | Example: Students may use graphing calculators and spreadsheets to create and perform operations on matrices. |
|  | • Find \(2A - B + C\) and \(A\times B\) given Matrices \(A, B,\) and \(C\) below. |
|  | Matrix A | Matrix B | Matrix C |
|  | \[
\begin{bmatrix}
-7 & 19 & 15 \\
41 & -63 & 20 \\
2 & 0 & -8 
\end{bmatrix}
\] | \[
\begin{bmatrix}
23 & 18 & 55 \\
-18 & -47 & 11 \\
39 & -6 & -8 
\end{bmatrix}
\] | \[
\begin{bmatrix}
-4 & 7 & 12 \\
51 & 9 & 80 \\
13 & 72 & 8 
\end{bmatrix}
\] |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷ N.VM.9 – Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.  
   *Example:* Students may use graphing calculators and spreadsheets to create and perform operations on matrices.  
   - Given \( A = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} \) and \( C = \begin{bmatrix} 6 & -2 \\ 9 & 7 \end{bmatrix} \); determine if the following statements are true:  
     - \( AB = BA \)  
     - \( (AB)C = A(BC) \) |
| ✷ N.VM.10 – Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. |
N.VM.11 – Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

Explanation: A matrix is a two-dimensional array with rows and columns; a vector is a one-dimensional array that is either one row or one column of the matrix.

Resources: Students will use matrices to transform geometric objects in the coordinate plane. Students may demonstrate transformations using dynamic geometry programs or applets. They will explain the relationship between the ordered pair representation of a vector and its graphical representation.

Examples:

- Multiply the following matrix, if possible: \[
\begin{bmatrix}
13 \\
5 \\
8
\end{bmatrix}
\begin{bmatrix}
6 \\
3 \\
7
\end{bmatrix}
\]

- Old MacDonald has three fruit farms. On these farms he grows peaches, apricots, plums, and apples. When picked, the fruit is sorted into layered boxes in which they will be sold. The chart below shows the number of boxes for each type of fruit.

<table>
<thead>
<tr>
<th>Location</th>
<th>Peaches</th>
<th>Apricots</th>
<th>Plums</th>
<th>Apples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farm 1</td>
<td>152</td>
<td>225</td>
<td>395</td>
<td>277</td>
</tr>
<tr>
<td>Farm 2</td>
<td>236</td>
<td>183</td>
<td>245</td>
<td>183</td>
</tr>
<tr>
<td>Farm 3</td>
<td>95</td>
<td>132</td>
<td>0</td>
<td>285</td>
</tr>
</tbody>
</table>

Suppose he sells peaches for $27 a box, apricots for $15 a box, plums for $34 a box, and apples for $17 a box. Find the income for each farm. How much will he make total?

- Find the coordinates of the vertices of each figure after the given transformation:
  - a. Translation: 2 units left and 1 unit down
    \[
    \begin{bmatrix}
    1 \\
    0
    \end{bmatrix}
    \begin{bmatrix}
    1 \\
    1 \\
    4 \\
    3
    \end{bmatrix}
    \begin{bmatrix}
    0 \\
    1 \\
    0 \\
    -4
    \end{bmatrix}
    \]
  - b. Rotation 270° counterclockwise about the origin
    \[
    \begin{bmatrix}
    -3 \\
    -3 \\
    0
    \end{bmatrix}
    \begin{bmatrix}
    -1 \\
    4 \\
    3
    \end{bmatrix}
    \]
  - c. Dilation of 0.25
    \[
    \begin{bmatrix}
    -1 \\
    2 \\
    -1
    \end{bmatrix}
    \begin{bmatrix}
    -5 \\
    -2 \\
    -1
    \end{bmatrix}
    \]
  - d. Reflection across \( y = x \)
    \[
    \begin{bmatrix}
    -2 \\
    0 \\
    -3
    \end{bmatrix}
    \]
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | N.VM.12 – Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.  

**Note:** DCAS will also assess $3 \times 3$ matrices.  
**Explanation:** Students should be able to utilize matrix multiplication to perform reflections, rotations, and dilations, and find the area of a parallelogram. Students may demonstrate these relationships using dynamic geometry programs or applets.  
**Example:**
- Let $S$ be the parallelogram determined by the vectors $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$. Let $A = \begin{bmatrix} 6 & -2 \\ -3 & 2 \end{bmatrix}$. Sketch $S$. Use determinants to find its area.
High School – Algebra Overview

**Seeing Structure in Expressions (A-SSE)**
- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

**Arithmetic with Polynomials and Rational Expressions (A-APR)**
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

**Creating Equations (A-CED)**
- Create equations that describe numbers or relationships

**Reasoning with Equations and Inequalities (A-REI)**
- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

**Mathematical Practices (MP)**
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Algebra: Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9-10 ✭ | A.SSE.1 – Interpret expressions that represent a quantity in terms of its context. <br> a. Interpret parts of an expression, such as terms, factors, and coefficients. <br> b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* $P$ *and a factor not depending on* $P$. <br> *Explanation:* Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret there meaning in terms of a context. <br> *Example:* <br> • A company uses two different sized trucks to deliver sand. The first truck can transport $x$ cubic yards, and the second $y$ cubic yards. The first truck makes $S$ trips to a job site, while the second makes $T$ trips. What do the following expressions represent in practical terms? <br> a. $S + T$  
 b. $x + y$  
 c. $xS + yT$  
 d. $\frac{xS + yT}{S + T}$ |
| 9-10 | A.SSE.2 – Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. <br> *Explanation:* Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. <br> *Example:* <br> • Factor $x^3 - 2x^2 - 35x$ <br> • Find a value for $a$, a value for $k$, and a value for $n$, so that $(3x + 2)(2x - 5) = ax^2 + kx + n$. |
### Write expressions in equivalent forms to solve problems

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9-10 ★ | A.SSE.3 – Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  
  a. Factor a quadratic expression to reveal the zeros of the function it defines.  
  b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  
  **Explanation:** Students will use the properties of operations to create equivalent expressions.  
  **Examples:**  
  - Express $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$ in factored form and use your answer to say for what values of $x$ the expression is zero.  
  - Write the expression below as a constant times a power of $x$ and use your answer to decide whether the expression gets larger or smaller as $x$ gets larger.  
    
    $$\frac{(2x^3)^2}{(3x^4)}$$  
  c. Use the properties of exponents to transform expressions for exponential functions. **For example,** the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.  
  **Examples:**  
  - If $x$ is positive and $x \neq 0$, simplify $\sqrt[3]{x}$.  
  - Simplify: $\sqrt{x + 2} + \sqrt{4x + 8}$.  

| ✪ ★ | A.SSE.4 – Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. **For example,** calculate mortgage payments.  
  **Example:**  
  - In February, the Bezanson family starts saving for a trip to Australia in September. The Bezanson’s expect their vacation to cost $5375. They start with $525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip?  

**Label Legend:** 9-10 = Standards for Grades 9 and 10; ✪ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
**Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)**

**Perform arithmetic operations on polynomials**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-10</td>
<td>A.APR.1 – Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</td>
</tr>
</tbody>
</table>

*Example:*
- Simplify: \( \frac{a^2-b^2}{a+b} \).
- Expand and simplify: \((x^3 + 3x^2 - 2x + 5)(x - 7)\).

**Understand the relationship between zeros and factors of polynomials**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>✷</td>
<td>A.APR.2 – Know and apply the Remainder Theorem: For a polynomial ( p(x) ) and a number ( a ), the remainder on division by ( x - a ) is ( p(a) ), so ( p(a) = 0 ) if and only if ( (x - a) ) is a factor of ( p(x) ).</td>
</tr>
</tbody>
</table>

*Explanation:* The Remainder theorem says that if a polynomial \( p(x) \) is divided by \( x - a \), then the remainder is the constant \( p(a) \). That is, \( p(x) = q(x)(x - a) + p(a) \). So, if \( p(a) = 0 \), then \( p(x) = q(x)(x - a) \).
- Let \( p(x) = x^5 - 3x^4 + 8x^2 - 9x + 30 \). What does your answer tell you about the factors of \( p(x) \)? [Answer: \( p(-2) = 0 \) so \( x + 2 \) is a factor.]

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>✷</td>
<td>A.APR.3 – Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</td>
</tr>
</tbody>
</table>

*Example:*
- Factor the expression \( x^3 + 4x^2 - 59x - 126 \) and explain how your answer can be used to solve the equation \( x^3 + 4x^2 - 59x - 126 = 0 \). Explain why the solutions to this equation are the same as the \( x \)-intercepts of the graph of the function \( f(x) = x^3 + 4x^2 - 59x - 126 \).
Use polynomial identifies to solve problems

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ♦ | A.APR.4 – Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* \((x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2\) *can be used to generate Pythagorean triples.*  

**Example:**  
- Use the distributive law to explain why \(x^2 - y^2 = (x - y)(x + y)\) for any two numbers \(x\) and \(y\).  
- Derive the identity \((x - y)^2 = x^2 - 2xy + y^2\) from \((x + y)^2 = x^2 + 2xy + y^2\) by replacing \(y\) by \(-y\).  
- Use an identity to explain the pattern.  
  \[2^2 - 1^2 = 3\]  
  \[3^2 - 2^2 = 5\]  
  \[4^2 - 3^2 = 7\]  
  \[5^2 - 4^2 = 9\]  
  [Answer: \((n + 1)^2 - n^2 + 1\) for any whole number \(n\).] |
| ♦ | A.APR.5 – Know and apply the Binomial Theorem for the expansion of \((x + y)^n\) in powers of \(x\) and \(y\) for a positive integer \(n\), where \(x\) and \(y\) are any numbers, with coefficients determined for example by Pascal's Triangle. *(The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)*  

**Examples:**  
- Use Pascal's Triangle to expand the expression \((2x - 1)^4\)  
- Find the middle term in the expansion of \((x^2 + 2)^{18}\)  
  \[\begin{array}{ccccccc}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
\end{array}\]  
  \[\begin{array}{ccc}
4C_0 & 4C_1 & 4C_2 & 4C_3 & 4C_4 \\
\end{array}\]  
  \[\text{\( (x + 1)^3 = x^3 + 3x^2 + 3x + 1\)}\]
### Algebra: Creating Equations ★ (A-CED)

Create equations that describe numbers or relationships

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷★ | A.CED.1 – Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*  
  
  **Explanation:** Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.  
  
  **Examples:**  
  • Given that the following trapezoid has area 54 cm², set up an equation to find the length of the base and solve the equation.  
    
    ![Trapezoid Diagram](image)
    
  • Lava coming from the eruption of a volcano follows a parabolic path. The height $h$ in feet of a piece of lava $t$ seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?  
| 9–10★ | A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  
  
  **Examples:**  
  • Gold is alloyed with different metals to make it hard enough to be used in jewelry. The amount of gold present in a gold ally is measured in 24ths called karats. 24-karat gold is 100% gold. Similarly, 18-karat gold is 75% gold. How many ounces of 18-karat gold should be added to an amount of 12-karat gold to make 4 ounces of 14-karat gold? Graph equations on coordinate axes with labels and scales.  
  • A metal alloy is 25% copper. How much of each alloy should be used to make 1000 grams of a metal alloy that is 45% copper? 

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<table>
<thead>
<tr>
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</tr>
</thead>
</table>
| 9–10  | A.CED.3 – Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*  
*Example:*  
- A club is selling hats and jackets as a fundraiser. Their budget is $1500, and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8.  
  - Write a system of inequalities to represent the situation.  
  - Graph the inequalities.  
  - If the club buys 150 hats and 100 jackets, will the conditions be satisfied?  
  - What is the maximum number of jackets they can buy and still meet the conditions? |
| 9–10  | A.CED.4 – Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law V = IR to highlight resistance R.*  
*Examples:*  
- The Pythagorean Theorem expresses the relation between the legs $a$ and $b$ of a right triangle and its hypotenuse $c$ with the equation $a^2 + b^2 = c^2$.  
  - Why might the theorem need to be solved for $c$?  
  - Solve the equation for $c$ and write a problem situation where this form of the equation might be useful.  
  - Solve $V = \frac{4}{3}\pi r^3$ for radius $r$.  
- Motion can be described by the formula below, where $t =$ time elapsed, $u =$ initial velocity, $a =$ acceleration, and $s =$ distance traveled.  
  $$s = ut + \left(\frac{1}{2}\right)at^2$$  
  - Why might the equation need to be rewritten in terms of $a$?  
  - Rewrite the equation in terms of $a$. |
### Algebra: Reasoning with Equations and Inequalities (A-REI)

**Understand solving equations as a process of reasoning and explain the reasoning**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10  | A.REI.1 – Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  
**Explanation:** Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.  
**Examples:**  
- Explain why the equation $\frac{x}{2} + \frac{7}{3} = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $\frac{x}{2} + \frac{7}{3}$ is equal to $3x + 14$?  
- Show that $x = 2$ and $x = -3$ are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning. |
|       | A.REI.2 – Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  
**Examples:**  
- $\sqrt{x + 2} = 5$  
- $\frac{7}{8} \sqrt{2x - 5} = 21$  
- $\frac{x+2}{x+3} = 2$  
- $\sqrt{3x - 7} = -4$ |
Solve equations and inequalities in one variable

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10 | A.REI.3 – Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Examples:**  
  7  
  \[ \frac{7}{3} y - 8 = 111 \]  
  3x > 9  
  \[ ax + 7 = 12 \]  
  \[ \frac{3 + x}{7} = \frac{x - 9}{4} \]  
  Solve for x:  \[ \frac{2}{3} x + 9 < 18 \] |
|        | A.REI.4 – Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form \( (x - p)^2 = q \) that has the same solutions. Derive the quadratic formula from this form. **Explanation:** Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to \( ax^2 + bx + c = 0 \) to the behavior of the graph of \( y = ax^2 + bx + c \).  

<table>
<thead>
<tr>
<th>Value of Discriminant</th>
<th>Nature of Roots</th>
<th>Nature of Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b^2 - 4ac = 0 )</td>
<td>1 real root</td>
<td>intersects x-axis once</td>
</tr>
<tr>
<td>( b^2 - 4ac &gt; 0 )</td>
<td>2 real roots</td>
<td>intersects x-axis twice</td>
</tr>
<tr>
<td>( b^2 - 4ac &lt; 0 )</td>
<td>2 complex roots</td>
<td>does not intersect x-axis</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| **b.** Solve quadratic equations by inspection (e.g., for \( x^2 = 49 \)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \( a \pm bi \) for real numbers \( a \) and \( b \). **Examples:**  
  • Are the roots of \( 2x^2 + 5 = 2x \) real or complex? How many roots does it have? Find all solutions of the equation.  
  • What is the nature of the roots of \( x^2 + 6x + 10 = 0 \)? Solve the equation using the quadratic formula and completing the square. How are the two methods related? |

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## Solve systems of equations

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10  | A.REI.5 – Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  
   **Example:**  
   - Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that they are two numbers, \( x \) and \( y \), satisfy the equations \( x + y = 10 \) and \( x - y = 4 \). |
| 9–10  | A.REI.6 – Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  
   **Explanation:** The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.  
   **Examples:**  
   - José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.  
     **Before:**  
     - José  
     - Phillipe  
     **After:**  
     - José  
     - Phillipe  
     - Solve the system of equations: \( x + y = 11 \) and \( 3x - y = 5 \). Use a second method to check your answer.  
     - Solve the system of equations: \( x - 2y + 3z = 5 \), \( x + 3z = 11 \), \( 5y - 6z = 9 \)  
     - The opera theater contains 1,200 seats, with three different prices. The seats cost $45 dollars per seat, $50 per seat, and $60 per seat. The opera needs to gross $63,750 on seat sales. There are twice as many $60 seats as $45 seats. How many seats in each level need to be sold? |
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<th>Standard</th>
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</table>
| ✷ A.REI.7 – Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  
For example, find the points of intersection between the line \( y = -3x \) and the circle \( x^2 + y^2 = 3 \).  
Example:  
- Two friends are driving to the Grand Canyon in separate cars. Suzette has been there before and knows the way, but Andrea does not. During the trip Andrea gets ahead of Suzette and pulls over to wait for her. Suzette is traveling at a constant rate of 65 miles per hour. Andrea sees Suzette drive past. To catch up, Andrea accelerates at a constant rate. The distance in miles (\( d \)) that her car travels as a function of time in hours (\( t \)) since Suzette’s car passed is given by \( d = 3500t^2 \).  
  Write and solve a system of equations to determine how long it takes for Andrea to catch up with Suzette. |
| + A.REI.8 – Represent a system of linear equations as a single matrix equation in a vector variable.  
Example:  
- Write the system \[
\begin{align*}
-a + 2c &= 4 \\
2a + b - c &= 0 \\
2a + 3c &= 11
\end{align*}
\]
  Identify the coefficient matrix, the variable matrix, and the constant matrix. |
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</table>
| +     | A.REI.9 – Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).  
**Explanation:** Students will perform multiplication, addition, subtraction, and scalar multiplication of matrices. They will use the inverse of a matrix to solve a matrix equation. Students may use graphing calculators, programs, or applets to model and find solutions for systems of equations.  
**Example:**  
- Solve the system of equations by converting to a matrix equation and using the inverse of the coefficient matrix.  
\[
\begin{align*}
5x + 2y &= 4 \\
3x + 2y &= 0
\end{align*}
\]  
**Solution:**  
\[
\begin{align*}
\text{Matrix } A &= \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix} \\
\text{Matrix } X &= \begin{bmatrix} x \\ y \end{bmatrix} \\
\text{Matrix } B &= \begin{bmatrix} 4 \\ 0 \end{bmatrix}
\end{align*}
\]
\[
\text{Matrix } A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}
\]
\[
X = A^{-1}B
\]
\[
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}
\]
| 9–10 | A.REI.10 – Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  
**Example:**  
- Which of the following points is on the circle with equation: \((x - 1)^2 + (y + 2)^2 = 5\)?  
  a. (1, -2) b. (2, 2) c. (3, -1) d. (3, 4) |
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<th>Standard</th>
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</table>
| ✷     | A.REI.11 – Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  

**Explanation:** Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.  

**Example:**
- Given the following equations determine the $x$-value that results in an equal output for both functions.
  
  $f(x) = 3x - 2$
  
  $g(x) = (x + 3)^2 - 1$ |
<table>
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<th>Label</th>
<th>Standard</th>
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</table>
| 9–10  | A.REI.12 – Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  
*Explanation:* Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities.  
*Examples:*  
- Graph the solution:  
  \[ y \leq 2x + 3. \]
- A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines, but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set.  
- Graph the system of linear inequalities below and determine if \((3, 2)\) is a solution to the system.  
  \[
  \begin{align*}
  x - 3y &> 0 \\
  x + y &\leq 2 \\
  x + 3y &> -3
  \end{align*}
  \]
*Solution:*  
\[
\begin{array}{c}
\text{(3, 2) is not an element of the solution set (graphically or by substitution).}
\end{array}
\]
High School – Functions Overview

**Interpreting Functions (F-IF)**
- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

**Building Functions (F-BF)**
- Build a function that models a relationship between two quantities
- Build new functions from existing functions

**Linear, Quadratic, and Exponential Models (F-LE)**
- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

**Trigonometric Functions (F-TF)**
- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

**Mathematical Practices (MP)**
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
**Functions: Interpreting Functions (F-IF)**

**Understand the concept of a function and use function notation**

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| 9–10  | F.IF.1 – Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.  
**Explanation:** The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain.  
**Example:** For the functions in a. through f. below:  
- List the algebraic operations in order of evaluation. What restrictions does each operation place on the domain of the function?  
- Give the function's domain.  
  a. $y = \frac{2}{x-3}$  
  b. $y = \sqrt{x - 5} + 1$  
  c. $y = 4 - (x - 3)^2$  
  d. $y = \frac{7}{4-(x-3)^2}$  
  e. $y = 4 - (x - 3)^{\frac{1}{2}}$  
  f. $y = \frac{7}{4 - (x-3)^2}$ |
| 9–10  | F.IF.2 – Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.  
**Examples:**  
- If $f(x) = x^2 + 4x - 12$, find $f(2)$.  
- Let $f(x) = 2(x + 3)^2$. Find $f(3), f\left(-\frac{1}{2}\right), f(a)$, and $f(a - h)$.  
- If $P(t)$ is the population of Tucson $t$ years after 2000, interpret the statements $P(0) = 487,000$ and $P(10) - P(9) = 5900$. |
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| ★     | F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include:* intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.  
*Resources:* Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.  
*Examples:*  
- A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where $t$ is measured in seconds and $h$ is height above the ground measured in feet.  
  - What is a reasonable domain restriction for $t$ in this context?  
  - Determine the height of the rocket two seconds after it was launched.  
  - Determine the maximum height obtained by the rocket.  
  - Determine the time when the rocket is 100 feet above the ground.  
  - Determine the time at which the rocket hits the ground.  
  - How would you refine your answer to the first question based on your response to the second and fifth questions?  
- Compare the graphs of $y = 3x^2$ and $y = 3x^3$.  
- Let $(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$.  
- Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.  
- It started raining lightly at 5 a.m., and then the rainfall became heavier at 7 a.m. By 10 a.m., the storm was over, with a total rainfall of 3 inches. It didn’t rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday. |
| 9–10 | F.IF.5 – Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.  
*Explanation:* Students may explain orally, or in written format, the existing relationships.  
*Example:*  
- Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders’ organization brings in as revenue is a function of the number of people, $n$, in attendance. If each ticket costs $30, find the domain and range of this function. |
F.IF.6 – Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Explanation: The average rate of change of a function $y = f(x)$ over an interval $[a, b]$ is: $\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (e.g., falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.

Examples:
- Use the following table to find the average rate of change of $g$ over the intervals $[-2, -1]$ and $[0, 2]$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-10</td>
</tr>
</tbody>
</table>
- The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40 and 50 meter mark on a test track.
  - For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1.
  - How does the velocity of car 1 compare to that of car 2?

<table>
<thead>
<tr>
<th></th>
<th>Car 1</th>
<th>Car 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>$t$</td>
<td>$t$</td>
</tr>
<tr>
<td>10</td>
<td>4.472</td>
<td>1.742</td>
</tr>
<tr>
<td>20</td>
<td>6.325</td>
<td>2.899</td>
</tr>
<tr>
<td>30</td>
<td>7.746</td>
<td>3.831</td>
</tr>
<tr>
<td>40</td>
<td>8.944</td>
<td>4.633</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>5.348</td>
</tr>
</tbody>
</table>

F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
### Label: Explanation for F.IF.7.a through d

**Key characteristics** include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.

**Examples:**

- Describe key characteristics of the graph of \( f(x) = |x - 3| + 5 \).
- Sketch the graph and identify the key characteristics of the function described below.

\[
F(x) = \begin{cases} 
  x + 2 & \text{for } x \geq 0 \\
  -x^2 & \text{for } x < -1 
\end{cases}
\]

- Graph the function \( f(x) = 2x \) by creating a table of values. Identify the key characteristics of the graph.
- Graph \( f(x) = 2 \tan x - 1 \). Describe its domain, range, intercepts, and asymptotes.
- Draw the graph of \( f(x) = \sin x \) and \( f(x) = \cos x \). What are the similarities and differences between the two graphs?
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

*Example:*

**Part 1:**
- Have students graph $y = 2^x$ by making a table of ordered pairs. Ask students how they would graph $y = \log_2 x$. What is the relationship between the two graphs? Hopefully students will remember they are inverses. How can we use the ordered pairs of $y = 2^x$ to graph $y = \log_2 x$?
- Have students graph $y = \log_2 x$.
- Have a discussion with the students about the properties of logarithmic graphs. Are there any asymptotes? Is the function increasing or decreasing? Is there an $x$-intercept? What is the end behavior?
- Have students repeat for $y = \log_{\frac{1}{2}} x$.
- You can repeat the earlier discussion for this function.
- Have students compare the shapes of the graphs where the base is between 0 and 1 and when the base is greater than 1.
- Have students practice graphing logarithmic functions with tables and using the previously used transformations.

**Part 2:**
- $y = -4\sin\frac{\pi}{4}(x + 3) - 2$

*Diagram:*

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<tr>
<td>$\blacklozenge$, $\star$</td>
<td>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</td>
</tr>
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*Example:*

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<tbody>
<tr>
<td>❖</td>
<td>F.IF.8 – Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</td>
</tr>
</tbody>
</table>
| ❖ | a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  
*Example:*  
- Which of the following equations could describe the function whose graph is shown below? Explain.  

\[ f_1(x) = (x + 12)^2 + 4 \quad f_2(x) = -4(x + 2)(x + 3) \]  
\[ f_3(x) = -(x - 2)^2 - 1 \quad f_4(x) = (x + 4)(x - 6) \]  
\[ f_5(x) = (x + 18)^2 - 40 \quad f_6(x) = (x - 12)(-x + 18) \]  
\[ f_7(x) = (x - 10)^2 - 15 \quad f_8(x) = (20 - x)(30 - x) \]

| ❖ | b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \( y = (1.02)^t \), \( y = (0.97)^t \), \( y = (1.01)^{12t} \), \( y = (1.2)^{t/10} \), and classify them as representing exponential growth or decay.  
*Examples:*  
- The projected population of Delroysville is given by the formula \( p(t) = 1500(1.08)^t \). You have been selected by the city council to help them plan for future growth. Explain what the formula \( p(t) = 1500(1.08)^t \) means to the city council members.  
- Which of the following functions will represent $500 placed into a mutual fund yielding 10% per year for 4 years.  
  a. \( A = 500(.10)^4 \)  
  b. \( A = 500(1.1)^4 \)  
  c. \( A = 500(4)(.10) \)  
  d. \( A = 500(1.04)^{10} \) |
<table>
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</table>
| 9–10  | F.IF.9 – Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Example:  
- Examine the functions below. Which function has the larger maximum? How do you know?  

\[ f(x) = -2x^2 - 8x + 20 \] |

![Graph of a quadratic function](image_url)
**Functions: Building Functions (F-BF)**

**Build a function that models a relationship between two quantities**

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>9–10</td>
<td>F.BF.1 – Write a function that describes a relationship between two quantities.</td>
</tr>
<tr>
<td>✷</td>
<td><em>Explanation:</em> Students will analyze a given problem to determine the function expressed by identifying patterns in the function’s rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function’s description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.</td>
</tr>
<tr>
<td>✷</td>
<td>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</td>
</tr>
<tr>
<td>✷</td>
<td>b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</td>
</tr>
<tr>
<td>✷</td>
<td>Examples for F.BF.1, F.BF.1.a, and F.BF.1.b:</td>
</tr>
<tr>
<td>✷</td>
<td>• You buy a $10,000 car with an annual interest rate of 6% compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation.</td>
</tr>
<tr>
<td>✷</td>
<td>• A cup of coffee is initially at a temperature of 93° F. The difference between its temperature and the room temperature of 68° F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.</td>
</tr>
<tr>
<td>✷</td>
<td>• The radius of a circular oil slick after $t$ hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \leq t \leq 10$. Find the area of the oil slick as a function of time.</td>
</tr>
<tr>
<td>✷</td>
<td>c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</td>
</tr>
<tr>
<td>✷</td>
<td>Example:</td>
</tr>
<tr>
<td>✷</td>
<td>• According to the U.S. Energy Information Administration, a barrel of crude oil produces approximately 20 gallons of gasoline. EPA mileage estimates indicate a 2011 Ford Focus averages 28 miles per gallon of gasoline.</td>
</tr>
<tr>
<td>✷</td>
<td>• Write an expression for $g(x)$, the number of gallons of gasoline produced by $x$ barrels of crude oil.</td>
</tr>
<tr>
<td>✷</td>
<td>• Write an expression for $M(x)$, the number of miles on average that a 2011 Ford Focus can drive on $x$ gallons of gasoline.</td>
</tr>
<tr>
<td>✷</td>
<td>• Write an expression for $M(g(x))$. What does $M(g(x))$ represent in terms of the context?</td>
</tr>
<tr>
<td>✷</td>
<td>• One estimate (from <a href="http://www.oilvoice.com">www.oilvoice.com</a>) claimed that the 2010 Deepwater Horizon disaster in the Gulf of Mexico spilled 4.9 million barrels of crude oil. How many miles of Ford Focus driving would this spilled oil fueled?</td>
</tr>
</tbody>
</table>
F.BF.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Explanation: An explicit rule for the \(n\)th term of a sequence gives \(a_n\) as an expression in the term’s position \(n\); a recursive rule gives the first term of a sequence, and a recursive equation relates \(a_n\) to the preceding term(s). Both methods of presenting a sequence describe \(a_n\) as a function of \(n\).

Examples:
- Generate the 5th–11th terms of a sequence if \(A_1 = 2\) and \(A_{(n+1)} = (A_n)^2 - 1\).
- Use the formula: \(A_n = A_1 + (d(n - 1))\) where \(d\) is the common difference to generate a sequence whose first three terms are: -7, -4 and -1.
- There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of the year. Find the population in five years. Also, find the long-term population.
- Given the formula \(A_n = 2n - 1\), find the 17th term of the sequence. What is the 9th term in the sequence 3, 5, 7, 9, …?
- Given \(a_1 = 4\) and \(a_n = a_{n-1} + 3\), write the explicit formula.
Build new functions from existing functions

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| ✷     | F.BF.3 – Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.  

Explanation: Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.  

Examples:  
- Is \( f(x) = x^3 - 3x^2 + 2x + 1 \) even, odd, or neither? Explain your answer orally or in written format.  
- Compare the shape and position of the graphs of \( f(x) = x^2 \) and \( g(x) = 2x^2 \), and explain the differences in terms of the algebraic expressions for the functions.  
- Describe the effect of varying the parameters \( a \), \( h \), and \( k \) on the shape and position of the graph \( f(x) = a(x - h)^2 + k \).
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</table>

**Examples for F.BF.3 continued:**

- Compare the shape and position of the graphs of \( f(x) = e^x \) to \( g(x) = e^{x-6} + 5 \), and explain the differences, orally or in written format in terms of the algebraic expressions for the functions.

![Graph of \( f(x) = e^x \) and \( g(x) = e^{x-6} + 5 \)](image)

- Describe the effect of varying the parameters \( a, h, \) and \( k \) on the shape and position of the graph \( f(x) = ab^{(x+h)} + k \) orally or in written format. How do the values between 0 and 1 effect the graph? What is the effect of negative values on the graph?

- Compare the shape and position of the graphs of \( y = \sin x \) to \( y = 2 \sin x \).

![Graph of \( y = \sin x \) and \( y = 2 \sin x \)](image)

---

**Label Legend:** 9-10 = Standards for Grades 9 and 10; ⚫ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>F.BF.4 – Find inverse functions.</td>
</tr>
<tr>
<td>❖</td>
<td>a. Solve an equation of the form ( f(x) = c ) for a simple function ( f ) that has an inverse and write an expression for the inverse. For example, ( f(x) = 2x^3 ) or ( f(x) = \frac{x+1}{(x-1)} ) for ( x \neq 1 ).</td>
</tr>
<tr>
<td>+</td>
<td>b. Verify by composition that one function is the inverse of another.</td>
</tr>
<tr>
<td>+</td>
<td>c. Read values of an inverse function from a graph or a table, given that the function has an inverse.</td>
</tr>
<tr>
<td>+</td>
<td>d. Produce an invertible function from a non-invertible function by restricting the domain.</td>
</tr>
</tbody>
</table>

**Resources and Examples for F.BF.4 b, c, and d:**

**Resources:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.

**Examples:**
- For the function \( h(x) = (x - 2)^3 \), defined on the domain of all real numbers, find the inverse function if it exists or explain why it does not exist.
- Graph \( h(x) \) and \( h^{-1}(x) \) and explain how they relate to each other graphically and algebraically.
- Find a domain for \( f(x) = 3x^2 + 12x - 8 \) on which it has an inverse. Explain why it is necessary to restrict the domain of the function.
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | F.BF.5 – Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.  
*Resources:* Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to solve problems involving logarithms and exponents.  
*Examples:*  
- Find the inverse of $f(x) = 3(10)^{2x}$.  
- Recall that $\log_b(x)$ is by definition the exponent which $b$ must be raised to in order to yield $x$ ($b > 0$).  
  - Part I  
    - Use this definition to compute $2^5$.  
    - Use this definition to compute $\log_{10}(0.001)$.  
    - Use this definition to compute $\ln(e^3)$.  
    - Explain why $\log_b(b^3) = y$ where $b > 0$.  
  
The above technique can be used to raise numbers to logarithmic powers by first simplifying the exponent.  
  - Part II  
    - Evaluate $10^{\log_{10}(100)}$.  
    - Evaluate $2^{\log_2(\sqrt{2})}$.  
    - Evaluate $e^{\ln(89)}$.  
    - Explain why $b^{\log_b(x)} = x$ where $b > 0$. |
Functions: Linear, Quadratic, and Exponential Models ★ (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✧, +, ✧ | F.LE.1 – Distinguish between situations that can be modeled with linear functions and with exponential functions.  
**Explanations:** Students can investigate functions and graphs modeling different situations involving simple and compound interest. Students can compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate. Spreadsheets and applets can be used to explore and model different interest rates and loan terms.  
**Resources:** Students can use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions. |
| ✧, ✧ | a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.  
**Examples:**  
- A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in Plan 1? Plan 2? Plan 3?  
  ✧ Plan 1: $59.95 per month for 700 minutes and $0.25 for each additional minute;  
  ✧ Plan 2: $39.95 per month for 400 minutes and $0.15 for each additional minute; and  
  ✧ Plan 3: $89.95 per month for 1,400 minutes and $0.05 for each additional minute.  
- A computer store sells about 200 computers at the price of $1,000 per computer. For each $50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit? |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷, ★ | b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.  
   *Example:*  
   - The data in the following table was taken from Wikipedia. |

<table>
<thead>
<tr>
<th>Year</th>
<th>World Population (Estimate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1804</td>
<td>1,000,000,000</td>
</tr>
<tr>
<td>1927</td>
<td>2,000,000,000</td>
</tr>
<tr>
<td>1960</td>
<td>3,000,000,000</td>
</tr>
<tr>
<td>1974</td>
<td>4,000,000,000</td>
</tr>
<tr>
<td>1987</td>
<td>5,000,000,000</td>
</tr>
<tr>
<td>1999</td>
<td>6,000,000,000</td>
</tr>
<tr>
<td>2012</td>
<td>7,000,000,000</td>
</tr>
</tbody>
</table>

a. Based on the data in the above table, would a linear function be appropriate to model the relationship between the world population and the year? Explain how you know.
b. Using only the data from 1960 onward in the above table, would a linear function be appropriate to approximate the relationship between the world population and the year? Explain how you know.
c. Based on your work in parts a. and b., would a linear function be appropriate to predict the world population in 2200? Explain.

| ✷, ★ | c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.  
   *Example:*  
   - Carbon 14 is a common form of carbon which decays exponentially over time. The half-life of Carbon 14, that is the amount of time it takes for half of any amount of Carbon 14 to decay, is approximately 5730 years.  
   Suppose we have a plant fossil and that the plat, at the time it died, contained 10 micrograms of Carbon 14 (one microgram is equal to one millionth of a gram). |

a. Using this information, make a table to calculate how much Carbon 14 remains in the fossilized plant after $5730 \times n$ years for $n = 0, 1, 2, 3, 4$.  
b. What can you conclude from part a. about when there is one microgram of Carbon 14 remaining in the fossil?  
c. How much carbon remains in the fossilized plant after $2865 = \frac{5730}{2}$ years? Explain how you know.  
d. Using the information from part c., can you give a more precise response to when there is one microgram of Carbon 14 remaining in the fossilized plant?
### Additional Examples for F.LE.1:
- A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How much will they need to save each month in order to meet their goal?
- Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has?
  - Lee borrows $9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest.
  - Lee borrows $9,000 from a bank to buy a car. The bank charges 5% interest compounded annually.
- Calculate the future value of a given amount of money, with and without technology.
- Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology.

### F.LE.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

**Resources:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

**Examples:**
- Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the key characteristics of the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

- Sara’s starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation.
F.LE.3 – Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Examples:
- Contrast the growth of the $f(x) = x^3$ and $f(x) = 3^x$.
- Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves:
  1st Choice: Two dollars for each bag of leaves.
  2nd Choice: She will be paid for the number of bags of leaves she rakes as follows: two cents for one bag, four cents for two bags, eight cents for three bags, and so on with the amount doubling for each additional bag.
- If Celia rakes five bags of leaves, should she opt for payment method 1 or 2? What if she rakes ten bags of leaves?
- How many bags of leaves does Celia have to rake before method 2 pays more than method 1?

F.LE.4 – For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

Resources: Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms.

Example:
- Solve $200e^{0.04t} = 450$ for $t$.

Solution:
  First, isolate the exponential part by dividing both sides of the equation by 200.
  $e^{0.04t} = 2.25$
  Next, take the natural logarithm of both sides.
  $\ln e^{0.04t} = \ln 2.25$
  The left-hand side simplifies to $0.04t$ by logarithmic identity 1.
  $0.04t = \ln 2.25$
  Lastly, divide both sides by 0.04
  $t = \frac{2.25}{0.04}$
  $t \approx 20.3$
Interpret expressions for functions in terms of the situation they model

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>❖ ★</td>
<td>F.LE.5 – Interpret the parameters in a linear or exponential function in terms of a context.</td>
</tr>
<tr>
<td>Resources: Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic, or exponential functions.</td>
<td></td>
</tr>
<tr>
<td>Examples:</td>
<td></td>
</tr>
<tr>
<td>• A function of the form ( f(n) = P(1 + r)^n ) is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where ( n ) is the number of years since the initial deposit. What is the value of ( r )? What is the meaning of the constant ( P ) in terms of the savings account? Explain either orally or in written format.</td>
<td></td>
</tr>
<tr>
<td>• Lauren keeps records of the distances she travels in a taxi and what she pays:</td>
<td></td>
</tr>
<tr>
<td>![Distance in miles</td>
<td>Fare in dollars](#)</td>
</tr>
<tr>
<td><img src="#" alt="Distance in miles" /></td>
<td><img src="#" alt="Fare in dollars" /></td>
</tr>
<tr>
<td>3</td>
<td>8.25</td>
</tr>
<tr>
<td>5</td>
<td>12.75</td>
</tr>
<tr>
<td>11</td>
<td>26.25</td>
</tr>
<tr>
<td>a. If you graph the ordered pairs ((d, F)) from the table, they lie on a line. How can you tell this without graphing them?</td>
<td></td>
</tr>
<tr>
<td>b. Show that the linear function in part a. has equation ( F = 2.25d + 1.5 ).</td>
<td></td>
</tr>
<tr>
<td>c. What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides?</td>
<td></td>
</tr>
</tbody>
</table>
## Functions: Trigonometric Functions (F-TF)

**Extend the domain of trigonometric functions using the unit circle**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✤ F.TF.1 – Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.  
  \[
  \text{Example:} \\
  \begin{align*}
  &\text{The minute hand on the clock at the City Hall clock in Stratford measures 2.2 meters from the tip to the axle.}\\
  &\text{a. Through what angle does the minute hand pass between 7:07 a.m. and 7:43 a.m.?}\\
  &\text{b. What distance does the tip of the minute hand travel during this period?}
  \end{align*}
  | ✤ F.TF.2 – Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.  
  \[
  \text{Resources: Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or in written format) their understanding.} \\
  \begin{align*}
  &\text{Example:}\\
  &\text{Using the Unit Circle, find the following:}\\
  &\sin (-30^\circ) = \\
  &\tan 420^\circ = \\
  &\cos (-90^\circ) = \\
  &\csc \left(\frac{\pi}{3}\right) = \\
  &\cot (-45^\circ) = \\
  &\sin 330^\circ = \\
  &\tan 60^\circ = \\
  &\cos 270^\circ = \\
  &\csc \left(\frac{13\pi}{3}\right) = \\
  &\cot (-765^\circ) = \\
  &\sec \left(\frac{16\pi}{3}\right) = \\
  &\csc \left(\frac{9\pi}{4}\right) = \\
  &\tan (4\pi) = \\
  &\sin \left(\frac{3\pi}{2}\right) = \\
  &\cos \left(-\frac{\pi}{6}\right) = \\
  &\sec \left(\frac{4\pi}{3}\right) = \\
  &\csc \left(\frac{\pi}{4}\right) = \\
  &\tan (10\pi) = \\
  &\sin \left(\frac{11\pi}{2}\right) = \\
  &\cos \left(\frac{11\pi}{6}\right) = \\
  &\text{Look at each pair of angles above. What do you notice about those pairs?} \\
  &\text{What conclusions can you draw about the trigonometric functions and how they work about the circle?}
  \end{align*}
<p>|</p>
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | F.TF.3 – Use special triangles to determine geometrically the values of sine, cosine, tangent for \(\frac{\pi}{3}\), \(\frac{\pi}{4}\), and \(\frac{\pi}{6}\), and use the unit circle to express the values of sine, cosine, and tangent for \(\pi-x\), \(\pi+x\), and \(2\pi-x\) in terms of their values for \(x\), where \(x\) is any real number.  
   * Examples:  
     - Evaluate all six trigonometric functions of \(\frac{\pi}{3}\).  
     - Evaluate all six trigonometric functions of \(\theta = 225^\circ\).  
     - Find the value of \(x\) in the given triangle where \(\overline{AD} \perp \overline{DC}\) and \(\overline{AC} \perp \overline{DB}\), \(m\angle A = 60^\circ\), \(m\angle C = 30^\circ\). Explain your process for solving the problem including the use of trigonometric ratios as appropriate. 
     - Find the measure of the missing segment in the given triangle where \(\overline{AD} \perp \overline{DC}\), \(\overline{AC} \perp \overline{DB}\), \(m\angle A = 60^\circ\), \(m\angle C = 30^\circ\), \(\overline{AC} = 12\), \(\overline{AB} = 3\). Explain, orally or in written format, your process for solving the problem, including use of trigonometric ratios as appropriate.  

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**Label Legend:**  9-10 = Standards for Grades 9 and 10; ♦ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
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<th>Label</th>
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</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>F.TF.4 – Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</td>
</tr>
</tbody>
</table>

**Resources:** Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or written format) their understanding of symmetry and periodicity of trigonometric functions.

**Example:**
- Using the Unit Circle, find the following:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin (-30^\circ)$</td>
<td>$\tan 60^\circ$</td>
<td>$\cos (-90^\circ)$</td>
<td>$\csc \left(\frac{\pi}{3}\right)$</td>
<td>$\cot (-45^\circ)$</td>
</tr>
<tr>
<td>$\sin 30^\circ$</td>
<td>$\tan (-60^\circ)$</td>
<td>$\cos 90^\circ$</td>
<td>$\csc \left(-\frac{\pi}{3}\right)$</td>
<td>$\cot 45^\circ$</td>
</tr>
<tr>
<td>$\sec \left(\frac{\pi}{3}\right)$</td>
<td>$\csc \left(-\frac{\pi}{4}\right)$</td>
<td>$\sin \left(\frac{\pi}{2}\right)$</td>
<td>$\tan \left(-\frac{3\pi}{2}\right)$</td>
<td>$\cos \left(-\frac{\pi}{6}\right)$</td>
</tr>
<tr>
<td>$\sec \left(-\frac{\pi}{3}\right)$</td>
<td>$\csc \left(\frac{\pi}{4}\right)$</td>
<td>$\sin \left(-\frac{\pi}{2}\right)$</td>
<td>$\tan \left(\frac{3\pi}{2}\right)$</td>
<td>$\cos \left(\frac{\pi}{6}\right)$</td>
</tr>
<tr>
<td>$\cos \pi$</td>
<td>$\cot \left(-\frac{\pi}{4}\right)$</td>
<td>$\tan \pi$</td>
<td>$\sin \left(-\frac{5\pi}{6}\right)$</td>
<td>$\sec \left(-\frac{4\pi}{3}\right)$</td>
</tr>
<tr>
<td>$\cos (-\pi)$</td>
<td>$\cot \left(\frac{\pi}{6}\right)$</td>
<td>$\tan (-\pi)$</td>
<td>$\sin \left(\frac{5\pi}{6}\right)$</td>
<td>$\sec \left(\frac{4\pi}{3}\right)$</td>
</tr>
</tbody>
</table>

- **Recall:** **Even Functions** are such that $f(-x) = f(x)$. **Odd Functions** are such that $f(-x) = -f(x)$. Would you consider sine an odd or even function or neither? Why or why not? Would you consider cosine an odd or even function or neither? Why or why not? Would you consider tangent an odd or even function or neither? Why or why not?
- What does this tell you about the graphs of sine, cosine, and tangent?
- What kind of functions do you think cosecant, secant, and cotangent are with respect to being odd, even, or neither? Explain.
# Model periodic phenomena with trigonometric functions

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷ | F.TF.5 – Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.  
   *Example:*  
   - The temperature of a chemical reaction oscillates between a low of 20°C and a high of 120°C. The temperature is at its lowest point when \( t = 0 \) and completes one cycle over a 6-hour period.  
     a. Sketch the temperature, \( T \), against the elapsed time, \( t \), over a 12-hour period.  
     b. Find the period, amplitude, and the midline of the graph you drew in part a.  
     c. Write a function to represent the relationship between time and temperature.  
     d. What will the temperature of the reaction be 14 hours after it began?  
     e. At what point during a 24-hour day will the reaction have a temperature of 60°C? |
| ✷ | F.TF.6 – Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.  
   *Resources:* Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions.  
   *Examples:*  
   - Identify a domain for the sine function that would permit an inverse function to be constructed.  
   - Describe the behavior of the graph of the sine function over this interval.  
   - Explain (orally or in written format) why the domain cannot be expanded any further. |
| ✷ | F.TF.7 – Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.  
   *Resources:* Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and solve trigonometric equations.  
   *Example:*  
   - Two physics students set up an experiment with a spring. In their experiment, a weighted ball attached to the bottom of the spring was pulled downward 6 inches from the rest position. It rose to 6 inches above the rest position and returned to 6 inches below the rest position once every 6 seconds. The equation \( h = -6 \cos \left( \frac{\pi}{2} t \right) \) accurately models the height above and below the rest position every 6 seconds. Students may explain, orally or in written format, when the weighted ball first will be at a height of 3 inches, 4 inches, and 5 inches above rest position. |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>☀</td>
<td>F.TF.8 – Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.</td>
</tr>
</tbody>
</table>

**Example:** Proving $\sin^2\theta + \cos^2\theta = 1$

- Given the Unit Circle below, complete the following steps:
  - Identify the center of the circle (the origin) as $O$
  - Label point $P$ on the unit circle in Quadrant I
  - Draw ray $\overrightarrow{OP}$ as the terminal ray of angle $\theta$
  - Draw $\overrightarrow{PR}$, which is a line segment perpendicular to the $x$-axis

- What is the length of $\overrightarrow{OP}$? Justify your answer
- What is the length of $\overrightarrow{PR}$? Justify your answer
- What is the length of $\overrightarrow{OR}$? Justify your answer
- Using the Pythagorean Theorem, justify the Identity $\sin^2\theta + \cos^2\theta = 1$. |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | F.TF.9 – Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.  
*Example:*  
- Using ΔABD, determine trig ratios. |

![Diagram of triangle ABD with angles and sides labeled.](image-url)  
- Using ΔEFG, determine trig ratios. |

![Diagram of triangle EFG with angles and sides labeled.](image-url)  
- Simplify without a calculator:  
  a. $\cos 30^\circ$  
  b. $\sin 30^\circ$  
  c. $\sin^2 60^\circ + \cos^2 60^\circ$  
  d. Is $\cos^2 60^\circ + \sin^2 60^\circ = \cos^2 45^\circ + \sin^2 45^\circ$?  
  e. Is $(\cos 60^\circ + \sin 60^\circ)^2 = (\cos 45^\circ + \sin 45^\circ)^2$?  
  f. Comment on what you noticed in items d. and e. |
High School – Geometry Overview

**Congruence (G-CO)**
- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

**Similarity, Right Triangles, and Trigonometry (G-SRT)**
- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

**Circles (G-C)**
- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

**Expressing Geometric Properties with Equations (G-GPE)**
- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

**Geometric Measurement and Dimension (G-GMD)**
- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

**Modeling with Geometry (G-MG)**
- Apply geometric concepts in modeling situations

**Mathematical Practices (MP)**
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
**Geometry: Congruence (G-CO)**

**Experiment with transformations in the plane**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10  | G.CO.1 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.  
**Examples:**  
- Have students write their own understanding of a given term.  
- Give students formal and informal definitions of each term and compare them.  
- Develop precise definitions through use of examples and non-examples.  
- Discuss the importance of having precise definitions. |
| 9–10  | G.CO.2 – Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).  
**Resources:** Students may use geometry software and/or manipulatives to model and compare transformations. |
| 9–10  | G.CO.3 – Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.  
**Resources:** Students may use geometry software and/or manipulatives to model transformations. |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example for G.CO.2 and G.CO.3:</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>1. Draw the shaded triangle after:</td>
<td></td>
</tr>
<tr>
<td>a. It has been translated $-7$ horizontally and $+1$ vertically. Label your answer $A$.</td>
<td></td>
</tr>
<tr>
<td>b. It has been reflected over the $x$-axis. Label your answer $B$.</td>
<td></td>
</tr>
<tr>
<td>c. It has been rotated $90^\circ$ clockwise around the origin. Label your answer $C$.</td>
<td></td>
</tr>
<tr>
<td>d. It has been reflected over the line $y = x$. Label your answer $D$.</td>
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</tr>
<tr>
<td>2. Describe fully the single transformation that:</td>
<td></td>
</tr>
<tr>
<td>a. Takes the shaded triangle onto the triangle labeled $E$.</td>
<td></td>
</tr>
<tr>
<td>b. Takes the shaded triangle onto the triangle labeled $F$.</td>
<td></td>
</tr>
<tr>
<td><strong>9–10</strong></td>
<td>G.CO.4 – Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.</td>
</tr>
<tr>
<td><strong>Resources:</strong> Students may use geometry software and/or manipulatives to model transformations. Students may observe patterns and develop definitions of rotations, reflections, and translations.</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong></td>
<td></td>
</tr>
<tr>
<td>- Perform a rotation, reflection, and translation with a given polygon and give a written explanation of how each step meets the definitions of each transformation using correct mathematical terms.</td>
<td></td>
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<tr>
<td>Label</td>
<td>Standard</td>
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</tbody>
</table>
| 9–10  | G.CO.5 – Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.  
**Resources:** Students may use geometry software and/or manipulatives to model transformations and demonstrate a sequence of transformations that will carry a given figure onto another.  
**Example:**  
- The triangle in the upper left of the figure below has been reflected across a line into the triangle in the lower right of the figure. Use a straightedge and compass to construct the line across which the triangle was reflected. |

![Diagram of geometric figure and transformation](image)
Understand congruence in terms of rigid motions

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10</td>
<td>G.CO.6 – Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</td>
</tr>
</tbody>
</table>

**Explanations:** A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.

**Resources:** Students may use geometric software to explore the effects of rigid motion on a figure(s).

**Example:** Composition of a Translation and a Rotation

- $\triangle ABC$ has vertices $A(-1, 0), B(4, 0), C(2, 6)$
  - a. Draw $\triangle ABC$ on the coordinate grid provided.
  - b. Translate $\triangle ABC$ using the rule $(x, y) \rightarrow (x - 6, y - 5)$ to create $\triangle A'B'C'$. Record the new coordinate grid (using a different color if possible).

  $A'$  $B'$  $C'$

  - c. Rotate $\triangle A'B'C'$ 90°CCW using the rule $(x, y) \rightarrow$ to create $\triangle A''B''C''$. Record the new coordinates below and add the triangle to your coordinate grid (using a different color if possible).

  $A''$  $B''$  $C''$

  - d. Write ONE rule below that would change $\triangle ABC$ to $\triangle A''B''C''$ in one step.

  ______________________________________________________
<table>
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<th>Label</th>
<th>Standard</th>
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</table>
| 9–10  | G.CO.7 – Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.  
**Explanations:**  
- A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.  
- Congruence of triangles – Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.  
**Example:**  
- How many ways can you construct a triangle congruent to the given triangle inside the rectangle? Demonstrate each. |
| 9–10  | G.CO.8 – Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.  
**Example:**  
- Josh is told that two triangles \(ABC\) and \(DEF\) share two sets of congruent sides and one pair of congruent angles: \(AB\) is congruent to \(DE\), \(BC\) is congruent to \(EF\), and angle \(C\) is congruent to angle \(F\). He is asked if these two triangles must be congruent. Josh draws the two triangles below and says, “They are definitely congruent because they share all three side lengths!”  
  - Explain Josh’s reasoning using one of the triangle congruence criteria: ASA, SSS, SAS.  
  - Give an example of two triangles \(ABC\) and \(DEF\), fitting the criteria of this problem, which are not congruent. |
Prove geometric theorems

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
</table>
| 9–10  | G.CO.9 – Prove theorems about lines and angles. *Theorems include:* vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. *Resources:* Students may use geometric simulations (computer software or graphing calculator) to explore theorems about lines and angles. *Example:* View the diagram below depicts the construction of a parallel line, above the ruler. The steps in the construction result in a line through the given point that is parallel to the given line. Which statement below justifies why the constructed line is parallel to the given line?

a. When two lines are each perpendicular to a third line, the lines are parallel.
b. When two lines are each parallel to a third line, the lines are parallel.
c. When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel.
d. When two lines are intersected by a transversal and corresponding angles are congruent, the lines are parallel. (Answer is d.)
G.CO.10 – Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Resources: Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles.

Example:
- For items 1 and 2, what additional information is required in order to prove the two triangles are congruent using the provided justification?
- Use the set of choices in the box below. Select a side or angle and place it in the appropriate region. Only one side or angle can be placed in each region.

<table>
<thead>
<tr>
<th>Side</th>
<th>Angle</th>
<th>Side</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>AC</td>
<td>AD</td>
<td>BC</td>
</tr>
<tr>
<td>BD</td>
<td>CD</td>
<td>CE</td>
<td>DE</td>
</tr>
<tr>
<td>∠ABC</td>
<td>∠ABD</td>
<td>∠ACB</td>
<td>∠ADB</td>
</tr>
<tr>
<td>∠BAC</td>
<td>∠CDE</td>
<td>∠CED</td>
<td>∠DCE</td>
</tr>
</tbody>
</table>

Key: Item 1 – $\angle ABD \cong \angle ABC$
Item 2 – $AC \cong CE$
G.CO.11 – Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

**Resources:** Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms.

**Example:**
- Suppose that $ABCD$ is a parallelogram, and that $M$ and $N$ are the midpoints of $AB$ and $CD$, respectively. Prove that $MN = AD$, and that the line $MN$ is parallel to $AD$. 

![Diagram](image.png)
### Make geometric constructions

<table>
<thead>
<tr>
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<th>Standard</th>
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</thead>
<tbody>
<tr>
<td>9–10</td>
<td>G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). <em>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</em></td>
</tr>
</tbody>
</table>

*Resources:* Students may use geometric software to make geometric constructions.

*Examples:*
- Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.
- Construct the circumcenter of a given triangle.
- You have been asked to place a warehouse so that it is an equal distance from the three roads indicated on the following map. Find this location and show your work.

![Map Diagram]

- Show how to fold your paper to physically construct this point as an intersection of two creases.
- Explain why the above construction works and, in particular, why you only needed to make two creases.
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
</table>
| 9–10 | G.CO.13 – Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  
*Resources:* Students may use geometric software to make geometric constructions.  
*Example:*  
- Find two ways to construct a hexagon inscribed in a circle as shown. |
### Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)

**Understand similarity in terms of similarity transformations**

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>9–10</td>
<td>G.SRT.1 – Verify experimentally the properties of dilations given by a center and a scale factor:</td>
</tr>
<tr>
<td></td>
<td>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</td>
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<tr>
<td></td>
<td>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</td>
</tr>
</tbody>
</table>

**Explanation:** A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

**Resources:** Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations.

**Example:**
- Suppose we apply a dilation by a factor of 2, centered at the point $P$ to the figure below.

![Dilation Diagram](image)

- In the picture, locate the images $A'$, $B'$, and $C'$ of the points $A$, $B$, and $C$ under this dilation.
- Based on your picture in part a., what do you think happens to the line $l$ when we perform the dilation?
- Based on your picture n part a., what appears to be the relationship between the distance $A'B'$ and the distance $AB$? How about the distances $B'C'$ and $BC$?
- Can you prove your observations in part c.?
<table>
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<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
</table>
| 9–10  | G.SRT.2 – Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.  
Explaination: A similarity transformation is a rigid motion followed by a dilation.  
Resources: Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.  
Example:  
• In the picture below, line segments $AD$ and $BC$ intersect at $X$. Line segments $AB$ and $CD$ are drawn, forming two triangles $\triangle AXB$ and $\triangle CXD$.  
![Diagram](image)

• In each part a–d below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar, and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one variable to the other. If not, explain why not.  
a. The lengths $AX$ and $XD$ satisfy the equation $2AX = 3XD$.  
b. The lengths $AX$, $BX$, $CX$, and $DX$ satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$.  
c. Lines $AB$ and $CD$ are parallel.  
d. $\angle XAB$ is congruent to $\angle XCD$.  

| 9–10  | G.SRT.3 – Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.  
Example:  
• What is the least amount of information needed to prove two triangles similar? How do you know?  
• Using a ruler and a protractor, prove $AA$ similarity.  

**Label Legend:**  9-10 = Standards for Grades 9 and 10; ♦ = Algebra 2 Standards; ✽ = STEM Standards, ★ = Standards Connected to Mathematical Modeling
Prove theorems involving similarity

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</table>
| 9–10  | G.SRT.4 – Prove theorems about triangles. *Theorems include:* a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.  
*Resources:* Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.  
*Example:*  
- How does the Pythagorean Theorem support the case for triangle similarity?  
  - View the video below and create a visual proving the Pythagorean Theorem using similarity.  
  http://www.youtube.com/watch?v=LrS5_l-gk94 |
| 9–10  | G.SRT.5 – Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.  
*Explanations:*  
- Similarity postulates include SSS, SAS, and AA.  
- Congruence postulates include SSS, SAS, ASA, AAS, and H-L.  
*Resources:* Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.  
*Example for G.SRT.4 and G.SRT.5:*  
- This diagram is made up of four regular pentagons that are all the same size.  

- Find the measure of $\angle AE$. Show your calculations and explain your reasons.  
- Find the measure of $\angle E/F$. Explain your reasons and show how you figured it out.  
- Find the measure of $\angle K/M$. Tell how you figured it out.
Define trigonometric ratios and solve problems involving right triangles

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
</table>
| 9–10  | G.SRT.6 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.  

*Resources:* Students may use applets to explore the range of values of the trigonometric ratios as \( \theta \) ranges from 0 to 90 degrees.  

\[
\begin{align*}
\text{sine of } \theta & = \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \\
\text{cosine of } \theta & = \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \\
\text{tangent of } \theta & = \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \\
\text{cosecant of } \theta & = \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} \\
\text{secant of } \theta & = \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} \\
\text{cotangent of } \theta & = \cot \theta = \frac{\text{adjacent}}{\text{opposite}}
\end{align*}
\]

| 9–10  | G.SRT.7 – Explain and use the relationship between the sine and cosine of complementary angles.  

*Resources:* Geometric simulation software, applets, and graphing calculators can be used to explore the relationship between sine and cosine.  

*Example:*  
- What is the relationship between cosine and sine in relation to complementary angles?  
- Construct a table demonstrating the relationship between sine and cosine of complementary angles.

*Label Legend:* 9-10 = Standards for Grades 9 and 10;  = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
<tbody>
<tr>
<td>9–10</td>
<td>G.SRT.8 – Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. &lt;br&gt;<strong>Resources:</strong> Students may use graphing calculators or programs, tables, spreadsheets, or computer algebra systems to solve right triangle problems. &lt;br&gt;<strong>Example:</strong> &lt;br&gt;• Find the height of a flagpole to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the flagpole is 50 feet.</td>
</tr>
</tbody>
</table>

![Diagram](image-url)
## Apply trigonometry to general triangles

<table>
<thead>
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<th>Label</th>
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</table>
| +     | G.SRT.9 – Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.  
*Example:*  
- When could a contractor use the equation $A = \frac{1}{2} ab \sin(C)$ to find the area of a triangle?  
  - Create a poster showing no less than three examples of finding the area of a non-right triangle using the formula $A = \frac{1}{2} ab \sin(C)$. |
| +     | G.SRT.10 – Prove the Laws of Sines and Cosines and use them to solve problems.  
*Example:*  
- Annie and Sashi are backpacking in the Sierra Nevada. They walk 8 km from their base camp at a bearing of 42°. After lunch, they change direction to a bearing of 137° and walk another 5 km.  
  a. How far are Annie and Sashi from their base camp?  
  b. At what bearing must Sashi and Annie travel to return to their base camp? |
| +     | G.SRT.11 – Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  
*Example:*  
- Tara wants to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78°. From the second position, the angle between the mountain and the first position is 53°. How can Tara determine the distance of the mountain from each position, and what is the distance from each position? |
### Geometry: Circles (G-C)

**Understand and apply theorems about circles**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
</table>
| 9–10  | G.C.1 – Prove that all circles are similar.  
*Resources:* Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.  
*Example:*  
- Show the two given circles are similar by stating the necessary transformations from $C$ to $D$.  
  
  $C$: center (2, 3) radius 5; $D$: center (−1, 4) radius 10 |
| 9–10  | G.C.2 – Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*  
*Examples:*  
- Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.  
  
  ![Diagram](image)
  
  - Find the unknown length in the picture below.  
  
  ![Diagram](image) |

*Label Legend:* 9-10 = Standards for Grades 9 and 10; ✶ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
<table>
<thead>
<tr>
<th>Label</th>
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<tbody>
<tr>
<td>9–10</td>
<td>G.C.3 – Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. &lt;br&gt;<strong>Resources:</strong> Students may use geometric simulation software to make geometric constructions. <strong>Example:</strong>&lt;br&gt;• The following diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long. &lt;br&gt;• Explain why triangles $AOX$ and $AOY$ are congruent.&lt;br&gt;a. What can you say about the measures of the line segments $CX$ and $CZ$?&lt;br&gt;b. Find $r$, the radius of the circle. Explain your work clearly and show all your calculations.</td>
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<td>Standard</td>
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<tr>
<td>3.</td>
<td>The following diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long. Draw radius lines as in the previous task and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.</td>
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</table>

![Diagram of a circle touching a right triangle](image)

<table>
<thead>
<tr>
<th>+</th>
<th>G.C.4 – Construct a tangent line from a point outside a given circle to the circle.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resources:</td>
<td>Students may use geometric simulation software to make geometric constructions.</td>
</tr>
</tbody>
</table>

**Example for G.C.1 through G.C.4:**
- A certain machine is to contain two wheels, one of radius 3 centimeters and one of radius 5 centimeters, whose centers are attached to points 14 centimeters apart. The manufacturer of this machine needs to produce a belt that will fit snugly around the two wheels, as shown in the diagram below. How long should the belt be?

![Diagram of a belt around two wheels](image)
### Find arc lengths and areas of sectors of circles

<table>
<thead>
<tr>
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<th>Standard</th>
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</thead>
<tbody>
<tr>
<td>9–10</td>
<td>G.C.5 – Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</td>
</tr>
</tbody>
</table>

**Resources:** Students can use geometric simulation software to explore angle and radian measures and derive the formula for the area of a sector.

**Example:** Consider the Ferris Wheel

- The amusement park has discovered that the brace that provides stability to the Ferris wheel has been damaged and needs work. The arc length of steel reinforcement that must be replaced is between the two seats shown below. If the sector area is 28.25 ft\(^2\) and the radius is 12 feet, what is the length of steel that must be replaced? Describe the steps you used to find your answer.

- If the amusement park owners wanted to decorate each sector of this Ferris wheel with a different color of fabric, how much of each color fabric would they need to purchase? The area to be covered is described by an arc length of 5.9 feet. The circle has a radius of 15 feet. Describe the steps you used to find your answer.

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Label Legend: 9-10 = Standards for Grades 9 and 10; ✶ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
## Geometry: Expressing Geometric Properties with Equations (G-GPE)

*Translate between the geometric description and the equation for a conic section*

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</table>
| 9–10  | G.GPE.1 – Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.  
*Resources:* Students may use geometric simulation software to explore the connection between circles and the Pythagorean Theorem.  
*Example:*  
- Write an equation for a circle with a radius of 2 units and center at (1, 3).  
- Write an equation for a circle given that the endpoints of the diameter are (−2, 7) and (4, −8).  
- Find the center and radius of the circle $4x^2 + 4y^2 − 4x + 2y − 1 = 0$. |

| 9–10  | G.GPE.2 – Derive the equation of a parabola given a focus and directrix.  
*Resources:* Students may use geometric simulation software to explore parabolas.  
*Examples:*  
- Write and graph an equation for a parabola with focus (2, 3) and directrix $y = 1$. |

| +     | G.GPE.3 – Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.  
*Resources:* Students may use geometric simulation software to explore conic sections.  
*Example:*  
- Write an equation in standard form for an ellipse with foci at (0, 5) and (2, 0) and a center at the origin. |
Use coordinates to prove simple geometric theorems algebraically

<table>
<thead>
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<tbody>
<tr>
<td>9–10</td>
<td>G.GPE.4 – Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point ( (1, \sqrt{3}) ) lies on the circle centered at the origin and containing the point ( (0, 2) ).&lt;br&gt;&lt;br&gt;<strong>Resources:</strong> Students may use geometric simulation software to model figures and prove simple geometric theorems.&lt;br&gt;&lt;br&gt;<strong>Example:</strong>&lt;br&gt;&lt;br&gt;• Use slope and distance formula to verify the polygon formed by connecting the points ( (-3, -2), (5, 3), (9, 9), (1, 4) ) is a parallelogram.</td>
</tr>
<tr>
<td>9–10</td>
<td>G.GPE.5 – Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).&lt;br&gt;&lt;br&gt;<strong>Explanation:</strong> Lines can be horizontal, vertical, or neither.&lt;br&gt;&lt;br&gt;<strong>Resources:</strong> Students may use a variety of different methods to construct a parallel or perpendicular line to a given line and calculate the slopes to compare the relationships.</td>
</tr>
<tr>
<td></td>
<td>Example for G.GPE.4 and G.GPE.5:&lt;br&gt;&lt;br&gt;• Draw a quadrilateral ( ABCD ). Try to draw your parallelogram so that no two sides are congruent and no two sides are parallel.&lt;br&gt;&lt;br&gt;a. Let ( P, Q, R, ) and ( S ) be the midpoints of sides ( AB, BC, CD, ) and ( DA ), respectively. Use a ruler to locate these points as precisely as you can and join them to form a new quadrilateral ( PQRS ). What do you notice about the quadrilateral ( PQRS )?&lt;br&gt;&lt;br&gt;b. Suppose your quadrilateral ( ABCD ) lies in the coordinate plane. Let ( (x_1, y_1) ) be the coordinates of vertex ( A ), ( (x_2, y_2) ) be the coordinates of ( B ), ( (x_3, y_3) ) be the coordinates of ( C ), and ( (x_4, y_4) ) be the coordinates of ( D ). Use coordinates to prove the observation you made in part a.</td>
</tr>
<tr>
<td>9–10</td>
<td>G.GPE.6 – Find the point on a directed line segment between two given points that partitions the segment in a given ratio.&lt;br&gt;&lt;br&gt;<strong>Resources:</strong> Students may use geometric simulation software to model figures or line segments.&lt;br&gt;&lt;br&gt;<strong>Examples:</strong>&lt;br&gt;&lt;br&gt;• Given ( A(3, 2) ) and ( B(6, 11) ).&lt;br&gt;&lt;br&gt;  ♦ Find the point that divides the line segment ( AB ) two-thirds of the way from ( A ) to ( B ).&lt;br&gt;&lt;br&gt;  The point two-thirds of the way from ( A ) to ( B ) has ( x )-coordinate two-thirds of the way from 3 to 6 and ( y ) —coordinate two-thirds of the way from 2 to 11.&lt;br&gt;&lt;br&gt;  So, ( (5, 8) ) is the point that is two-thirds from point ( A ) to point ( B ).&lt;br&gt;&lt;br&gt;  ♦ Find the midpoint of line segment ( AB ).</td>
</tr>
<tr>
<td>Label</td>
<td>Standard</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>9–10</td>
<td>G.GPE.7 – Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. Resources: Students may use geometric simulation software to model figures. Example: • Find the perimeter and area of a rectangle with vertices at C(−1, 1), D(3, 4), E(6, 0), F(2, −3). (Hint: graph the rectangle.) Round your answers to the nearest hundredth when necessary.</td>
</tr>
</tbody>
</table>
Geometry: Geometric Measurement and Dimension (G-GMD)

Explain volume formulas and use them to solve problems

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10  | G.GMD.1 – Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
|       | *Explanation:* Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. |
| +     | G.GMD.2 – Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. *Explanation:* Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. |

*Example for G.GMD.1 and G.GMD.2:*
- People who live in isolated or rural areas have their own tanks of natural gas to run appliances like stoves, washers, and water heaters. These tanks are made in the shape of a cylinder with hemispheres on the ends.

![Diagram of a tank](image)

The Insane Propane Tank Company makes tanks with this shape in different sizes. The cylinder part of every tank is exactly 10-feet long, but the radius of the hemispheres, r, will be different depending on the size of the tank.

The company wants to double the capacity of their standard tank, which is 6 feet in diameter. What should the radius of the new tank be? Explain your thinking and show your calculations.
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10</td>
<td>G.GMD.3 – Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.</td>
</tr>
<tr>
<td>*</td>
<td>Explanation: Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius.</td>
</tr>
<tr>
<td></td>
<td>Example:</td>
</tr>
<tr>
<td></td>
<td>• Janine is planning on creating a water-based centerpiece for each of the 30 tables at her wedding reception. She has already purchased a cylindrical vase for each table. The radius of the vases is 6 cm, and the height is 28 cm. She intends to fill them half way with water and then add a variety of colored marbles until the waterline is approximately three-quarters of the way up the cylinder. She can buy bags of 100 marbles in 2 different sizes, with radii of 9 mm or 12 mm. A bag of 9 mm marbles costs $3, and a bag of 12 mm marbles costs $4.</td>
</tr>
<tr>
<td></td>
<td>a. If Janine only bought 9 mm marbles how much would she spend on marbles for the whole reception? What if Janine only bought 12 mm marbles? (Note: 1 cm³ = 1 ml)</td>
</tr>
<tr>
<td></td>
<td>b. Janine wants to spend at most $d$ dollars on marbles. Write a system of equalities and/or inequalities that she can use to determine how many marbles of each type she can buy.</td>
</tr>
<tr>
<td></td>
<td>c. Based on your answer to part b., how many bags of each size marble should Janine buy if she has $180 and wants to buy as many small marbles as possible?</td>
</tr>
</tbody>
</table>
Visualize relationships between two-dimensional and three dimensional objects

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10</td>
<td>G.GMD.4 – Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</td>
</tr>
</tbody>
</table>

*Resources:* Students may use geometric simulation software to model figures and create cross sectional views.

*Examples:*

- Identify the shape of the vertical, horizontal, and other cross sections of a cylinder.
- The official diameter of a tennis ball, as defined by the International Tennis Federation, is at least 2.575 inches and at most 2.700 inches. Tennis balls are sold in cylindrical containers that contain three balls each. To model the container and the balls in it, we will assume that the balls are 2.7 inches in diameter and that the container is a cylinder the interior of which measures 2.7 inches in diameter and $3 \times 2.7 = 8.1$ inches high.
  a. Lying on its side, the container passes through an X-ray scanner in an airport. If the material of the container is opaque to X-rays, what outline will appear? With what dimensions?
  b. If the material of the container is partially opaque to X-rays and the material of the balls is completely opaque to X-rays, what will the outline look like (still assuming the can is lying on its side)?
  c. The central axis of the container is a line that passes through the centers of the top and bottom. If one cuts the container and the balls by a plane passing through the central axis, what does the intersection of the plane with the container and balls look like? (The intersection is also called a cross section. Imagine putting the cut surface on an inkpad and then stamping a piece of paper. The stamped image is a picture in the intersection.)
  d. If the can is cut by a plane parallel to the central axis, but at a distance of 1 inch from the axis, what will the intersection of this plane with the container and balls look like?
  e. If the can is cut by a plane parallel to one end of the can—a horizontal plane—what are the possible appearances of the intersections?
  f. A cross section by a horizontal plane at a height of $1.35 + w$ inches from the bottom is made, with $0 < w < 1.35$ (so the bottom ball is cut). What is the area of the portion of the cross section inside the container but outside the tennis ball?
  g. Suppose the can is cut by a plane parallel to the central axis but at a distance of $w$ inches from the axis $(0 < w < 1.35)$. What fractional part of the cross section of the container is inside of a tennis ball?
Geometry: Modeling with Geometry (G-MG)

Apply geometric concepts in modeling situations

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10 ★</td>
<td>G.MG.1 – Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).&lt;br&gt;&lt;br&gt;&lt;em&gt;Resources:&lt;/em&gt; Students may use simulation software and modeling software to explore which model best describes a set of data or situation.&lt;br&gt;&lt;br&gt;&lt;em&gt;Example:&lt;/em&gt;&lt;br&gt;• Suppose that even in perfect visibility conditions, the lamp at the top of a lighthouse is visible from a boat on the sea at a distance of up to 32 km.&lt;br&gt;• If the “distance” in question is the straight-line distance from the lamp itself to the boat, what is the height above sea level of the lamp on top of the lighthouse?&lt;br&gt;• What are two other interpretations of the distance being investigated in this problem? Describe how to solve the alternate versions.</td>
</tr>
<tr>
<td>9–10 ❖ ★</td>
<td>G.MG.2 – Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).&lt;br&gt;&lt;br&gt;&lt;em&gt;Resources:&lt;/em&gt; Students may use simulation software and modeling software to explore which model best describes a set of data or situation.&lt;br&gt;&lt;br&gt;&lt;em&gt;Example:&lt;/em&gt;&lt;br&gt;• Take a look at the two boxes below. Each box has the same volume. If each ball has the same mass, which box would weigh more? Why?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Block I</th>
<th>Block II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass = 79.4 grams</td>
<td>Mass = 25.4 grams</td>
</tr>
<tr>
<td>Volume = 29.8 cubic cm</td>
<td>Volume = 29.8 cubic cm</td>
</tr>
<tr>
<td>Label</td>
<td>Standard</td>
</tr>
<tr>
<td>-------</td>
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</tr>
</tbody>
</table>
| 9–10 ★ | G.MG.3 – Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  

*Resources:* Students may use simulation software and modeling software to explore which model best describes a set of data or situation.  

*Example:*  
- You have been hired by the owner of a local ice cream parlor to assist in his company’s new venture. The company will soon sell its ice cream cones in the freezer section of local grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones. Use a real ice cream cone or the dimensions of a real ice cream cone to complete the following tasks.  
  a. Sketch a wrapper like the one described above, using the actual size of your cone. Ignore any overlap required for assembly.  
  b. Use your sketch to help you develop an equation the owner can use to calculate the surface area of a wrapper (including the lid) for another cone given its base had a radius of length, , and a slant height, .  
  c. Using measurements of the radius of the base and slant height of your cone and your equation from the previous step, find the surface area of your cone.  
  d. The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit your cone that could be cut from this one piece of paper. Explain your estimate.
High School – Statistics and Probability Overview

Interpreting Categorical and Quantitative Data (S-ID)
- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions (S-IC)
- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Conditional Probability and the Rules of Probability (S-CP)
- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions (S-MD)
- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Mathematical Practices (MP)
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Label Legend: 9-10 = Standards for Grades 9 and 10; ♦ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID)
Summarize, represent, and interpret data on a single count or measurement variable

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10 ★</td>
<td>S.ID.1 – Represent data with plots on the real number line (dot plots, histograms, and box plots).</td>
</tr>
<tr>
<td></td>
<td>Example:</td>
</tr>
<tr>
<td></td>
<td>• Step 1–Directions: Create a double stem and leaf chart for the following sets of Data.</td>
</tr>
<tr>
<td></td>
<td>Height of 8th Graders in First Period Algebra (in inches)</td>
</tr>
<tr>
<td></td>
<td>60 58 63 55</td>
</tr>
<tr>
<td></td>
<td>67 72 59 62</td>
</tr>
<tr>
<td></td>
<td>60 54 64 70</td>
</tr>
<tr>
<td></td>
<td>68 66 54 60</td>
</tr>
<tr>
<td></td>
<td>63 57 62 68</td>
</tr>
<tr>
<td></td>
<td>Height of 8th Graders in Second Period Algebra (in inches)</td>
</tr>
<tr>
<td></td>
<td>61 59 63 56</td>
</tr>
<tr>
<td></td>
<td>57 70 58 61</td>
</tr>
<tr>
<td></td>
<td>69 57 66 70</td>
</tr>
<tr>
<td></td>
<td>63 66 59 64</td>
</tr>
<tr>
<td></td>
<td>66 57 62 69</td>
</tr>
<tr>
<td></td>
<td>• Step 2–Directions: Calculate the 5 number summary for each set of data above.</td>
</tr>
<tr>
<td></td>
<td>• Step 3–Directions: Create a double box and whisker plot to display each set of data above.</td>
</tr>
<tr>
<td>9–10 ★</td>
<td>S.ID.2 – Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</td>
</tr>
<tr>
<td></td>
<td>Resources: Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets.</td>
</tr>
<tr>
<td></td>
<td>Examples:</td>
</tr>
<tr>
<td></td>
<td>• The two data sets below depict the housing prices sold in the Overland Park area and Olathe areas of Johnson County, Kansas. Based on the prices below which price range can be expected for a home purchased in Overland Park? In the Olathe area? In Johnson County?</td>
</tr>
<tr>
<td></td>
<td>• Overland Park area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}</td>
</tr>
<tr>
<td></td>
<td>• Olathe area {5 million, 154000, 250000, 250000, 200000, 160000, 190000}</td>
</tr>
<tr>
<td></td>
<td>• Given a set of test scores: 99, 96, 94, 93, 90, 88, 86, 77, 70, 68, find the mean, median and standard deviation. Explain how the values vary about the mean and median. What information does this give the teacher?</td>
</tr>
<tr>
<td>Label</td>
<td>Standard</td>
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</tr>
</tbody>
</table>
| 9–10  | S.ID.3 – Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).  
*Resources:* Students may use spreadsheets, graphing calculators and statistical software to statistically identify outliers and analyze data sets with and without outliers as appropriate.  
*Examples:*  
The ages of the students in a certain high school are to be graphed on a set of parallel box plots according to the following:  
- Set I – All seniors in the school (grade 12).  
- Set II – All students in the school (grades 9–12).  
In the figure below, drag each of the two box plots into position above the number line to approximate the ages of the two sets of students. To do this:  
- First, move each box plot at an appropriate location according to its center.  
- Then, drag each endpoint to stretch the box plot to represent the spread.  
- Please note that there are no outliers in either set.  
| Set I – Senior only | ![Box Plot] |
| Set II – All students | ![Box Plot] |

*Solution:*  
- Seniors only  
- All students  

Graphs should show: Median of I > Median of II Range of I < Range of II Max of I ≤ Max of II
### Label Legend:
- 9-10 = Standards for Grades 9 and 10
- ✤ = Algebra 2 Standards
- + = STEM Standards
- ⭐ = Standards Connected to Mathematical Modeling

### Examples for S.ID.3 continued:
- The dot plots below compare the number of minutes 30 flights made by two airlines arrived before or after their scheduled arrival times.

#### Airline P
- Negative numbers represent the minutes the flight arrived before its scheduled time.
- Positive numbers represent the minutes the flight arrived after its scheduled time.
- Zero indicates the flight arrived at its scheduled time.

#### Airline Q
- Based on these data, from which airline will you choose to buy your ticket? Use the ideas of center and spread to justify your choice.

**Sample Top-Score Response:** I would choose to buy the ticket from Airline P. Both airlines are likely to have an on-time arrival since they both have median values at 0. However, Airline Q has a much greater range in arrival times. Airline Q could arrive anywhere from 35 minutes early to 60 minutes late. For Airline P, this flight arrived within 10 minutes on either side of the scheduled arrival time about 2/3 of the time, and for Airline Q, that number was only about 1/2. For these reasons, I think Airline P is the better choice.
Label Legend: 9-10 = Standards for Grades 9 and 10;  = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>9–10 ★</td>
<td>S.ID.4 – Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Resources: Students may use spreadsheets, graphing calculators, statistical software, and tables to analyze the fit between a data set and normal distributions and estimate areas under the curve. Examples: • The bar graph below gives the birth weight of a population of 100 chimpanzees. The line shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000–3999 grams.</td>
</tr>
</tbody>
</table>

Birth Weight Distribution for a Population

- Determine which situation(s) is best modeled by a normal distribution. Explain your reasoning.  
  - Annual income of a household in the U.S.  
  - Weight of babies born in one year in the U.S.
Summarize, represent, and interpret data on two categorical and quantitative variables

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-10 ★</td>
<td>S.ID.5 – Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. Resources: Students may use spreadsheets, graphing calculators, and statistical software to create frequency tables and determine associations or trends in the data. Examples: Two-Way Frequency Table • A two-way frequency table is shown below displaying the relationship between age and baldness. We took a sample of 100 male subjects and determined who is or is not bald. We also recorded the age of the male subjects by categories.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bald</th>
<th>Age</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>35</td>
<td>11</td>
</tr>
<tr>
<td>Yes</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>59</td>
<td>41</td>
</tr>
</tbody>
</table>

The total row and total column entries in the table above report the marginal frequencies, while entries in the body of the table are the joint frequencies.

Two-Way Relative Frequency Table
• The relative frequencies in the body of the table are called conditional relative frequencies.

<table>
<thead>
<tr>
<th>Bald</th>
<th>Age</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>0.35</td>
<td>0.11</td>
</tr>
<tr>
<td>Yes</td>
<td>0.24</td>
<td>0.30</td>
</tr>
<tr>
<td>Total</td>
<td>0.59</td>
<td>0.41</td>
</tr>
<tr>
<td>Label</td>
<td>Standard</td>
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<td>-------</td>
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</tbody>
</table>
| 9–10 | S.ID.6 – Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.  
*Explanation:* The residual in a regression model is the difference between the observed and the predicted \( y \) for some \( x \) (\( y \) the dependent variable and \( x \) the independent variable).  
So, if we have a model \( y = ax + b \), and a data point \( (x_i, y_i) \) the residual for this point is: \( r_i = y_i - (ax_i + b) \). Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals.  
*Example:*  
- Measure the wrist and neck size of each person in your class and make a scatter plot. Find the least squares regression line.  
- Calculate and interpret the correlation coefficient for this linear regression model. Graph the residuals and evaluate the fit of the linear equations.  
  a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.  
  b. Informally assess the fit of a function by plotting and analyzing residuals.  
  c. Fit a linear function for a scatter plot that suggests a linear association. |

**Interpret linear models**

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10 | S.ID.7 – Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.  
*Resources:* Students may use spreadsheets or graphing calculators to create representations of data sets and create linear models.  
*Example:*  
- Lisa lights a candle and records its height in inches every hour. The results recorded as \((\text{time, height})\) are \((0, 20)\), \((1, 18.3)\), \((2, 16.6)\), \((3, 14.9)\), \((4, 13.2)\), \((5, 11.5)\), \((7, 8.1)\), \((9, 4.7)\), and \((10, 3)\). Express the candle's height \((h)\) as a function of time \((t)\) and state the meaning of the slope and the intercept in terms of the burning candle.  
*Solution:*  
- \( h = -1.7t + 20 \)  
- Slope: The candle's height decreases by 1.7 inches for each hour it is burning.  
- Intercept: Before the candle begins to burn, its height is 20 inches. |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷     | S.ID.8 – Compute (using technology) and interpret the correlation coefficient of a linear fit.  
**Resources:** Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients.  
**Example:**  
- Collect height, shoe-size, and wrist circumference data for each student. Determine the best way to display the data. Answer the following questions:  
  - Is there a correlation between any two of the three indicators?  
  - Is there a correlation between all three indicators?  
  - What patterns and trends are apparent in the data?  
  - What inferences can be made from the data? |
| 9–10  | S.ID.9 – Distinguish between correlation and causation.  
**Resources:** Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment.  
**Example:**  
- Diane did a study for a health class about the effects of a student's end-of-year math test scores on height. Based on a graph of her data, she found that there was a direct relationship between students' math scores and height. She concluded that "doing well on your end-of-course math tests makes you tall." Is this conclusion justified? Explain any flaws in Diane's reasoning. |
**Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)**

*Understand and evaluate random processes underlying statistical experiments*

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10 ★ | S.IC.1 – Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.  
**Example:**  
- From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school advisory panel. Here are four different methods of making the selection.  
  - Select the first three names on the class roll.  
  - Select the first three students who volunteer.  
  - Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix.  
  - Select the first three students who show up for class tomorrow.  
Which is the best sampling method, among these four, if you want the school panel to represent a fair and representative view of the opinions of your class. Explain the weaknesses of the three you did not select as the best. |
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10 ★</td>
<td>S.IC.2 – Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</td>
</tr>
</tbody>
</table>

**Resources:** Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials.

**Explanation:** The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process.

**Examples:**
- Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group’s results will most likely approach the theoretical probability?
- A random sample of 100 students from a specific high school resulted in 45% of them favoring a plan to implement block scheduling. Is it plausible that a majority of the students in the school actually favor the block schedule? Simulation can help answer the questions.

The accompanying plot shows a simulated distribution of sample proportions for samples of size 100 from a population in which 50% of the students favor the plan, and another distribution from a population in which 60% of the students favor the plan. (Each simulation contains 200 runs.)

a. What do you conclude about the plausibility of a population proportion of 0.50 when the sample proportion is only 0.45?
b. What about the plausibility of 0.60 for the population proportion?
## Make inferences and justify conclusions from sample surveys, experiments, and observational studies

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>9–10 ★</td>
<td>S.IC.3 – Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</td>
</tr>
</tbody>
</table>

**Explanations:**
- Students should be able to explain techniques/applications for randomly selecting study subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment.
- In statistics, an observational study draws inferences about the possible effect of a treatment on subjects, where the assignment of subjects into a treated group versus a control group is outside the control of the investigator (for example, observing data on academic achievement and socioeconomic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment.

**Example:**
- Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as “strict.” They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students.
  - a. Describe the parameter of interest and a statistic the students could use to estimate the parameter.
  - b. Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
  - c. The students quickly realized that, as there is no definition of “strict,” they could not simply ask a student, “Are your parents or guardians strict?” Write three questions that could provide objective data related to strictness.
  - d. Describe an appropriate method for obtaining a sample of 100 students based on your answer in part a. above.
Label | Standard
--- | ---
9–10 ★ | S.IC.4 – Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

*Resources:* Students may use computer generated simulation models based upon sample surveys results to estimate population statistics and margins of error.

*Example:* Who Is Hotter?

- Is normal body temperature the same for men and women? Medical researchers interested in this question collected data from a large number of men and women. Random samples from that data are recorded in the table below.
  - a. Use a 95% confidence interval to estimate the mean body temperature of men.
  - b. Use a 95% confidence interval to estimate the mean body temperature of women.
  - c. Find the margin of error for the men and for the women.
  - d. Which margin of error is larger? Why is it larger?

<table>
<thead>
<tr>
<th>Body Temperature (°F)</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.9</td>
<td>97.8</td>
<td></td>
</tr>
<tr>
<td>97.4</td>
<td>98.0</td>
<td></td>
</tr>
<tr>
<td>97.8</td>
<td>98.2</td>
<td></td>
</tr>
<tr>
<td>97.8</td>
<td>98.2</td>
<td></td>
</tr>
<tr>
<td>97.9</td>
<td>98.6</td>
<td></td>
</tr>
<tr>
<td>98.0</td>
<td>98.8</td>
<td></td>
</tr>
<tr>
<td>98.1</td>
<td>98.8</td>
<td></td>
</tr>
<tr>
<td>98.6</td>
<td>99.2</td>
<td></td>
</tr>
<tr>
<td>98.8</td>
<td>99.4</td>
<td></td>
</tr>
</tbody>
</table>

Two random samples of body temperatures.

Label | Standard
--- | ---
+ | + S.I.C.5 – Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

*Resources:* Students may use computer generated simulation models to decide how likely it is that observed differences in a randomized experiment are due to chance.

*Explanation:* Treatment is a term used in the context of an experimental design to refer to any prescribed combination of values of explanatory variables. For example, one wants to determine the effectiveness of weed killer. Two equal parcels of land in a neighborhood are treated; one with a placebo and one with weed killer to determine whether there is a significant difference in effectiveness in eliminating weeds.

*Example:* 
- Using a completely randomized design, 20 students counted the number of times they blinked their eyes and the number of breaths they took in one minute. The data is shown in the table below.

<table>
<thead>
<tr>
<th>Number of Breaths</th>
<th>Number of Blinks</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>27</td>
</tr>
<tr>
<td>9</td>
<td>28</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>20</td>
<td>31</td>
</tr>
<tr>
<td>12</td>
<td>29</td>
</tr>
<tr>
<td>13</td>
<td>30</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
</tr>
</tbody>
</table>

Compute the mean and standard deviation for both the number of breaths and number of blinks. What are the similarities and differences in the results?
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| 9–10★ | S.IC.6 – Evaluate reports based on data.  
  **Explanations:**  
  - Explanations can include but are not limited to sample size, biased survey sample, interval scale, unlabeled scale, uneven scale, and outliers that distort the line-of-best-fit. In a pictogram the symbol scale used can also be a source of distortion.  
  - As a strategy, collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed.  
  **Example:**  
  - A reporter used the two data sets below to calculate the mean housing price in Kansas as $629,000. Why is this calculation not representative of the typical housing price in Kansas?  
    - Wichita area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}  
    - Overland Park homes {5 million, 154000, 250000, 250000, 200000, 160000, 190000} |

Understand independence and conditional probability and use them to interpret data

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
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</thead>
<tbody>
<tr>
<td>❆</td>
<td>S.CP.1 – Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</td>
</tr>
</tbody>
</table>

Explanations:

- **Intersection**: The intersection of two sets $A$ and $B$ is the set of elements that are common to both set $A$ and set $B$. It is denoted by $A \cap B$ and is read “$A$ intersection $B$.”
  - $A \cap B$ in the diagram is {1,5}

- **Union**: The union of two sets $A$ and $B$ is the set of elements which are in $A$ or in $B$ or in both. It is denoted by $A \cup B$ and is read “$A$ union $B$.”
  - $A \cup B$ in the diagram is {1,2,3,4,5,7}
  - Could be both

- **Complement**: The complement of the set $A \cup B$ is the set of elements that are members of the universal set $U$ but are not in $A \cup B$. It is denoted by $(A \cup B)$. 
  - $(A \cup B)$ in the diagram is {8}

Example: Fill in a Venn diagram for the following problem:

- Of 100 Aggies surveyed concerning where they prefer to go on Friday nights:
  - 35 like Harry’s and Sadow Canyon
  - 69 like The Chicken
  - 59 like more than one of the three places
  - 30 like Harry’s or Shadow Canyon but not The Chicken
  - 15 like only Harry’s and The Chicken
  - 30 like all three places
  - 15 like Harry’s but not The Chicken
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ✷ ★   | S.CP.2 – Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.  

*Example:*  
- In the cafeteria, there are 10 chocolate milk cartons and 5 regular milk cartons. There are also 8 packages of chocolate chip cookies and 12 packages of peanut butter cookies. If you grab a milk and package of cookies without looking, what is the probability you get regular milk and chocolate chip cookies? |
| ✷ ★   | S.CP.3 – Understand the conditional probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.  

*Example:*  
- A die is thrown twice. Determine the probability that the sum of the rolls is less than 4 given that:  
  a. At least one of the rolls is a 1  
  b. The first roll is a 1 |
| ✷ ★   | S.CP.4 – Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.  

*Resources:* Students may use spreadsheets, graphing calculators, and simulations to create frequency tables and conduct analyses to determine if events are independent or determine approximate conditional probabilities.  

*Example:* Collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in $10^{th}$ grade. Do the same for other subjects and compare the results. |
| ✷ ★   | S.CP.5 – Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.  

*Examples:*  
- Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.  
- What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced? Are these events independent or dependent?  
- At your high school, the probability that a student takes Business Essentials and Spanish is 0.062. The probability that a student takes Business Essentials is 0.43. What is the probability that a student takes Spanish given that the student is taking Business Essentials? |
Use the rules of probability to compute probabilities of compound events in a uniform probability model

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| ❖ ★ | S.CP.6 – Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.  
*Resources:* Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.  
*Example:*  
- A local restaurant asked 1000 people, “Did you cook dinner last night?” The results of this survey are shown in the table below.  

<table>
<thead>
<tr>
<th>“Did You Cook Dinner Last Night?”</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>115</td>
<td>480</td>
</tr>
<tr>
<td>No</td>
<td>327</td>
<td>78</td>
</tr>
</tbody>
</table>

Determine what the probability is of a person chosen at random from the 1000 surveyed:  
a. cooked dinner last night  
b. was a male and did not cook dinner  
c. was a male  
d. was a female and cooked dinner last night  

| ❖ ★ | S.CP.7 – Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model.  
*Resources:* Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.  
*Example:*  
- In a math class of 32 students, 18 are boys and 14 are girls. On a unit test, 5 boys and 7 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?  

*Label Legend:*  9-10 = Standards for Grades 9 and 10; ❖ = Algebra 2 Standards; + = STEM Standards, ★ = Standards Connected to Mathematical Modeling
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
</table>
| +     | S.CP.8 – Apply the general Multiplication Rule in a uniform probability model, \( P(A \text{ and } B) = P(A)P(B/A) = P(B)P(A/B) \), and interpret the answer in terms of the model.  

*Resources:* Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.  

*Example:*  
- Someone draws a card at random out of a deck, replaces it, and then draws another card at random. What is the probability that the first card is the ace of clubs and the second card is a club (any club). Since there is only one ace of clubs in the deck, the probability of the first event is \( \frac{1}{52} \). Since \( \frac{13}{52} = \frac{1}{4} \) of the deck is composed of clubs, the probability of the second event is \( \frac{1}{4} \). Therefore, the probability of both events is: \( \frac{1}{52} \times \frac{1}{4} = \frac{1}{208} \). |
| +     | S.CP.9 – Use permutations and combinations to compute probabilities of compound events and solve problems.  

*Resources:* Students may use calculators or computers to determine sample spaces and probabilities.  

*Example:*  
- You and two friends go to the grocery store and each buys a soda. If there are five different kinds of soda, and each friend is equally likely to buy each variety, what is the probability that no one buys the same kind? |
Statistics and Probability: Using Probability to Make Decisions ★ (S-MD)

Calculate expected values and use them to solve problems

<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ ★</td>
<td>S.MD.1 – Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.</td>
</tr>
</tbody>
</table>

**Resources:** Students may use spreadsheets, graphing calculators and statistical software to represent data in multiple forms.

**Example:**
- Suppose you are working for a contractor who is designing new homes. She wants to ensure that the home models match the demographics for the area. She asks you to research the size of households in the region in order to better inform the floor plans of the home.

**Solution:**
- A possible solution could be the result of research organized in a variety of forms. In this case, the results of the research are shown in a table and graph. The student has defined their variable as $x$ as the number of people per household.

<table>
<thead>
<tr>
<th>People per Household</th>
<th>Proportion of Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.026</td>
</tr>
<tr>
<td>2</td>
<td>0.031</td>
</tr>
<tr>
<td>3</td>
<td>0.132</td>
</tr>
<tr>
<td>4</td>
<td>0.567</td>
</tr>
<tr>
<td>5</td>
<td>0.181</td>
</tr>
<tr>
<td>6</td>
<td>0.048</td>
</tr>
<tr>
<td>7</td>
<td>0.015</td>
</tr>
<tr>
<td>Label</td>
<td>Standard</td>
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<tr>
<td>-------</td>
<td>----------</td>
</tr>
<tr>
<td>+ ★</td>
<td>S.MD.2 – Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.</td>
</tr>
<tr>
<td></td>
<td><em>Explanation</em>: The expected value of an uncertain event is the sum of the possible points earned multiplied by each point's chance of occurring.</td>
</tr>
<tr>
<td></td>
<td><em>Resources</em>: Students may use spreadsheets or graphing calculators to complete calculations or create probability models.</td>
</tr>
<tr>
<td></td>
<td><em>Example</em>: In a game, you roll a six-sided number cube numbered with 1, 2, 3, 4, 5, and 6. You earn 3 points if a 6 is rolled, 6 points if a 2, 4, or 5 is rolled, and nothing otherwise. Since there is a 1/6 chance of each number coming up, the outcomes, probabilities, and payoffs look like this:</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Outcome Probability Points Table" /></td>
</tr>
<tr>
<td></td>
<td>The expected value is sum of the products of the probability and points earned for each outcome (the entries in the last two columns multiplied together):</td>
</tr>
<tr>
<td></td>
<td>[ \left( \frac{1}{6} \right) \times 0 + \left( \frac{1}{6} \right) \times 6 + \left( \frac{1}{6} \right) \times 0 + \left( \frac{1}{6} \right) \times 6 + \left( \frac{1}{6} \right) \times 6 + \left( \frac{1}{6} \right) \times 3 = 3.50 \text{ points} ]</td>
</tr>
<tr>
<td>+ ★</td>
<td>S.MD.3 – Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.</td>
</tr>
<tr>
<td></td>
<td><em>Resources</em>: Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic, or exponential functions.</td>
</tr>
<tr>
<td></td>
<td><em>Example</em>: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.</td>
</tr>
</tbody>
</table>
### S.MD.4 – Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

**Resources:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic, or exponential functions.

**Example:**
- Find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

### S.MD.5 – Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

**Explanation:** Different types of insurance to be discussed include but are not limited to: health, automobile, property, rental, and life insurance.

**Resources:** Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic, or exponential functions.

**Examples:**
- Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
- Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
<table>
<thead>
<tr>
<th>Label</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>S.MD.6 – Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). &lt;br&gt;&lt;br&gt;<em>Resources:</em> Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic, or exponential functions. &lt;br&gt;&lt;br&gt;<em>Example:</em> &lt;br&gt;&lt;br&gt;- In the Rock-Paper-Scissors game, two players show a hand-sign at the same time. If the players make the same hand-sign, it is a tie. The rest of the rules are shown in the diagram. &lt;br&gt;&lt;br&gt;• What is the probability of winning? Losing? Tying? &lt;br&gt;&lt;br&gt;• Is the game fair? Explain.</td>
</tr>
<tr>
<td>✷</td>
<td>S.MD.7 – Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). &lt;br&gt;&lt;br&gt;<em>Resources:</em> Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic, or exponential functions. &lt;br&gt;&lt;br&gt;<em>Example:</em> &lt;br&gt;&lt;br&gt;- Blackburn Cedar, Inc. manufactures cedar fencing material in Marysville, Washington. The company’s quality manager inspected 5,900 boards and found that 4,100 could be rated as a #1 grade. &lt;br&gt;&lt;br&gt;a. If the manager wanted to assess the probability that a board being produced will be a #1 grade, what method of assessment would he likely use? &lt;br&gt;&lt;br&gt;b. Referring to your answer in part a, what would you assess the probability of a #1 grade board to be?</td>
</tr>
</tbody>
</table>