Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

<table>
<thead>
<tr>
<th>Unit Title:</th>
<th>The Overland Trail</th>
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<tbody>
<tr>
<td>Designed by:</td>
<td>Michelle Hawley, Innovative Schools</td>
</tr>
<tr>
<td></td>
<td>from Interactive Mathematics Program Year 1</td>
</tr>
<tr>
<td>Content Area:</td>
<td>Mathematics (Algebra)</td>
</tr>
<tr>
<td>Grade Level(s):</td>
<td>9</td>
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Summary of Unit
Anchoring back to previous learning about linear functions and their representations, students work with multiple representations of data including graphs, situations, tables and rules. Students are expected to work fluidly among all of the representations, creating graphs, tables, equations and even situations based upon the information presented to them. While the use of calculators continues to develop, students also spend time graphing by hand as a means of building greater understanding of linear functions and how they operate.

Stage 1 – Desired Results
What students will know, do, and understand

Delaware Content Standards
- Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. CC.N-Q.1
- Define appropriate quantities for the purpose of descriptive modeling. CC.N-Q.2
- Interpret parts of an expression, such as terms, factors, and coefficients. CC.A-SSE.1a
- Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. CC.A-SSE.2
- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. CC.A-SSE.3
- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. CC.A-CED.1
- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. CC.A-REI.1
- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CC.A-REI.3
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). CC.A-REI.10
• Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **CC.A-REI.11**

• Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **CC.F-IF.2**

• For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include:** intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **CC.F-IF.4**

• Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.** **CC.F-IF.5**

• Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **CC.F-IF.6**

• Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **CC.F-IF.7**

• Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **CC.F-IF.7b**

• Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. **CC.F-LE.1a**

• Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **CC.F-LE.1b**

• Interpret the parameters in a linear or exponential function in terms of a context. **CC.F-LE.5**

• Represent data on two quantitative variables on a scatter plot and describe how the variables are related. **CC.S-ID.6**

• Use a model function fitted to the data to solve problems in the context of the data. **Use given model functions or choose a function suggested by the context. Emphasize linear and exponential models.** **CC.S-ID.6a**

• Fit a linear function for scatter plots that suggest a linear association. **CC.S-ID.6c**

• Interpret the slope (rate of change) and the intercept (constant term) of a linear fit in the context of the data. **CC.S-ID.7**

**Big Idea(s)**

• Making decisions based upon given constraints
• Using algorithms to solve problems
• Appropriate mathematical communication skills
• Creating and using graphs
• Linear equations, graphs, tables and situations
• Using technology to solve problems
**Unit Enduring Understanding(s)**
- In the real world there can be multiple “correct” solutions to a given problem.
- Changing the constraints on a situation can impact the chosen solution and/or path to a solution.
- Linear relationships can be used to interpret and predict much of the world around us.
- Linear relationships can be represented in multiple ways: tables, graphs, equations, written situations.
- Understanding multiple representation of information help us become better-informed members of society.
- Technology can be useful when solving real world problems, if we understand the underlying concepts.

**Unit Essential Questions(s)**
- How do graphs help us represent situations?
- How can you determine whether a graph and an equation are related?, (emphasis on linear relationships)
- How do you recognize systems of equations?
- How can graphs help us solve systems of equations?
- What benefit do we get from fitting equations to data, both with and without graphing calculators?
- Why is it important to use principles for equivalent expressions (including the distributive property) when solving problems?
- How does the distributive property help us solve problems?
- How does a using equivalent equation enable us to solve equations?
- What strategies help us solve linear equations?
- How do graphs tell the story of a linear equation?
- What strategies help us match a graph with an equation?
- What are the relationships between the algebraic expression defining a linear function and the graph of that function?

**Knowledge and Skills**

**Students will know...**
- Strategies for solving problems with given conditions
- The distributive property
- Linear functions represent situations with constant change
- Changing the representation method does not affect the linear relationship
- When to use a calculator
- The importance of scale on a graph
- When to approximate answers and when to be exact
- Strategies for changing representations of linear relationships

**Key Vocabulary:**
- Equivalent expressions
- Formula
- Function
- Linear function
- Sequence
• Graph
• Equation
• Coordinate system
• Term
• Rate
• Mean
• Median
• Independent variable
• Dependent variable
• Axis
• Continuous graph/function
• Discrete graph/function
• Rate of change
• Quadrant
• Ordered pair
• Scale
• Line of best fit
• Coordinate system

Students will be able to...
• Use information given to draw a graph by hand
• Create a table to match a given situation
• Create a linear equation to match a given situation
• Write a situation to match given information
• Create an example to fit a set of constraints
• Find solutions that fit a set of constraints
• Make estimates using tables and graphs with lines of best fit
• Apply concepts of mean and median to solve problems
• Apply algorithms to solve problems
• Shift from one representation to another
• Find and interpret lines of best fit
• Solve linear equations
• Use a calculator to create a graph to match given information
• Use features on a calculator to obtain information
• Apply the distributive property

Stage 2 – Assessment Evidence
Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)
(from Integrated Mathematics Program Year 1)

1. Fuel Economies
   Your family is traveling on superhighways with a large rental truck and a van. Each time one of the vehicles stops for gas, you make a note of how much gas the vehicle has used and how many miles it has traveled.

   a) Using the data in the tables below, draw two graphs on the same set of axes comparing total miles traveled to total gallons of gas used for each vehicle.
b) Reading from the graph, estimate how many gallons of gas the truck used by the time you traveled 800 miles. Do the same for the van.

c) Estimate how many miles each vehicle can travel on a gallon of gas.

2. Van Repairs
One morning, you discover that the van won’t start. The mechanic says that the fuel pump needs replacement and that the work can be done in about two hours.

The family decides that you and one parent will head out in the truck toward that night’s stop, 400 miles down the highway. The rest of the family will remain behind and either catch up with you during the day or meet you tonight at the motel where you have reservations.

a) Suppose the truck travels 45 miles per hour, the van goes 55 miles per hour (once it is fixed), and the repair takes two hours. For simplicity, assume that neither vehicle makes any stops along the way.
   
i. Which vehicle will reach the motel first?

   ii. How far from the motel will the other vehicle be when the first vehicle arrives?

b) Of course, the repair might not take the full two hours, or it might take longer. So now consider the case in which the repair actually takes four hours. In this case, the truck definitely arrives first.

   How far from the motel will the van be when the truck arrives?

c) Now generalize from parts a and b. Suppose that \( h \) represents the number of hours the van is delayed. Assume that \( h \) is a number large enough to allow the truck to arrive first. Develop a formula or an equation in terms of \( h \) that tells how far the van will be from the motel when the truck arrives.

<table>
<thead>
<tr>
<th>Van</th>
<th>Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Gallons of gas used so far</td>
<td>Total gallons of gas used so far</td>
</tr>
<tr>
<td>Total Miles traveled so far</td>
<td>Total miles traveled so far</td>
</tr>
<tr>
<td>18</td>
<td>36</td>
</tr>
<tr>
<td>400</td>
<td>300</td>
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<td>52</td>
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<tr>
<td>1250</td>
<td>1100</td>
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**Rubric(s)**

<table>
<thead>
<tr>
<th></th>
<th>Van Graph:</th>
<th>Truck Graph:</th>
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</thead>
<tbody>
<tr>
<td>1a</td>
<td>Axis Labels (1 point)</td>
<td>Axis Labels (1 point)</td>
</tr>
<tr>
<td></td>
<td>Axis units (1 point)</td>
<td>Axis units (1 point)</td>
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<tr>
<td></td>
<td>Title (1 point)</td>
<td>Title (1 point)</td>
</tr>
<tr>
<td></td>
<td>At least two points plotted (2 points)</td>
<td>At least two points plotted (2 points)</td>
</tr>
<tr>
<td></td>
<td>Correct sketch of graph (1 point)</td>
<td>Correct sketch of graph (1 point)</td>
</tr>
<tr>
<td></td>
<td>/6</td>
<td>/6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Van:</th>
<th>Truck:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1b</td>
<td>Estimate is reasonable based on the graph (1 point)</td>
<td>Estimate is reasonable based on the graph (1 point)</td>
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<tr>
<td></td>
<td>Estimate is reasonable based on the table (1 point)</td>
<td>Estimate is reasonable based on the table (1 point)</td>
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<tr>
<td></td>
<td>/2</td>
<td>/2</td>
</tr>
</tbody>
</table>

|   | Student provides reasonable estimate of the unit rate | /2 |
|---|----------------------------------------------------|
| 1c | Van: (1 point) |
|    | Truck: (1 point) |

|   | Truck arrives first (1 point) | /2 |
|---|------------------------------|
| 2a | Reasonable explanation/work (1 point) |
|    | Van will be 21 miles away (1 point) | /2 |
|    | Reasonable explanation/work (1 point) |
|    | /2 |

|   | Van will be 111 miles away (1 point) | /2 |
|---|-------------------------------------|
| 2b | Reasonable explanation/work (1 point) |
|    | /2 |

|   | D=400-(488.95-55h) or another appropriate equation is written | /4 |
|---|---------------------------------------------------------------|

**Total Possible Points** | /28

Meets expectations: 25-28 points
Nearing expectations: 20-25 points
Below expectations: below 20 points

**Other Evidence**
- Class discussions
- Class work
- Homework
- Problem of the weeks
- Student math journals
- Collaborative discussions
- Exit Tickets
- Curriculum-Based assessment opportunities:
  - Creating Families: This assignment will give you information on how well students can deal with verbal constraints.
  - Laced Travelers: This activity will tell you whether students can put arithmetic processes into words.
  - Ox Expressions at Home: This assignment will help you assess how well students understand meaningful algebraic expressions.
  - Graph Sketches: This activity will give you a sense of how well students understand graphs.
  - Who Will Make It? This activity can help you gauge students’ ability to make meaningful inferences from graphs.
  - All Four, One—Linear Functions: This assignment will give you information about students’ understanding of the connections among different ways to represent a situation.
  - Straight-Line Reflections: This activity will give you a sense of how well students understand concepts related to straight-line graphs.
  - More Fair Share for Hired Hands: This assignment can provide information on student understanding of the connection between graphs and equations.
  - Family Comparisons by Algebra: This activity will help you evaluate students’ ability to represent situations using equations and their facility with solving linear equations.

**Student Self-Assessment and Reflection**
- Students will keep math portfolios where they respond to problem of the weeks and other mathematical prompts
- Students will work in an interactive classroom environment, where collaboration, discussion and feedback are everyday occurrences.

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**Stage 3 – Learning Plan**
*(Design learning activities to align with Stage 1 and Stage 2 expectations)*

**Key learning events needed to achieve unit goals**

**Lesson 1: Setting Out with Variables**

**Shoelaces**

Students extend their work with variables and algebraic expressions in *Family Constraints* to examine how to use variables to represent complex arithmetic algorithms. Students are introduced to the formal process of substitution into and evaluation of algebraic expressions, emphasizing the two steps in the process. Students work in small groups on their own Overland Trail families, while listening to classmate’s methods for figuring the total length of shoelaces needed. Students share verbal and written statements about procedures for calculating shoelace length. The teacher leads a discussion to convert the generalization to an algebraic expression and test it by substitution.
Key Questions:
- What similarities do you see between each of the methods for answering Question 4?
- How much shoelace did your Overland Trail family need?
- Describe in words how you found your answer to Question 4.
- Does anyone have a suggestion for abbreviating these verbal descriptions by using variables?
- How can you use the expression $224W + 256M + 144C$ to find the amount of shoelace needed for a family with four women, six men, and five children?

Laced Travelers (homework)
Students work through and describe the arithmetic used to solve several problems, setting the stage for writing algebraic expressions to describe situations. After working on their own, students focus on the verbal descriptions of their computations. Class discussion is oriented once again, as in Shoelaces, to translating verbal descriptions into algebraic expressions. This activity leads to more practice with substitution in To Kearny by Equation and sets the stage for students to focus on the meaning of algebraic expressions in Ox Expressions.

Key Questions:
- Describe in words how you found the answer.
- How can you rewrite this sentence without reference to specific numbers?
- How can you use variables to rewrite this sentence as an algebraic expression?

To Kearny by Equation
Students begin with a formula for profit and are asked to explain why it makes sense. They then adjust the formula to represent half the total profit, a process that can lead to two equivalent expressions and create a context for bringing forth the idea of equivalence and the distributive property. Students conclude the activity with substitution and evaluation practice. Students work individually and in their groups on the task, coming together as a class to review the ideas of equivalence and the distributive property.

The Vermillion Crossing (homework)
Students are asked to explain how a new formula for profit—a linear function of the number of wagons, men, women, and children—makes sense. They conclude the activity with substitution and evaluation practice. Students will work on this activity individually, followed by a class discussion of various approaches.

Ox Expressions
Students have created algebraic expressions to describe calculation procedures. Now they will move in the other direction, interpreting algebraic expressions in terms of the concepts that the variables represent. Students explain and evaluate algebraic expressions, generate meaningful algebraic expressions, and interpret algebraic expressions using summary phrases. The equivalent meaning of the expressions developed with a given summary phrase will help students to recognize that the expressions are equivalent, offering another opportunity to
identify the distributive property at work. Students work in groups to create meaningful algebraic expressions and describe them with summary phrases. The activity concludes with sharing findings and discussing the experience of writing algebraic expressions and summary phrases.

Key Questions:
- What is a summary phrase for the expression FC?
- What does M+W+C represent in terms of the problem?
- What questions can you express using these variables?
- What is one of your expressions, and what is the summary phrase that goes with it?
- Which letters appear most frequently in the expressions?
- What makes an expression meaningful?
- Do you think you found all the meaningful expressions?

**Ox Expressions at Home (homework)**
As in Ox Expressions, students approach the use of variables from another perspective, generating meaningful algebraic expressions from a given set of variables. They see that, when used in context, combinations of variables can represent more than a sequence of numeric steps. Students also strengthen their skill with substitution into algebraic expressions. Finally, the activity offers another context for working with the distributive property. Students complete the activity on their own. After this individual work, while students share their expressions and summary phrases, the teacher can gauge their understanding to determine what to debrief and what to emphasize. The activity concludes with identification of examples of the distributive property.

Key Question:
- How could you get the total amount of water by finding the separate totals for the women, for the men, and for the children?

**If I Could See this Thing**
In this activity, students are asked to create equations to represent the relationship between future and present populations. They are encouraged to use numeric examples to clarify their use of variables. The equivalent expressions that emerge offer another opportunity to review the distributive property. Students work individually and in their groups before coming together as a class to share their thinking about converting from written problems to algebraic equations.

Key Questions:
- What did you do to come up with an equation for Question 1?
- How did you get the numeric results in Questions 2a and 2b?

**Lesson 2: The Graph Tells a Story**
**Wagon Train Sketches and Situations**
Students begin to look at graphical representations of the relationship between two quantities, developing meaningful connections between a graph and the situation it represents. They interpret and create graph sketches for given situations. They recognize the distinction between discrete and continuous graphs and identify dependent and independent variables. This exploration sets the stage for the need to meaningfully scale graphs, followed by plotting points and graphing equations.
Students will be interpreting graphs in a qualitative way, focusing on rise and fall. They will also create sketches of graphs to reflect information describing various relationships. (The activity uses the term graph sketches to describe these more qualitative graphs.) In later activities, they will add scales, plot points, and draw graphs of equations on coordinate axes.

After the teacher introduces the activity, students work in groups, coming together as a class to share observations about the graphs. The teacher will use these observations to introduce such vocabulary as constant rate, and linear, discrete, and continuous graphs. The teacher also introduces independent and dependent variables and the graphing convention for placement of these variables on the x- and y-axes, respectively.

Key Question:
- What does the “shoes or boots versus people” graph tell you?

Graph Sketches (homework)
This activity will give you a sense of how well students can interpret ideas about the relationship between two variables as communicated in a graph. The activity also sets the stage for assigning numbers to the axes of a graph.

To interpret the “story” that a graph tells, and to create a graph to represent a story, students must focus on how a relationship between two quantities can be expressed in this visual form. Along the way, they are encouraged to continue to employ the language of graphing—in particular, independent variable and dependent variable.

Following their explanation of Wagon Train Sketches and Situations, students work individually to interpret graphs and then to create their own graphs. In a class discussion, they share ideas and interpret each other’s work.

In Need of Numbers
In this activity, students are asked to add reasonable numeric scales to the axes of graph sketches from previous activities. The key idea is that each axis is to be treated as a number line, or part of one.

This activity continues the development of graphing strategies by introducing the process of quantifying graphs using scales. Students begin to understand that each axis is a number line. They read information from graphs and use this information to add scales to graph sketches. Students must justify the reasonableness of their scales. In the process, they employ basic ideas and terminology about coordinate graphing, in particular the term coordinate. Scaling the axes of a graph and reading quantitative information from graphs are conceptual sticking points for many algebra students, so the groundwork laid in this activity is particularly important for the mathematical development of the unit.

After students have shared ideas from Graph Sketches, the teacher introduces the process of scaling the axes on a graph. Students then do the activity in groups and begin to wrestle with the difficulties associated with scaling axes. The Issues Involved is an immediate follow-up activity in which issues about scaling are raised and more conventions for graphing are defined.

Key Questions:
• What would be an appropriate scale for this axis?
• Is it okay if “10 days” and “10 pounds” are represented by different lengths on the axes?
• How much coffee is left after 10 days? After 20 days? How do you find these values from the graph?

The Issues Involved (homework)
This activity raises some general issues about the scaling of graphs, in particular that scales should be consistent—even spaced—and that axes are usually assumed to begin at zero unless otherwise marked. The discussion of the activity offers opportunities to emphasize important vocabulary—such as continuous graph, discrete graph, first coordinate, second coordinate, coordinate axes, and ordered pair—and to relate graphs back to In-Out tables.

After students work individually, the teacher leads a discussion to clarify ideas, using student observations to communicate the conventions associated with scaling graphs.

Key Questions:
• What problems or questions did you have scaling the axes?
  How did you resolve them?
• According to this graph, how much does an average boy grow in his first year? In his second year?

Out Numbered
This activity will help students draw connections between what they know about reading information from graphs and what they know about In-Out tables and rules. In addition, several standard terms and conventions are introduced. The activity prepares students to reflect on how to move from rules to graphs, the focus of the next activity.

This activity will further the connections students are building between situations, graphs, tables, and rules. Students read data from scaled graphs of linear situations and find rules for the data, pausing to consider that the straightness of a graph corresponds to a constant rate of change. They also review the four-quadrant coordinate system, using the terms rectangular (or Cartesian) coordinate system and quadrant.

Students work on the activities in groups, sharing ideas about how to respond to the questions. Sometime after groups have completed Question 2, the teacher brings the class together to discuss observations, highlighting the mathematical connections students observe between representations and introducing new vocabulary.

Key Questions:
• How does the fact that 8 more people can be carried by each additional wagon affect your table?
• Can this number tell you how many people could be carried by 100 wagons?
• What would you do to calculate the number of people for w wagons?
• Why does the rule Out = 8 · In make sense in Question 1?
• How much coffee is consumed each day in Question 2?

From Rules to Graphs (homework)
This activity reverses the process of moving from graph to rule followed in Out Numbered, helping students gain fluency interpreting representations and transferring between representations.

The multiple representations of a function—through situations, tables, graphs, and equations—is an important theme that will be revisited often in this unit and throughout the IMP curriculum. Later in this unit, the activity *All Four, One* provides an opportunity for students to synthesize their understanding of the connections among these representations.

Key Questions:
- How did you interpret the instruction “sketch the graph”?
- Do you think this graph will be discrete or continuous?
- Why do some rules give linear graphs that go up to the right while others give graphs that go down to the right?
- How is the fact that the graph is not a straight line related to the table?

In this activity, students create graphs for equations that are given symbolically. In the process, they are reminded that graphs can continue into quadrants other than the first, that a graph is like a picture of an In-Out table, that linear graphs have a constant rate of change (increase, decrease, or no change), and that not every graph is linear. The vocabulary and conventions of graphing are emphasized during discussion of the activity, encouraging students to use these terms to communicate their ideas.

*POW 7: Around the Horn*
(Allow one week for students to complete outside of class)
This POW, like others, is designed to engage students in exploring a larger problem over time. Writing about the problem—discussing the process of their investigation as well as justifying their solutions—is an important component of the problem-solving activity.

Ideas associated with constant rate are in play in this activity. In addition, students often demonstrate creativity and ingenuity in employing spatial and relational skills to model the proposed situation.

Students will be introduced to the problem and will begin to explore it in class. After a few days, if time allows, let them share ideas and generate methods for further exploration at home. After approximately one week, have a few students present the POW as a concluding activity.

*You’re the Storyteller: From Rules to Equations* (homework)
This activity brings to a close the emphasis in *The Graph Tells a Story* on developing connections among the four representations of a function—rules, situations, graphs, and tables. It also continues the focus on the meaning of variables and algebraic expressions.

Students will draw upon the operations and numbers defining the relationship between two unknown quantities to create meaningful situations that given equations could define. They will also find solutions, via any method at their disposal, to make the equations true.

Key Question:
• What situation might lead to the equation 2x = 8?

Lesson 3: Traveling at a Constant Rate
Previous Travelers
Students bring together many of the graphing techniques they have developed to plot data presented in tabular form and to analyze the data to make predictions. They assign a line of best fit to graphed data and then read and interpret new points along this line in order to make predictions and to identify a rule for the line. The intention is for students to recognize the graph of a function as the set of all points that satisfy the conditions of the function.

In this activity, students will
• transform information from tabular form or verbal description to graphical form
• look for a straight line that reasonably approximates the graph
• use a table for that straight line as an aid in finding a rule for the graph
• use the graph or the rule to make predictions and estimates about the situation

Although a linear model makes sense for this situation, the data do not perfectly fit a linear equation. Therefore, students will need to estimate and use common sense. Through this activity, they will be introduced to the concept of a line of best fit. After the class does an example together, students work collaboratively to plot points, assign a line of best fit, and estimate a rule for that line.

Key Questions:
• How might you use the data about beans to plan for 20 people?
• Will this technique yield the same prediction for 20 people if applied to the data for groups of 5 people?
• How can you use all the data about beans to make a prediction?
• How would you scale the axes?
• Would you be able to use the scale you have assigned to this graph to predict the amount for your family?
• How many pounds of beans would you estimate each person will use?
• Based on the line of best fit, how many pounds of beans will each of your families need?
• Can you write an algebraic rule for your line of best fit?

Broken Promises (homework)
Students create a set of data by estimating areas. They plot the data and then interpret and make predictions from the resulting graph. That the data involve unusual spacing and very large numbers challenges students to be thoughtful about the scales they select. That the data don’t lend themselves to a linear model will encourage students to consider when and why to use a line of best fit. Students work in their groups to help one another complete the process of plotting and interpreting the data, followed by a class discussion of the nonlinearity of the data.

Key Questions:
• How did you estimate the areas?
• What are your predictions for the future of Native American land?
**Sublette’s Cutoff**
Students make predictions and estimates based on limited data. This begins the development of their awareness of the connections between rate and starting point and the equation of a linear graph. The activity also gives students their first, informal experience reasoning about a system of linear functions. Students continue to develop awareness of and flexibility with connections among situations, tables, and graphs. Though no rules are asked for per se in this activity, students will make predictions from the data, possibly employing some sort of curve-fitting technique. They also interpret graphs to answer informal, context-based questions related to intercepts, intersections, and rate.

**Who Will Make It? (homework)**
This activity gauges students’ ability to make meaningful inferences from graphs.

Students examine another situation in which the data are approximately linear. They make estimates and predictions based on graphs of the data and linear models informally fit to these graphs. Students will also consider the meaning of the starting point as well as the downward trend in the data with regard to the situation and the graph.

**Key Questions:**
- What location on the graph represents the starting point for each family?
- How is the downward trend of the data evident in the graph?

**The Basic Student Budget**
This activity extends students’ work with graphing and predicting to writing rules, setting the stage for building a contextually meaningful understanding of symbolic representations of linear functions.

In a linear function of the form \( f(x) = ax + b \), the y-intercept, \( b \), may be thought of as the starting value (when considering first-quadrant data such as in applied contexts like those in this unit), and the slope, \( a \), is the rate of change of \( y \) with respect to the change in \( x \). This activity treats these concepts and connections informally.

Students will work in groups to plot data, sketch a line of best fit, and make predictions based on the line in order to answer questions about the data. The class discussion will involve some analysis of the numbers used in the rules and their association with the situation.

**Key Questions:**
- How did you find a rule for the amount of money each person would have?
- What connections do you see between the rule and the situation?
- How can you see in the graph that Cal started with the most money? Can you see that in the rule as well?
- Who spends the most money on average per day? Can you see this in the rule? How?
- How does each student’s graph tell you who is spending the most per day?
Following Families on the Trail (homework)
In this activity, students begin to study straight-line graphs in more depth. They identify connections between the coefficient in the symbolic form of a linear function and the related graph and situation.

Students’ work shifts from fitting lines to data to working explicitly with a linear function of two variables. Students create and compare graphs involving constant rates, focusing on how the starting values and rates affect the graphs. They also create rules for constant-rate situations and examine how such rules depend on the rate and starting value.

Key Questions:
- Why do the graphs from Question 1 both rise as they go to the right while the graphs from Question 2 both fall as they go to the right?
- How did this group use the information in the situation to write this rule?
- What information did this group use to write this rule?
- Is there information in the rule that shows up in the graph? How?

Graphing Calculator In-Outs
Students employ technology to graph linear and nonlinear functions and use the graphs to answer questions. The trace feature of the calculator allows students to find In-Out pairs. The ability to use this technology is required for later investigations in the unit and throughout the IMP curriculum.

Key Questions:
- What scales have you assigned to the calculator’s display? What do you want to see?
- What sort of numbers do you want to see that aren’t in your viewing window? Are these in the x- or the y-direction?

Fort Hall Businesses (homework)
In this activity, students examine linear situations in which they need to determine the starting value and the rate. Rather than developing a formalized method to derive rules from each type of situation, the goal in this activity is developing fluency in reasoning about linear functions and their representations.

Students use various kinds of information to find equations for linear functions. In one situation, they are given the rate and one data point and need to find the starting value in order to write the rule. In another situation, they are given two data points and must determine the rate, then the starting value, and finally the rule. In each case, they examine how the rule is related to the data and see the crucial role of the assumption that the rate is constant.

Students work individually and then compare results and methods in a class discussion.

Key Questions:
- How are these situations different from those in Following Families on the Trail?
Sublette’s Cutoff Revisited
Students reexamine the situation from Sublette’s Cutoff using calculator-based curve-fitting techniques to make predictions. The emphasis is on developing the skills and techniques needed to use this technology. Students also see the graphical effects of changing the values of the linear and constant coefficients of a linear function.

Students learn how to use the calculator to plot points and then explore the results of adjusting linear functions to improve their fit.

Key Questions:
- What might be a good function for these data points?

The Basic Student Budget Revisited (homework)
Students practice the calculator method for curve fitting. The class discussion explores the effects on the graph of changing the coefficients of the linear function.

Students use the graphing calculator to fit a line to data and then use the line to make predictions. In the activity, students must use a negative linear term, which they connect to the context of the situation. They also identify the effects on the graph of \( ax + b \) as \( a \) and \( b \) take on different values.

Students work individually and then come together to share results and to make connections between the symbolic form of the function, the situation, and the graph.

Key Questions:
- What is different about the lines in this graph compared to the lines in Sublette’s Cutoff?
- What did you have to do in the equations to make the graphs turn downward?

POW 8: On Your Own
(Allow one week for students to complete outside of class)
Students work on a research, planning, and budgeting problem involving thinking about their own future. This helps them to put mathematics to practical use and to develop their organizational skills.

To engage in budgeting and forecasting, the mathematical aspects of this activity, students must gather information and make decisions based upon those data.

Students are introduced to the activity and encouraged to begin to gather information immediately. Some class time midway through the project for students to share information and resources can be quite valuable. The project concludes with students sharing their lifestyle plans and budgets.

All Four, One (homework)
Students examine the connections among graphical, tabular, and symbolic representations of situations.

Travel on the Trail
Students consider how changes in starting value and rate affect graphs and rules.
This activity requires students to develop algebraic rules for given situations. They will use graphs or other representations of the resulting linear functions to answer questions about rate and solutions to a system of two linear equations.

Students will work collaboratively in their groups to put to use the connections they have made among situations, tables, graphs, and rules. Some class discussion will follow, primarily to emphasize the role of rate and starting value in each type of representation of the situations.

Key Questions:
- What clues do the instructions in Question 1a give you about the labels for the x- and y-axes?
- How does the 12 show up on the graph? In the rule? Why?
- How does the 23 show up on the graph? In the rule? Why?

*Moving Along* (homework)
This activity continues to engage students in solving problems involving four representations of linear functions. They are specifically asked to write equations and to identify rates and starting values, with the goal of strengthening connections between the situation and the rule for linear functions.

Students continue to draw upon their understanding of the relationships among the four views of a function in order to develop equations for situations in which two data pairs are provided. The roles of rate and starting value are emphasized, setting the stage for formalization of the standard form of a linear function.

After students work individually on the activity, they come together as a class to discuss methods for determining the rate and starting value and for writing the equation.

*All Four, One—Linear Functions*
In this summative activity, students describe, in writing, methods for moving from one representation of a linear function to another.

This activity draws upon and strengthens the connections among the four representations of a function, with a particular focus on linear functions. Students are encouraged to work from rate of change and starting value in identifying these connections. They learn that *linear* has an algebraic as well as a geometric meaning and are introduced to a common form of the linear function, \( y = ax + b \).

Students work on this activity in groups, perhaps submitting a first draft submitted for feedback, with a final draft due at a later date.

Key Questions:
- How can you tell from looking at a rule whether its graph is a straight line?
- Why do situations with constant rates have rules of this special type?
- Do you remember an activity from this unit in which you had to create an In-Out table from a graph?

*Straight-Line Reflections* (homework)
This activity introduces the idea of equivalent forms of linear functions by drawing on students’ ability to move among the various representations of a function.
Students draw upon their understanding of the four representations of linear functions to informally become acquainted with the notion of equivalent equations. In particular, they graph an equation and then use what they have learned to determine a linear equation of the form $y = ax + b$ from the graph. Students will then apply the distributive property as well as other basic symbol manipulation to demonstrate the equivalence of the two equations.

Students work individually on the activity and then debrief their work as a class, making observations and conjectures about various symbolic forms of equivalent linear equations.

Key Question:
- What did you interpret the word equivalent to mean in Questions 3b and 4b? Where have you heard the term before?

Lesson 4: Reaching the Unknown
Fair Shares on Chores
In this activity, students write equations, express them in equivalent form, and graph them. Two important concepts are reinforced in the process: abstracting a problem to an algebraic equation and reading numeric information from a graph. Students convert a linear equation into $Y= \text{form}$, conducive to use on the graphing calculator. The activity sets the stage for students to make meaning of graphical solutions to systems of equations.

Students examine possible solutions to a linear equation that involves two variables and explore how to express that equation by giving one variable in terms of the other. They also solve equations for one variable in terms of the other, based on reasoning in the problem context. Finally, they graph linear conditions on the calculator and use this tool to find solutions to an equation.

Students will work on the activity in groups and then share their results with the class. This activity sets the stage for lending graphical meaning to solving systems of equations. Students will revisit this situation in More Fair Share on Chores.

Key Questions:
- If each girl’s shift was four hours, how would you find the length of each boy’s shift?
- How can you check whether a number pair fits the condition?
- Why might you want to express one variable in terms of the other?

Fair Share for Hired Hands (homework)
Students examine possible solutions to a linear equation in two variables and explore how to express that equation as one variable in terms of the other. The activity focuses on the connection between an equation and its graph to further set the stage for considering the graphical meaning of the solution to a system of equations.

Students continue work with all four representations of linear functions. Most importantly, they write a rule in $Y= \text{form}$ and consider the meaning of specific points on the line.

Students use a context to derive a set of ordered pair solutions, plot these pairs, and determine an equation for the linear relationship. Again, students are asked to
convert the equation into $Y = \text{form}$. Finally, a class discussion emphasizes the meaning of the points along the graph of the line. This sets the stage for the next activities, in which students consider the meaning of a point of intersection.

Key Questions:
- How did you get $Y$ from $X$?
- What is the relationship between fitting the equation and being on the graph?
- Is every solution to the problem on the graph?
- Does every point on the graph represent a solution to the problem?

More Fair Share on Chores
Students examine how to use graphs to find the solution that fits two linear conditions.

Students write and graph an equation representing a new linear condition for the situation first encountered in Fair Share on Chores. Next, they are asked to find a solution that fits both this new condition and the original condition; that is, they are asked to find the solution to a system of two linear equations in two unknowns (although that formal mathematical terminology is not used).

Students will work on the activity in groups and then share their solution methods with the class.

Key Questions:
- How did you represent the first condition graphically?
- How did you represent the second condition graphically?
- What points fit both conditions?

More Fair Share for Hired Hands (homework)
This activity, a follow-up to More Fair Share on Chores that builds on students’ work on Fair Share for Hired Hands, will provide information on students’ understanding of the connection between graphs and equations.

Students write and graph an equation for a linear situation. Next they determine a solution to a pair of conditions represented by linear equations and are asked to find the solution to a system of two linear equations in two unknowns.

Students will work on this activity individually and then share their techniques as a class. The activity concludes with a discussion of graphical methods that emphasizes that every point on a line fits the equation for that line.

Key Questions:
- What equation represents the information in Question 3?
- How can you rewrite the equation $3X + 4Y = 30$ so you can graph it on the calculator?
- How would you find the rate of pay for each more-experienced hand if you knew the rate for each less-experienced hand?
- How might you convince someone that the coordinates of the point where these two lines intersect tells where both conditions in this problem are satisfied?
**Water Conservation**
This activity gives students more experience in writing rules for situations and in finding the common solution to a pair of equations.

Students continue their exploration of linear functions. In particular, they examine ways to find the input for which two linear functions give the same output.

Students work on the activity in groups and then discuss their work as a class. The equation developed by the class for Question 5 will be returned to later in the unit when students have the symbol-manipulation tools necessary to solve it algebraically.

**Key Question:**
- How can you use your expressions from Questions 1 and 2 to get an equation for finding the day when the families have equal amounts of water?

**The Big Buy** (homework)
This activity continues, in a present-day context, students’ work writing equations to represent linear conditions and then graphing to solve for given sets of conditions.

Students continue their work with linear functions and use a pair of graphs to compare different rates of pay. In particular, they graph to find the input for which two linear functions give the same output. The concept of the point of intersection plays an important role in this problem. The activity concludes with students writing an equation whose solution would solve the problem. Although students continue to solve simple systems of equations in this activity, no formal work is done to name this process or to reduce it to a rote procedure.

Students work individually to develop and graph two equations for a given situation and then to answer questions about the situation. They then share ideas in their groups and also write an equation that determines the x-coordinate of the point of intersection of the two lines.

**Key Questions:**
- What equation would tell us how many hours of work it would take for Jillian and Max to earn an equal amount of money?

**Getting the Gold** (homework)
In this activity, students extend their work with linear functions beyond the first quadrant, giving meaning to the x-intercept of a graph. This activity also generates one more equation obtained by setting two linear expressions equal, setting the stage for the development of methods for solving simple linear equations.

Students continue their work with representing situations by equations. In this case, they consider the starting values and rates related to the profit a miner might make using two different methods for collecting gold. They compare the associated linear functions both graphically and symbolically, considering questions that can be answered by the x-intercepts and the point of intersection or by setting up an equation for finding when the two functions are equal.

Students work individually to explore two methods for collecting gold and the profit associated with each, using graphs to answer questions about break-even and
equal-profit points. In a class debriefing, they develop and solve linear equations to answer the same questions.

The Mystery Bags Game
This activity introduces a model for thinking about solving simple linear equations through symbol manipulation. Students summarize the model’s procedure and begin to convert it to an algebraic method for solving simple linear equations.

Students learn a mathematical model for thinking about solving linear equations. The features of the pan-balance model correspond to the mathematical steps of solving a linear equation.

The weight of a mystery bag corresponds to the unknown. The number of mystery bags corresponds to the coefficient of the unknown. The weights correspond to constants. Removing bags and weights corresponds to subtracting from both sides of the equation or to dividing both sides of the equation by the same amount.

Students read the introduction and do one or two examples as a class before working in small groups. The teacher introduces an algebraic representation of the game, with some student practice. Next, students describe their general strategy in words and convert this method into a symbolic representation of the steps necessary to solve the equations.

In More Mystery Bags, students will practice these techniques, continuing to emphasize the model to justify the steps of solving linear equations. They will also return to equations generated and solved graphically in recent activities in order to solve them symbolically.

Key Questions:
• How can you represent Question 3 algebraically?
• How could you record algebraically the arithmetic steps you followed to solve for the weight of a mystery bag?

More Mystery Bags (homework)
This activity offers students more experience with the pan-balance model and encourages them to extend this reasoning toward a more general strategy for solving equations.

Students continue work with simple linear equations using the pan-balance method. The method is then generalized into more formal algebraic principles for solving equations. The concept of equivalent equations is also developed.

Students work individually with the pan-balance model before returning to equations identified in previous activities to solve them symbolically and confirm their solutions. Then, in a class discussion, students are introduced to a more formal consideration of equivalent equations as the broad idea that governs techniques for solving equations.

Key Questions:
• What does that step mean in terms of the situation?
• How can you use the pan-balance method or a similar technique to solve these equations algebraically?
• What do you think the term equivalent expressions means?
• Why is it okay to make this change in the equation?
• What did you do with the mystery-bag problems to get equivalent equations?
• What are some things we can do to an equation to get a different but equivalent equation?

Scrambling Equations
This activity will strengthen students’ understanding of equivalent equations and some of the symbol-manipulation processes that maintain this equivalence.

Students create linear equations by “scrambling” an equation of the form \( x = c \), using the rules developed in class for maintaining equivalent equations. Students take a simple equation, such as \( x = 5 \), and write a series of more complex equations in which each is equivalent to the preceding one. The focus thus shifts from simplifying a complex equation to “complexifying” a simple one, emphasizing the “doing” and “undoing” aspects of algebraic thinking.

Students work in groups to turn simple equations into more complex, equivalent equations and then try to retrace one another’s steps by “unscrambling” their equations. This work is followed up in More Scrambled Equations and Mystery Bags.

More Scrambled Equations and Mystery Bags (homework)
This activity further develops students’ abilities to simplify and solve equations. At the conclusion of this activity, students will have two related ways to think about solving equations. The pan-balance model provides a more concrete metaphor for manipulating symbols while the set of rules developed to maintain equivalent equations is useful in more varied situations.

The algebraic thinking developed during this unit emphasizes reasoning, multiple representations, and the connections among these representations. By this point in the unit, students will have solved a variety of equations, using both the pan-balance model and some general principles for symbol manipulation that maintain equivalent equations. These symbol-manipulation methods should be thought of as complementing other techniques students have for solving equations, including using a graph, a table, estimation, and other reasoning processes.

After students work on their own to use the rules of equivalent equations and the pan-balance model, they come together to share results and ask questions of one another. The teacher emphasizes the “doing” and “undoing” nature of these problems, some of which may remain unsolved by some students at this time.

Family Comparisons by Algebra
In this activity, students reexamine the questions raised in Following Families on the Trail and apply their new understanding of equations and equation-solving techniques to those questions. This activity concludes the part of the unit devoted to solving equations algebraically and emphasizing the connection to other types of solution methods. It will help you evaluate students’ ability to represent situations using linear equations as well as their facility with solving them.

Students create equations that were originally derived in Following Families on the Trail and solve them using the algebraic methods they have been developing. Each
situation had previously been modeled by two linear functions and solved by determining the coordinates of the point of intersection of their graphs.

Students work on the activity in small groups. Subsequent class discussion reemphasizes important ideas from the past several activities and helps students to confirm their understanding of methods for solving simple linear equations.

Key Questions:
- What two things are you trying to set equal?
- What arithmetic do you have to do to figure the Buck family’s distance from Green River after 5 days?
- In what ways does your solution to this algebraic equation relate to the graphs you built in Following Families on the Trail?

*Starting Over in California* (homework)
This activity provides some challenging problems in a linear context for students to solve in any manner that makes sense to them. The work will refresh students’ ideas about much of the unit’s mathematics and help prepare them for the portfolio and unit assessments.

The first two questions provide information about a linear situation. Students will need to apply the tools developed throughout the unit to solve them, including some or all of these important ideas: rate of change, starting value, In-Out tables, graphs, rules, and algebraic solution methods. Students are then asked to develop a situation and problem of their own.

Students will work individually to analyze and make predictions about linear situations and to write their own problems. When they come together to share their work, they will have the opportunity to see a variety of approaches to each problem and to share their own—and solve another’s—problem.
Resources and Teaching Tips

- Interactive Mathematics Program site: http://imp.moodle.its-about-time.com/
  Technology activities available for download from IMP site:
  - Family_Constraints.doc: Family Constraints (p. 186): This handout is for a follow-up activity in which students use Excel to explore the combinations of ages that add to 90. As Excel formulas are always written in terms of the output, this activity allows students to explore how these \( y = \) equations relate to those in the book that are sums equal to 90.
  - Oexcel_Expressions.doc: Ox Expressions (p. 198): This handout is for a follow-up activity in which students use Excel to test the effects of different values for their variables. They explore the representation of variables as cells in spreadsheets. By assigning new values to those cells, the results are automatically updated in their expressions. Students should also realize the limits of Excel with respect to determining how meaningful their expressions are.
  - In_Need_of_Numbers.doc: In Need of Numbers (p. 208): This handout is for a Fathom replacement activity in which students explore scales of axes by creating data sets that reproduce given graphs. The activity assumes some familiarity with Fathom and lets students see how changing individual data points change the graph and it’s scale. For some graphs, it could allow for the alternative of entering a formula.
  - Out_Numbered.doc Out Numbered (p. 211): This handout is for a Fathom replacement activity that uses a prepared Fathom document containing graphs of data. Students first use the Plot Function tool to experiment with equations until they match the data in the graph, after which they construct their own tables and graphs to match the original graphs.
  - Out_Numbered.ftm: Out Numbered (p. 211): This is the Fathom document required for the last activity.
  - Previous_Travelers.doc: Previous Travelers (p. 217): This handout is for a Fathom replacement activity that directs students to find the line of best. By having to adjust the moveable line differently for rotation and translation, students begin to isolate those components in ways they don’t with a by-hand sketch or adjustment of a piece of spaghetti. Since Fathom displays the equation of the movable line, you might ask students if they see any patterns in how the equation changes as they move the line around.
  - Following_Families.doc: Following Families on the Trail (p. 224): This handout leads students to use Fathom to compare graphs and interpret the point of intersection. It is a supplemental activity that gives student more opportunity to work with systems of equations. The main benefit of Fathom in this activity is in adjusting scales. Students can see lots of different scales more quickly than they can be hand.
  - On_Your_Own.doc: On Your Own (p. 231): This replacement activity for POW 8 has students use Excel to create their budget and learn some practical applications of spreadsheets: easy modification (adding and deleting line items), immediate updating of calculations when input values are changed, and the use of formatting to present data in a neat and well-organized way. Students can go further and use Excel to create graphs, which can be used for POW presentations.
o Fair_Share.doc: *Fair Share for Hired Hands* (p. 243): This handout is a follow-up activity in which students use Excel to investigate the relationship between pay rates. The activity introduces students to transforming equations and creating equations in \( y = \) form. Using these equations in Excel provides students with another reason for learning to use this equation form and another outlet for applying it.

o Pay_Rate_Comparison.xls: *Fair Share for Hired Hands* (p. 243): This prepared Excel spreadsheet contains the end result of the previous activity. It can be used as a demonstration.

o Balancing.gsp *The Mystery Bag Game* (p. 252): This prepared Sketchpad model of a pan balance can be used by students or as a demonstration of solving equations. In addition to weights, there are balloons that pull up on the pan, allowing for the inclusion of negative values.

o High-Low_Differences.doc: *High-Low Differences* (p.282): This handout tells students to use Excel as a tool for analyzing this supplemental activity. It gives some hints, but little direct instruction. Students are encouraged to use Excel to generate a lot of data, to help get at the patterns beneath the surface, and to eventually generalize the patterns in high-low differences to algebraic equations. The activity draws attention to place value and how it can be incorporated into writing expressions from digits.

o Three_Digit_Differences.xls *High-Low Differences* (p.282): This prepared Excel spreadsheet asks for three digits and then performs the high-low difference until the pattern repeats.

- Fathom software
- Geometer’s Sketchpad software
- Graphing calculators
- Graph paper, rulers, colored pencils
- Online graphing calculator: [http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html](http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html)

**Differentiation**

- Reinforcement Opportunities:
  o *Pick Any Answer* (reinforcement) This activity, which brings out the importance of the sequence in which arithmetic operations are done, provides a simple context for which students can create an algebraic expression. It might be an appropriate activity to use in connection with students’ work with variables.
  o *Substitute, Substitute* (reinforcement) This activity provides examples through which students can strengthen their understanding of the basic ideas of substitution. The activity uses a variety of phrases for referring to the process. One question uses substitution to reinforce the idea of combining terms.
  o *Classroom Expressions* (reinforcement) Students create summary phrases and algebraic expressions using a set of variables that relate to a classroom setting. The activity also introduces the notation of subscripts.
  o *Variables of Your Own* (reinforcement) Students make up a set of variables, and write algebraic expressions and summary phrases, in a context of their own.
- **Spilling the Beans** (reinforcement or extension) An essential part of this activity is the need to clearly understand any assumptions one makes. The activity also involves proportional reasoning, which plays an important role in the last unit of Year 1, *Shadows*.
- **More Graph Sketches** (reinforcement) This activity provides a variety of contexts for which students can create graph sketches like those in *Graph Sketches*.
- **What We Needed** (reinforcement) This activity asks students to figure out how long it took for their families to travel from Ft. Laramie to Ft. Hall and how much of two commodities they would need to bring with them.
- **Mystery Graph** (reinforcement) In this graph-interpretation activity, students are given the graph of a nonlinear function, but not its equation, and are asked to find a number of values for the function.
- **POW: High Low Differences** This activity is an additional open-ended POW in which students investigate sequences of calculations and look for patterns.
- **Keeping Track, A Special Show, and Keeping Track of Sugar** (reinforcement)
- These activities offer additional opportunities for students to work with situations involving constant rates of change. All three involve constructing and using equations given two data points. You might use them early in *Traveling at a Constant Rate* if students are having difficulty creating equations to describe such situations.

- **Extension Opportunities:**
  - **From Numbers to Symbols and Back Again** (extension) This activity uses formulas from two settings from earlier units (*The Game of Pig* and *Patterns*) as the context for work with substitution. Students will have to guess and test solutions to Questions 1c and 2b, as the equations are quadratic. Furthermore, the solution to Question 1c is not integral (or even rational), so students will have to determine a reasonable approximation.
  - **Integers the General Cube** (extension) In this activity, students create equations to describe a geometric situation.
  - **Integers Only** (extension) The activity *If I Could See This Thing* involves a function with outputs that should only be integers. *Integers Only introduces* the greatest integer function. Interested students might try to use this function to devise a formula for Question 2c of *If I Could See This Thing*.
  - **More Bales of Hay** (extension) This activity makes a good follow-up to presentations of *POW 6: The Haybaler Problem*. Experimentation with numbers is the essence of this activity. In particular, students are asked to consider whether such problems always have unique answers and in which cases the answers are whole numbers.
  - **Movin’ West** (extension) This activity deals with questions about migration patterns in the United States and whether one can expect such patterns to continue, so it is a suitable thematic follow-up to *Broken Promises*. In addition to developing a general algebraic formula for a rate-of-change situation, students must think about comparative rates of change and do some commonsense reasoning to decide how well the model suggested by their formula might work in the future.
o High Low Proofs (extension) This activity asks students to prove conjectures they made in the supplemental POW: High Low Differences explorations.

o The Growth of Westville (extension) The work early in Traveling at a Constant

o Rate focuses on situations involving constant rates of change and that led to linear graphs. This activity provides a western setting for examining situations that may appear to involve constant growth but do not lead to linear graphs, so this is a good follow-up to the series of activities.

o Westville Formulas (extension) This activity is a follow-up to the supplemental activity The Growth of Westville.

o The Perils of Pauline (extension) This is a well-known but challenging puzzle problem. Students are given information about the speed of an oncoming train and the position of a person in a tunnel that the train is approaching and are asked to determine the person’s speed given that she made it out of the tunnel on time.

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### Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.

- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.

- **21st Century Learning** – the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow’s workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

**Universal Design:** Throughout the unit, students are expected to approach algebraic concepts through visual, written and oral formats. Students are given situations in verbal or written formats, and expected to translate these situations into graphs and equations. Conversely, students are expected to “bring life” to tables and graphs by creating a situation that would create the given table or graph. The use of graphing calculators is integral to the work in this unit.

**21st Century Learning:** This curriculum focuses very strongly upon the use of 21st century skills in mathematics. Students engage in a problem-based curriculum, and are expected to think critically about the problems presented, and to develop strategies to solve these problems. Throughout the problem-solving process, students must be able to build on previous knowledge to build new information and continue to make decisions based on the mathematics at hand. This process is done largely in cooperative groupings. Students have access to technology, and are given multiple opportunities to extend their work into new situations to create generalizations.
• **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century (SETDA, 2003).

Students engage daily with graphing calculators as a tool to solve problems. The use of technology allows students to experiment with best-fit lines on their own, as well as pinpoint intersections of lines easily. Students must learn how to use these features in order to develop fluency in algebra. Further use of graphing and other available software helps students continue to make sense of algebraic concepts.

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**Content Connections**

Content Standards integrated within instructional strategies

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Analyzing multiple representations of data and using lines of best fit to approximate relationships is highly applicable to earth science content.

Sample Activity:

NGSS Related Earth Science Standards:
HS-ESS3-1 Construct an explanation based on evidence for how the availability of natural resources, occurrence of natural hazards, and changes in climate have influenced human activity

HS-ESS3-2 Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.

HS-ESS3-3 Create a computational simulation to illustrate the relationships among management of natural resources, the sustainability of human populations, and biodiversity.

HS-ESS3-5 Analyze geoscience data and the results from global climate models to make an evidence-based forecast of the current rate of global or regional climate change and associated future impacts to Earth systems.

HS-ESS3-6 Use a computational representation to illustrate the relationships among Earth systems and how those relationships are being modified due to human activity.

NGSS Related Engineering, Technology and Applications of Science Standards:
HS-ESS3-1. Construct an explanation based on evidence for how the availability of natural resources, occurrence of natural hazards, and changes in climate have influenced human activity.
HS-ESS3-2. Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.*

HS-ESS3-3. Create a computational simulation to illustrate the relationships among management of natural resources, the sustainability of human populations, and biodiversity.

HS-ESS3-4. Evaluate or refine a technological solution that reduces impacts of human activities on natural systems.*

HS-ESS3-5. Analyze geoscience data and the results from global climate models to make an evidence-based forecast of the current rate of global or regional climate change and associated future impacts to Earth systems.

HS-ESS3-6. Use a computational representation to illustrate the relationships among Earth systems and how those relationships are being modified due to human activity.

Social Studies: This unit looks at the mid-nineteenth century western migration across the United States in terms of the many linear relationships involved. These relationships grow out of the study of planning what to take on the 2,400-mile trek, estimating the cost of the move, and studying rates of consumption and of travel.
Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

<table>
<thead>
<tr>
<th>Unit Title:</th>
<th>Patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designed by:</td>
<td>Michelle Hawley, Innovative Schools</td>
</tr>
<tr>
<td></td>
<td>from <em>Integrated Mathematics Program, Year 1</em></td>
</tr>
<tr>
<td>Content Area:</td>
<td>Mathematics (Algebra)</td>
</tr>
<tr>
<td>Grade Level(s):</td>
<td>9</td>
</tr>
</tbody>
</table>

**Summary of Unit**

The primary focus of this unit is understanding and representing functions. The work begins with informal problem solving methods to make sense of functions. Students are exposed to a variety of problems that elicit deeper understanding of the essential concepts of functions, especially that of 1:1 correspondence. Within every lesson, students create tables, expressions, and give verbal explanations of the information provided in a problem. One of the major tools introduced in this unit is the In-Out table, which helps students make sense of the relationship represented by the given information. Through this lens students begin to understand that different expressions can be equivalent, meaning that they yield the same outputs. Through this all, the unit emphasizes that algebraic notation is born out of the need for a mathematical shorthand, one which can simplify work with patterns.

**Stage 1 – Desired Results**

What students will know, do, and understand

**Delaware Content Standards**

- Interpret expressions that represent a quantity in terms of its context. **CC.A-SSE.1**
- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \). **CC.F-IF.1**
- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \) for \( n \geq 1 \). **CC.F-IF.3**
- Write a function that describes a relationship between two quantities. **CC.F-BF.1**
- Determine an explicit expression, a recursive process, or steps for calculation from a context. **CC.F-BF.1a**
• Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. CC.F-BF.2
• Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. CC.G-CO.1

**Big Idea(s)**
• Finding, analyzing, and generalizing geometric and numeric patterns
• Analyzing and creating In-Out tables as a problem solving strategy
• Using variables in many ways, including to express generalizations
• Developing and using general principles for working with variables, including the distributive property
• Order of operations
• Understanding positive and negative integers
• Applying algebraic ideas, including In-Out tables, in geometric settings
• Developing proofs concerning consecutive sums and other topics

**Unit Enduring Understanding(s)**
- Notational systems are used to express complex ideas in a compact form.
- Patterns can be used to make sense of the world around us, and can be represented in various forms.
- Functions help us understand relationships.
- There are a few problem-solving strategies that can be applied to a variety of problem solving situations.
- Real-world problems are not “neat” – sometimes they are rather ambiguous.

**Unit Essential Questions(s)**
• How do variables and algebraic expressions help us to represent concrete situations, generalize results, and describe functions?
• How does using different representations of functions—symbolic, graphical, situational, and numerical—build our understanding of the connections between these representations?
• Why do we use function notation?
• How can we model and compute with signed numbers?
• When is trial and error a useful problem solving strategy?

**Knowledge and Skills**
- Needed to meet Content Standards addressed in Stage 3 and assessed in Stage 2

**Students will know...**
• Order of operations
• In-out tables as a strategy for creating expressions
• Variables are used to make generalizations
• Specific problem-solving strategies
• Strategies for finding patterns
• Algebraic concepts can help us make sense of geometry
• Calculators are a useful tool for increasing efficiency
Key Vocabulary:
- Function
- Proof
- Input
- Output
- Conclusion
- Justify
- Solution
- Unique
- Range
- Whole numbers
- Integers
- Natural Numbers
- Order of Operations
- Conjectures
- Counterexamples
- Summation notation
- Exponent
- Square root
- Mathematical model
- Variable
- Expression
- Coefficient
- Term
- Constant term
- Solution
- Strategy
- Equivalent expressions
- Square
- Linear Function

**Students will be able to...**
- Create in-out tables
- Analyze in-out tables
- Create expressions
- Apply the order of operations, including the use of integers
- Apply appropriate problem-solving strategies
- Use a graphing calculator

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**Stage 2 – Assessment Evidence**
Evidence that will be collected to determine whether or not Desired Results are achieved

**Suggested Performance/Transfer Task(s)**
http://www.illustrativemathematics.org/illustrations/241
Kimi and Jordan are each working during the summer to earn money in addition to their weekly allowance. Kimi earns $9 per hour at her job, and her allowance is $8 per week. Jordan earns $7.50 per hour, and his allowance is $16 per week.

a. Jordan wonders who will have more income in a week if they both work the same number of hours. Kimi says, "It depends." Explain what she means.
b. Is there a number of hours worked for which they will have the same income? If so, find that number of hours. If not, why not?
c. What would happen to your answer to part (b) if Kimi were to get a raise in her hourly rate? Explain.
d. What would happen to your answer to part (b) if Jordan were no longer to get an allowance? Explain.

**Rubric(s)**

<table>
<thead>
<tr>
<th>Response</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Student is able to represent the two sets of earnings: K=9h + 8</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Student determines that the critical point in the system is at $5\frac{1}{3}$ hours (5 hours and 20 minutes), where they both earn $56. Student response need not specifically state the earned dollar amount.</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Student explains that, if they both work less than $5\frac{1}{3}$ hours, Jordan makes more money; working more than $5\frac{1}{3}$ hours means Kimi makes more money. (1 point each)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Student shows his or her work for obtaining $5\frac{1}{3}$ hours as a response.</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Student provides mathematical reasoning for the response (table, graph, equations)</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Student explanation is clear</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Student recognizes the intersection as the point at which Kimi and Jordan earn the same amount of money.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Student solves the system of equations to</td>
<td>1</td>
</tr>
</tbody>
</table>
determine that the intersection is $5\frac{1}{3}$ hours. Student may use work completed in part a as justification for this response.

<table>
<thead>
<tr>
<th>c.</th>
<th>Student explanation includes the following concepts:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>o Her slope increases (1 point)</td>
</tr>
<tr>
<td></td>
<td>o Her y-intercept will not change (1 point)</td>
</tr>
<tr>
<td></td>
<td>o The intersection of the two lines will move left on the graph, implying that the new intersection point occurs at some point less than $5\frac{1}{3}$ hours (2 points)</td>
</tr>
</tbody>
</table>

Student explanation is clear /2

<table>
<thead>
<tr>
<th>d.</th>
<th>Student explanation includes the following concepts (1 point each):</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>o Jordan’s slope does not change (1 point)</td>
</tr>
<tr>
<td></td>
<td>o Jordan’s y-intercept moves to zero (1 point)</td>
</tr>
<tr>
<td></td>
<td>o They will never have the same totals - there will no longer be an intersection for these lines in the first quadrant (1 point)</td>
</tr>
</tbody>
</table>

Student explanation is clear /2

<table>
<thead>
<tr>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>/27</td>
</tr>
</tbody>
</table>

Meets expectations: 23-27 points
Nearing expectation: 19-23 points
Below expectation: less than 19 points

Other Evidence
- Class discussions
- Class work
- Homework
- Problem of the weeks
- Student math journals
- Collaborative discussions
- Exit Tickets
- Curriculum-Based assessment opportunities:
• **Presentations on Calculator Exploration:** These presentations will give you information on how comfortable students are with calculators and open-ended investigation.

• **Pulling Out Rules:** This activity will help you gauge how well students understand the basic ideas of In-Out tables and evaluate their ability in writing rules to describe tables.

• **You’re the Chef:** This summary activity will tell you how well students understand the arithmetic of positive and negative integers.

• **Presentations on Consecutive Sums:** These presentations will indicate how students are developing in their ability to conduct independent mathematical investigations.

• **An Angular Summary:** This activity will help you gauge students’ understanding of the sum of the angles in a polygon and related formulas.

• **Border Varieties:** This activity will reflect students’ understanding of the use of variables.

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**Student Self-Assessment and Reflection**

- Students will keep math portfolios where they respond to problem of the weeks and other mathematical prompts
- Students will work in an interactive classroom environment, where collaboration, discussion and feedback are everyday occurrences.

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**Stage 3 – Learning Plan**

(Design learning activities to align with Stage 1 and Stage 2 expectations)

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**Key learning events needed to achieve unit goals**

**Lesson 1: The Importance of Patterns**

*What’s Next?*

This activity introduces the mathematical idea of a sequence. Students are asked to find patterns that fit a given sequence and then to use these patterns to predict the next few terms of the sequence. The search for patterns is a recurring theme in this unit and throughout the IMP curriculum. These early activities also build a foundation for the concept of function, one of the truly big ideas of algebra. Students will work on this activity in a small group of peers. They will be encouraged to be creative in describing their patterns and to share ideas with group members. The activity concludes with students discussing some of the patterns they identified.

*Who’s Who (homework)*

This activity presents interlocking sets of conditions. Using these conditions, students must identify “who’s who” and are asked to provide a convincing argument—a proof—of their conclusion. Issues of proof arise repeatedly throughout the curriculum and in daily interactions. Most significantly, students are always expected to justify their solutions, to convince others, and to be convinced by others. The reasoning students will use to analyze the stated conditions, to make conjectures about the solution, to test those conjectures to convince themselves
that their solution meets the stated conditions, and then to determine whether their solution is unique—that is, to prove their solution—is at the heart of what it means to do mathematics.

**Inside Out**

This activity introduces students to a powerful representation of functions, In-Out tables, which will be used repeatedly throughout the IMP curriculum. The In-Out machine metaphor is used to introduce In-Out tables and the terms input and output. In their exploration, students experiment, make conjectures, and work toward getting clear verbal statements of the rules. A powerful way to think about a function is as a machine. The thing put in is called the input (the In, for short); the thing that comes out is called the output (the Out). An In-Out table is one representation of a function. Other representations include graphs and symbolic rules. (Students will study these representations later in *The Overland Trail.* ) We often use the word function in such phrases as “the Out is a function of the In,” which means that the Out value depends on, or is determined by, the In value.

**Calculator Exploration**

Given the versatility and power of the graphing calculator, and the wide variety of prior experiences students are likely to bring to this open-ended activity, students’ explorations will probably range widely. This activity gives students more opportunities to find and express rules for In-Out tables, both in words and symbolically, and to use an In-Out table as a problem-solving tool. However, there are some important mathematical issues students will encounter:

- **How to handle order of operations on a calculator:**
  - Calculators will evaluate such expressions as $17 - 6 ÷ 3 + 4 \times 92$, without the need for parentheses, by doing the exponent first, then multiplication and division, and finally addition and subtraction.

- **How to use the built-in mathematical functions:**
  - To find the square root of 3 you must access the square root function before the number 3, but to find 5! you must access the factorial function after the number 5.
  - $\sin(30)$ will not be 0.5 unless the mode is set to degrees.

- **The graphing capabilities of the calculator.**

**Lonesome Llama**

Students will, within certain constraints, be trying to identify the unique card in a stack of 46 cards. The characteristics that distinguish the cards are mathematical—such as the number, type, and orientation of geometric figures—so students will be communicating about mathematics. The mathematical goals of this activity are for students to develop ways to describe the distinctive features within a set of diagrams that are largely alike and to develop a procedure for sorting the diagrams by those features.

After students, working in small groups, have found the singleton card or made sufficient progress, a whole-class discussion can focus on how they worked and what they discovered.

**Role Reflections**
Upon completion of *Lonesome Llama* (or as a filler while groups finish *Lonesome Llama*), students should privately reflect on the activity to identify actions taken by a student engaged in the various roles. A class discussion of responses helps to highlight the value of the variety of contributions that class members made to the successful completion of the activities.

**Pulling Out Rules** (homework)

This activity gives students more opportunities to find and express rules for In-Out tables, both in words and symbolically, and to use the In-Out table as a problem-solving tool. Students are first asked to find rules for In-Out tables that each contain four pairs of numbers. Then they are asked to generate many possible rules that fit tables with only one or two pairs of numbers. Finally, they are presented with a problem for which an In-Out table is a helpful solution tool. Students work on the following skills:

- Finding rules for In-Out tables.
- Developing symbol sense. For example, “The *Out* is 2 times the *In*, then add 3,” \( \text{Out} = 2 \text{In} + 3 \), \( \text{Out} = 2x + 3 \), and \( y = 2x + 3 \) are equivalent and increasingly abstract ways to use symbols to summarize the rule for Question 1a.
- Confronting the idea that the number of data points in a table affects the number of rules that will explain all the data.
- Using a function as a mathematical model of a quantitative situation, and then using the model to solve a problem related to that situation.
- Using the terms variable, algebraic expression, coefficient, and constant term.

Students are first asked to find rules for In-Out tables that each contain four pairs of numbers. Then they are asked to generate many possible rules that fit tables with only one or two pairs of numbers. Finally, they are presented with a problem for which an In-Out table is a helpful solution tool.

**Lesson 2: Communicating About Mathematics**

*Marcella’s Bagels*

*Marcella’s Bagels* gives students an opportunity to use a variety of problem-solving strategies. They might guess at the original number of bagels, examine what happens when they work through the steps in the problem, and then revise their initial guesses accordingly. The problem also lends itself to the powerful strategy of working backward. Thinking of the story as a movie, students can begin with the number of bagels Marcella has at the end and “run the movie backward,” undoing each action she took and arriving at the number of bagels she had at the start. At each step, Marcella gives away half of her bagels plus 2, so in reverse she would add 2 and then double the total.

**Key Questions:**

- How did you find the answer?
- How do you know your answer is correct?
- What is helpful in this write-up?
- What is missing?
- What isn’t needed?
- How does the process used here differ from the solution?

*Extended Bagels* (homework)
This activity extends *Marcella’s Bagels* by posing the question of how altering the final number of bagels would change the initial number of bagels. This is a “functions” question: the starting number is a function of the ending number. Students are asked to find that functional relationship by trying several ending values, organizing their findings in an In-Out table, and then searching for a rule.

**Key Questions:**
- How does the solution to Marcella’s Bagels depend on the number of bagels Marcella has when she gets home?

**The Chef’s Hot and Cold Cubes**

The “hot and cold cubes” sequence of activities offers a model for the operations of integer arithmetic, key tools in high school mathematics. This activity reviews some basics about negative numbers and reaffirms some conventions before students begin to make sense of the “hot and cold cubes” model for integer arithmetic. (The IMP program assumes that most students have had prior exposure to negative numbers and have been taught—but may not remember or understand—basic rules for arithmetic with integers. In this set of activities, students are introduced to a model that embodies these rules and serves as a metaphor for thinking about integer arithmetic. Rather than simply reviewing the rules for such arithmetic, this model provides a frame of reference for the rules and will allow students, if necessary, to reconstruct the rules for themselves in the future.)

The basic operations for natural numbers are defined, at least intuitively, in terms of putting sets of objects together and taking objects away from a set, but this definition doesn’t make sense for negative numbers. In moving from whole numbers to integers, numbers are no longer simply a magnitude, but also a direction. Treating an integer as merely an opposite of a whole number, as do such commonly memorized rules as “subtracting a negative is the same as adding a positive,” does not encourage a more powerful understanding of integer. The hot and cold cubes model emphasizes both the magnitude and the direction of an integer and encourages awareness of the meaning of the operation involved.

The activity begins with discussion of the need to justify solutions when doing integer arithmetic and is followed by a brief review of notation, language, and conventions. Students are then introduced to the model and, in their groups, perform some integer arithmetic to help make sense of the model. They are asked to translate the chefs’ moves (using hot and cold cubes to change the temperature in the cauldron) into integer arithmetic and to translate integer arithmetic into chefs’ moves.

**Key Questions:**
- What is the answer to \((-3)(-5)\)? How do you know your answer is right?

**Do It the Chefs’ Way** (homework)

Students use the hot and cold cubes model to understand arithmetic with integers. This is also a good time to introduce the concept of absolute value and to explore patterns in operations with integers. Students have worked in their groups to make sense of the “hot and cold cubes” model. Now they will spend some individual time practicing with and confirming their understanding of the model. After comparing their work with one another and discussing questions that arise, students review a few more basic ideas related to operations with integers.

**Key Questions:**
- What's the difference between the operation of subtraction and the negative sign?
- What should come next in this sequence?
- How does this pattern relate to the hot and cold cubes model?
- How might you represent the situation with objects?

1-2-3-4 Puzzle and its companion activity, Uncertain Answers (homework), help students gain insight into the need for rules for order of operations and provide additional experience with the algebraic logic of graphing calculators. This open-ended exploration highlights the importance of order-of-operations rules for communicating mathematically and gives students an opportunity to explore order of operations on the graphing calculator. The activity is also an ideal time to establish the conventional order of operations.

These precedence rules have been established to remove the ambiguity from the meaning of such written expressions as 3 × 7 + 2², which might otherwise be evaluated by multiplying 3 by 7, adding 2, and then squaring the result to obtain 529. Using the precedence rules above, the value of this expression is 25, because 2² = 4, 3 × 7 = 21, and 21 + 4 = 25.

Key Questions:
- What methods did you use to find your expressions?
- Did you proceed in numeric order or did you jump around?
- Did you get an expression for one number by adjusting the expression for another?
- Did you use any patterns that you saw in the expressions?

POW 2: Checkerboard Squares
(Allow students one week to complete out of class)

You're the Chef (homework)
This activity concludes work with the "hot and cold cubes" model for integer arithmetic, for now. This activity requires students not only to make meaning of the model but to create a thorough explanation of the model, using examples. Their write-ups of this activity will be part of their Patterns portfolios. Throughout the rest of the program, students are expected to use negative numbers where appropriate.

Lesson 3: Investigations
Consecutive Sums
The core for a series of activities that open Investigations, Consecutive Sums poses an open-ended situation in which students are encouraged to make and test conjectures, construct proofs, and find counterexamples. This activity helps to establish a classroom environment of student-student interaction during the exploration of a challenging mathematical investigation.

Students examine complex, open-ended mathematical questions, develop and test ideas, write proofs using logical reasoning and algebraic notation, and disprove conjectures using counterexamples.
Students are also introduced to summation notation. Consecutive sums are defined as sums of consecutive natural numbers, such as 6 + 7 + 8 + 9 = 30, 35 + 36 =
71, and \(1 + 2 + 4 + \ldots + 10 = 55\). The third example can be written, using summation notation, as \(\sum_{i=1}^{10} i = 55\).

Key Questions:
- What numbers can be written as consecutive sums?
- What numbers can be written as more than one consecutive sum?
- If a number can be written as a consecutive sum, is that consecutive sum unique?
- What numbers are not answers to some consecutive sum? Are there patterns in these numbers?
- Are there patterns to the answers to consecutive sums that are two terms long (such as 4 + 5), three terms long, or four terms long?

**Add it Up**
This activity introduces the mathematical symbol for summation notation. Students begin to understand the utility of this notation by working with both numeric and geometric examples. One of the challenges of secondary mathematics teaching is helping students to understand the notational systems used to express complex ideas in a compact form. This activity introduces one such system, summation notation, and offers students opportunities to start to make sense of it.

This activity challenges students to evaluate whether a conjecture is true or false. The conjecture in question is “If an odd number is greater than 1, then it can be written as the sum of two consecutive numbers.” Students are asked to find a counterexample if they think the statement is false or to devise a set of instructions for writing any odd number greater than 1 as a sum of two consecutive numbers if they think the statement is true.

**Key Questions:**
- Is the conjecture in the activity true? How confident are you about your answer?
- How would you write 397 as the sum of two consecutive numbers?
- How would you write 4913 as the sum of two consecutive numbers?
- How would you write 157,681 as the sum of two consecutive numbers?

**Pattern Block Investigations**
Students are introduced to pattern blocks, determining angle measures, and using a protractor. Some students may already be familiar with these, and some will benefit from a refresher. Pattern blocks are polygons that share side and angle relationships. By fitting and stacking these blocks, students can observe many geometric relationships inherent in these special manipulatives.

**Key Questions:**
- How far have I turned?
- How many degrees are in that turn?

**That’s Odd!** (homework)
The purpose of this activity is not for students to learn a proof that odd numbers greater than 1 can be written as a sum of two consecutive numbers. Rather, it is intended to help students to do the following:

- begin to learn what a proof is
- learn to distinguish between specific examples and a general argument
- gain experience in communicating complex, abstract ideas
- become familiar with a more precise way of thinking than they may have encountered before

**Degree Discovery (homework)**

The central mathematical idea underlying the next three activities—Degree Discovery, Polygon Angles, and An Angular Summary—is that there is a functional relationship between the number of sides of a polygon and the sum of the measures of its interior angles: sum of angle measures is equal to 180 degrees multiplied by (number of sides – 2).

Students will draw several triangles, measure and sum the angles in them, and then do the same for quadrilaterals. Their observations are noted in class and initiate a sequence of activities in which students derive and prove the angle sum formula for polygons.

**Key Questions:**

- What if your protractor measurements are not exact?
- Why might measurement results vary for some triangles?

**Polygon Angles**

In this activity, students generalize the results from triangles and quadrilaterals to all polygons. This mathematical investigation is a valuable opportunity for them to learn about what doing mathematics is and to see themselves doing mathematics. Students work in groups on a rather open task to explore and record what they notice about the sums of polygon angles. If they record their observations about different types of polygons in an In-Out table comparing number of angles to angle sum, they may observe another pattern.

**Key Questions:**

- Think about different ways to organize your data to see whether there might be patterns in your findings.
- What do you notice about your table?
- What would be your conjecture for the angle sum for a 10-sided polygon? A 12-sided polygon? A 100-sided polygon?
- Is there a general formula connecting the In to the Out in this table?
- All angle sums are a multiple of 180 degrees, but what multiple?
- What do you think is the sum of the angles in a 10-sided polygon?
- What should you multiply 180° by to get the sum of the angles in a 100-sided polygon?
- Why must your rule be true for all polygons?
- Why should the triangle sum for quadrilaterals be exactly twice that for triangles?

**An Angular Summary (homework)**
In this activity, students reflect on and apply their knowledge of the relationship of sides and angles in polygons. This activity emphasizes the important mathematical relationships they have worked on recently, including the unproved fact that the sum of the angles in a triangle is 180° and, based on this conjecture, the proven polygon angle sum formula.

This activity draws upon the polygon angle sum formula to introduce the concept of a regular polygon, a polygon in which all angles have the same measure and all sides are the same length. Students draw and measure angles using a protractor one more time. This activity serves as a wrap-up for the angle and polygon investigation sequence. After recalling and writing about what they know about polygon angles, students extend this knowledge to a regular pentagon and regular octagon and then draw these polygons.

Lesson 4: Putting it Together

Squares and Scoops

This activity challenges students to draw on their work with patterns and their explorations in Consecutive Sums to generalize rules for two important patterns. Students are also challenged to consider appropriate range and domain for a given function.

- Question 1 involves consecutive sums starting with 1. Students investigate the relationship between the height of a stack of squares and the number of squares in the stack. The stack of squares is arranged in a triangular pattern, with each row of squares one unit longer than the one above. A 1-high stack contains 1 square, a 2-high stack contains 1 + 2 = 3 squares, and a 3-high stack contains 1 + 2 + 3 = 6 squares. The numbers 1, 3, 6, and so on are called the triangular numbers. In general, an n-high stack will contain 1 + 2 + 3 +...+n squares.

- Question 2 involves an analogous idea for consecutive products starting with 1. It poses a combinatorial question: How many ways are there to arrange n scoops of ice cream on a cone? There is 1 way to arrange 1 scoop and 2 ways to arrange 2 scoops. However, there are 6 ways to arrange 3 scoops. To make the problem easier to think about, imagine that each scoop is a different flavor. For 4 scoops, once the first flavor is chosen, we know there are 6 ways to arrange the rest, and with 4 ways to choose the first flavor, there are 4(6) = (3)(2) = 24 arrangements altogether. In general, there are n(n - 1)(n - 2)...(2)(1) = n! ways to arrange n flavors.

Key questions:
  - Is 1.5 an appropriate input for either of these tables?
  - What do you call the set of possible inputs for an In-Out table?

What other examples have you seen in which only certain Diagonally Speaking
This activity revisits and pulls together ideas in the Patterns unit. It reinforces In-Out tables and the search for patterns as powerful problem-solving tools. Students use an In-Out table in a geometric context to find a functional relationship and are challenged to prove why their pattern holds.

Key Questions:
  - Can you predict how many diagonals a polygon has?
  - Can you use the table to predict the number of diagonals for an 8-sided polygon, without drawing one?
  - Why must all 5-sided polygons have the same number of diagonals?
Another In-Outer (homework)
In this activity, students practice integer arithmetic and finding and using rules for In-Out tables. They also return to the focus on language and symbolic notation begun in the earlier activities Inside Out and Pulling Out Rules. The six questions in this activity give students additional opportunities to express the relationships between the In and the Out in an In-Out table representation of a function. Students write algebraic equations for expressing the Out as a function of the In and use their rules to find both the Out given the In and the In given the Out.

The Garden Border
The algebra-geometry connection is again a key mathematical element. In this activity, students derive general approaches for counting the tiles along the border of a square garden. The number of tiles is a linear function of the size of the garden. Students begin this two-activity set by creating as many ways as they can think of to count border tiles for a 10-by-10 garden, without counting one tile at a time. They discuss and compare the variety of counting methods they find, setting the stage for developing a symbolic representation of each method and for recognizing the equivalence of the expressions created.

In a follow-up activity, students begin by creating as many ways as they can think of to count border tiles for a 10-by-10 garden, without counting one tile at a time. They discuss and compare the variety of counting methods they find, setting the stage for developing a symbolic representation of each method and for recognizing the equivalence of the expressions created. This activity gives students additional experience using algebraic language and symbols to represent geometric situations. It also strengthens their understanding of equivalent expressions and skill in working with the distributive property.

Border Varieties (homework)
This activity gives students additional experience using algebraic language and symbols to represent geometric situations. It also strengthens their understanding of equivalent expressions and skill in working with the distributive property.

In The Garden Border, students perceived a geometric context in many ways. The variety of approaches they developed lead to different-looking, but equivalent, methods for counting tiles. Here are three such methods for counting border tiles, along with rules that reflect these three ways of viewing the problem.

\[4(s - 1) = 2(s) + 2(s - 1) = 4(s - 2) + 4\]
These rules are equivalent because each gives the same total number of tiles for any value of s. In this activity students compare a variety of methods for counting border tiles for a 10-by-10 garden and then review and generalize these methods for gardens of any size. The classroom conversation emphasizes the equivalence of the resulting expressions and the occurrence of the distributive property.

Key Questions:
- Would your method work for a 100-by-100 square?
- How can you be sure these expressions are equivalent?

**Resources and Teaching Tips**


  Technology Activities Available for Download from IMP site:
  - In-Out.xls *Inside Out* (p. 10): This demonstration can be used to introduce In-Out Machines. A number can be entered into the “In” cell of an In-Out Machine illustrated in the student book and the result will appears in the Out cell. Highlighting the Out cell and reading the formula from the Formula Bar can check rules.
  - Stump _Your Friends.ftm* Inside Out* (p. 10): This Fathom file can get students started using Fathom in place of a calculator to stump their friends with an In-Out machine.
  - 1-2-3-4_Puzzle.xls *1-2-3-4 Puzzle* (p. 18): This file contains some answers to the group activity as well as some answers that are possible using Excel functions beyond those mentioned in the problem.
  - Uncertain_Ansers.doc *Uncertain Answers* (p. 19): This activity explores the use of parentheses and other symbols in Excel.
  - Positive_Negative.xls *The Chef’s Hot and Cold Cubes* (p. 21): This Excel demonstration can be used to reveal patterns in the operations on integers.
  - Arithmetic_Machines.gsp *The Chef’s Hot and Cold Cubes* (p. 21): This Sketchpad demonstration can be used to explore the operations on integers with the number line model. The teacher can show the sketches, or students can explore them on their own and then write a summary of what they saw.
  - Mystery_Machines.gsp *The Chef’s Hot and Cold Cubes* (p. 21): These mystery machines challenge students to find the 0 and the 1 on an invisible number line by exploring how the product of two numbers relates to the two numbers. Let students explore this file after the Arithmetic Machines demonstration.
  - Chef_and_Patterns.doc *The Chef’s Hot and Cold Cubes* (p. 21): This Excel activity can be used by students to look for patterns in the rules for operating on integers.
  - Consecutive_Sums.doc *Consecutive Sums* (p. 28): This Fathom activity can be used to introduce students to Fathom.
  - Angles_on_the_Screen.doc *Degree Discovery* (p. 36): This handout is for a replacement activity that directs students to explore the sum of the interior angles of a triangle and other polygons using Sketchpad.
  - Degree_Discovery.gsp *Degree Discovery* (p. 36): This sketch has separate pages showing the angle measurements for triangles, quadrilaterals, and pentagons.
Border.gsp Border Varieties (p. 44): This is a series of activities using multiple pages in a Geometer’s Sketchpad document that replace and extend the Garden Border and Border Varieties homework assignments.

- Fathom software
- Geometer’s Sketchpad software
- Graphing calculators
- Graph paper, rulers, colored pencils
- Online graphing calculator: http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html

Differentiation

Reinforcement Opportunities:

- *Keep It Going* (reinforcement) Students use patterns to find the next few terms of four number sequences and then describe the patterns they found.
- *The Number Magician* (reinforcement) In essence, this activity describes a multistep In-Out machine. Students determine the original number that produces one particular answer and analyze the method used to determine the original number so quickly.
- *A Fractional Life* (reinforcement) This activity is part of The Greek Anthology, a group of problems collected by ancient Greek mathematicians. It will help reinforce students’ work with In-Out tables and can be used any time after In-Out tables are introduced.
- *It’s All Gone* (reinforcement) In a variation on Marcella’s Bagels, a man goes from store to store getting and spending money, winding up with no money in the end. Students are asked to determine how much money he had when he started.
- *1-2-3-4 Varieties* (reinforcement) This activity adds a rule to those used in 1-2-3-4 Puzzle: now the digits must appear in numeric order. In addition to finding expressions for the first 25 whole numbers, students are asked to find the greatest possible answer given.
- *Getting Involved* (reinforcement) Several activities in this unit—such as Role Reflections and Group Reflection—ask students to reflect on the process of working in groups. In this related activity, students are asked to reflect on a situation in which one person in a group is not contributing.
- *A Protracted Engagement* (reinforcement) In this open-ended activity, students are asked to decode a message created using angles of different sizes to correspond to different letters of the alphabet, and then to code a message of their own.
- *A Proof Gone Bad* (reinforcement) Students are asked to explain the contradictions in another student’s proof. Assign the activity after students have worked on developing proofs.

Extension Opportunities:

- *Whose Dog Is That?* (extension) This logic puzzle, much like Who’s Who?, gives students another opportunity to use organized thinking and to write clear explanations. Students are given several interlocking conditions and must use logical reasoning to determine a set of conclusions. Give students several days to work on the activity and to write up their results.
- *Counting Llama Houses* (extension) Students identify the ways in which the houses in Lonesome Llama differed and then determine how many different houses could have been created using these variations.
• **Positive and Negative Ideas** (extension) This activity extends students’ work with hot and cold cubes. It asks them to consider other ways they might model integer arithmetic.

• **Chef Divisions** (extension) This activity extends ideas introduced through the “hot-and-cold-cubes” model. While modeling division with hot and cold cubes, students think more deeply about the model and the reasoning involved in working with negative numbers.

• **More Broken Eggs** (extension) In *The Broken Eggs*, students found a possible number of eggs the farmer might have had when her cart was knocked over. The task now is to look for other solutions, to find and describe a pattern for obtaining all the solutions, and to explain why all the solutions fit that pattern.

• **Three in a Row** (extension) Students explore sums of three consecutive numbers as well as sums of other lengths. The activity is appropriate following the discussion of *That’s Odd!*

• **Any Old Sum** (extension) In this variation on the *Consecutive Sums* investigation, students examine sums that are not consecutive. In addition to extending ideas in *Consecutive Sums*, this activity gives students more experience with open-ended problems.

• **The General Theory of Consecutive Sums** (extension) Students explore consecutive sums of integers. You may want to allow students several days to work on this challenging activity.

• **Infinite Proof** (extension) Students are asked to prove that the square of every odd number is odd and that every prime number greater than 10 must have 1, 3, 7, or 9 as its units digit. The activity gives students the opportunity to see that proofs are possible in situations involving infinitely many cases.

• **Different Kinds of Checkerboards** (extension) In this follow-up to *POW 2: Checkerboard Squares*, students find the number of squares on nonsquare checkerboards and search for a general rule for checkerboards of dimensions *m* by *n*.

• **Lots of Squares** (extension) In this substantial investigation, students are asked to divide a square into different numbers of smaller squares. The goal is to determine which numbers of smaller squares are impossible and which are possible, and to prove their results. Assign the activity after students have worked on developing proofs.

• **From Another Angle** (extension) This activity extends students’ work with pattern blocks and generalizes ideas in *An Angular Summary*.

• **From One to *N*** (extension) The task in this activity, which is a natural outgrowth of the ideas in *Squares and Scoops*, is to find a simple expression in terms of *n* that allows one to find a sum without repeated addition. If students find such an expression, they look for a proof that their answer is correct.

• **Diagonals Illuminated** (extension) This follow-up activity to *Diagonally Speaking* draws the distinction between recursive and closed-form rules and asks students to develop a closed-form rule for the number of diagonals of any polygon. Students are then asked to explain why this rule makes sense.

• **More About Borders** (extension) This activity contains variations on the *Border Varieties* activity.

• **Programming Borders** (extension) Building on ideas in the supplemental activity *More About Borders*, this activity asks students to write a program that answers some or all of the questions posed in *More About Borders*. 

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5-C-46
Design Principles for Unit Development
At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.

- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.

- **21st Century Learning** – the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow’s workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

**Universal Design:** Throughout the unit, students are expected to approach algebraic concepts through visual, written and oral formats. Students are given situations in verbal or written formats, and expected to translate these situations into graphs and equations. Conversely, students are expected to “bring life” to tables and graphs by creating a situation that would create the given table or graph. The use of graphing calculators is integral to the work in this unit.

**21st Century Learning:** This curriculum focuses very strongly upon the use of 21st century skills in mathematics. Students engage in a problem-based curriculum, and are expected to think critically about the problems presented, and to develop strategies to solve these problems. Throughout the problem-solving process, students must be able to build on previous knowledge to build new information and continue to make decisions based on the mathematics at hand. This process is done largely in cooperative groupings. Students have access to technology, and are given multiple opportunities to extend their work into new situations to create generalizations.

**Technology Integration**
The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century(SETD, 2003).

Students engage daily with graphing calculators as a tool to solve problems. Graphing calculators allow students the opportunity to work with multiple representations in reduced time, allowing them to build a greater understanding of the relationships between all of the representations. Students will also have the opportunity to work with additional software and sites that enhance learning.
Earth Science: In this unit, students are expected to study patterns to make mathematical sense of the world around them. Through this process, students create statements, rules, expressions and equations that capture these patterns. The study of earth science also allows for representing patterns mathematically. There are also abundant opportunities for students to interpret multiple representations of data and make informed decisions based upon the data presented. While the mathematics unit focuses on linear relationships, in the context of earth science and real-world application, students should be able to extend this process to include some discussion with nonlinear relationships, with scaffolded support.

**NGSS Related Earth Science Standards:**
HS –ESS3-1 Construct an explanation based on evidence for how the availability of natural resources, occurrence of natural hazards, and changes in climate have influenced human activity.

HS-ESS3-2 Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.

HS-ESS3-3 Create a computational simulation to illustrate the relationships among management of natural resources, the sustainability of human populations, and biodiversity.

HS-ESS3-5 Analyze geoscience data and the results from global climate models to make an evidence-based forecast of the current rate of global or regional climate change and associated future impacts to Earth systems.

HS-ESS3-6 Use a computational representation to illustrate the relationships among Earth systems and how those relationships are being modified due to human activity.

**NGSS Related Engineering, Technology and Applications of Science Standards:**
HS-ESS3-1. Construct an explanation based on evidence for how the availability of natural resources, occurrence of natural hazards, and changes in climate have influenced human activity.

HS-ESS3-2. Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.*

HS-ESS3-3. Create a computational simulation to illustrate the relationships among management of natural resources, the sustainability of human populations, and biodiversity.

HS-ESS3-4. Evaluate or refine a technological solution that reduces impacts of human activities on natural systems.*

HS-ESS3-5. Analyze geoscience data and the results from global climate models to make an evidence-based forecast of the current rate of global or regional climate change and associated future impacts to Earth systems.
HS-ESS3-6. Use a computational representation to illustrate the relationships among Earth systems and how those relationships are being modified due to human activity.
Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

<table>
<thead>
<tr>
<th>Unit Title:</th>
<th>Cookies</th>
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<tbody>
<tr>
<td>Designed by:</td>
<td>Michelle Hawley</td>
</tr>
<tr>
<td></td>
<td>from <em>Interactive Mathematics Program, Year 2</em></td>
</tr>
<tr>
<td>Content Area:</td>
<td>Mathematics (Algebra)</td>
</tr>
<tr>
<td>Grade Level(s):</td>
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Summary of Unit

This unit engages students in the use of linear equations and inequalities as a problem solving strategy. The main problem centers on running a cookie business. Students work collaboratively on problems that involve constraints—perhaps ingredients or timing—and must find solutions that fit the given constraints. Since this is the first exposure to the use of multiple constraints, the constraints are all represented as linear relationships. This provides students a reasonable entry into the concepts of maximization and minimization. The structure of the unit requires that students develop a deep understanding of manipulating and interpreting systems of equations and inequalities. Lastly, students are given the opportunity to write and share their own linear programming problems.

Stage 1 – Desired Results
What students will know, do, and understand

Delaware Content Standards

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* CC.A-CED.1

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. CC.A-CED.2

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* CC.A-CED.3

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. CC.A-REI.3

*Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.* CC.A-REI.5 – supplementary lesson is being developed by the publisher
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **CC.A-REI.6**

Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line \( y = -3x \) and the circle \( x^2 + y^2 = 3 \). **CC.A-REI.7**

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding halfplanes. **CC.A-REI.12**

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include:** intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **CC.F-IF.4**

Graph linear and quadratic functions and show intercepts, maxima, and minima. **CC.F-IF.7a**

**Big Idea(s)**
- Using variables to represent linear problems
- Working with variables, equations and inequalities to solve problems
- Solving problems involving systems of equations and inequalities
- Using the concept of equivalence to solve problems
- Using graphs to solve problems
- Creating and analyzing graphs to solve problems
- Optimizing solutions based on constraints

**Unit Enduring Understanding(s)**
- Systems of equations and inequalities can help make use make sense of real-world problems.
- Graphing information can help us solve problems.
- While a system may have many solutions, sometimes one solution is the best solution when given constraints.
- Being able to communicate well allows us to share our mathematical findings with others.

**Unit Essential Questions(s)**
- How can we express real-world situations in terms of equations and inequalities?
- What role does the distributive property play when solving equations?
- What strategies can we use to solve systems of linear equations in two variables?
- How do we classify systems as dependent, inconsistent, and independent when given two linear equations?
• When are graphing calculators useful for problem solving?
• What strategies help us write and graph linear inequalities in two variables?
• What strategies help us fine the optimized solution(s) in a situation involving inequalities?
• How can we apply principles of linear programming with two variables?
• How can we create our own linear programming problems with two variables?

Knowledge and Skills

Students will know...
• Equivalence means that the values are the same
• The properties of operations
• Strategies for solving systems of equations and inequalities
• Equivalence must be maintained while solving equations
• Multiplying or dividing inequalities by a negative number reverses the relationship between the values
• Linear relationships are represented as straight lines on a graph
• The feasible region determines which values hold true for both equations
• Sometimes problems have unwritten constraints (i.e., when looking at a solution involving animals, the feasible region would not include negative numbers)

Key Vocabulary:
• Linear equality
• Linear function
• Linear programming
• Constraint
• Inequality
• Equivalent inequality
• Linear equation
• Half plane
• Feasible region
• System (of equations)
• Polygon
• Profit line
• Vertex (of feasible region)
• Edge (of feasible region)
• Equivalent equations
• Independent system
• Inconsistent system
• Dependent system
• Term
• Independent equations
• Dependent equations
• Inconsistent equations

Students will be able to...
• Write linear equations and inequalities
• Graph linear equations and inequalities
• Interpret data from a graph
• Manipulate equations and inequalities to find a solution or solutions
• Use substitution as a method for solving systems
• Test solutions to confirm that they fall in the correct section of the graph
• Construct a feasible region for a set of inequalities
• Determine which of the possible solutions provides the maximum and minimum results
• Use a graphing calculator to solve systems of linear equations and inequalities
• Distinguish between independent, dependent and inconsistent systems
• Present their findings and thinking to others verbally and in writing

Stage 2 – Assessment Evidence
Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)
http://www.illustrativemathematics.org/illustrations/610

The relation between quantity of chicken and quantity of steak if chicken costs $1.29/lb and steak costs $3.49/lb, and you have $100 to spend on meats for the barbecue.
   (i) write a constraint equation
   (ii) determine two solutions
   (iii) graph the equation and mark your solutions

Rubric(s)

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
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<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Student writes a correct equation 1.29c+3.49s=100</td>
<td>• Student writes a correct equation 1.29c+3.49s=100</td>
<td>• Student does not provide a correct equation combining the two variables, but does demonstrate some understanding of the relationships</td>
<td>• Student equation demonstrates little understanding of the problem</td>
</tr>
<tr>
<td></td>
<td>• Student identifies the variables used in the equation</td>
<td>• Student identifies the variables used in the equation</td>
<td>• Units might not be provided for the variables</td>
<td>• Units are not provided (pounds)</td>
</tr>
<tr>
<td></td>
<td>• Student includes units on labels for the variables (pounds)</td>
<td>• Student might omit units on labels for the variables (pounds)</td>
<td>• Student might provide a correct equation without any labels for the variables</td>
<td>• Student provides at most one correct combination of chicken and beef, possibly found through trial and error</td>
</tr>
<tr>
<td></td>
<td>• Student provides two reasonable (and mathematically accurate) combinations for chicken and beef</td>
<td>• Student provides two reasonable (and mathematically accurate) combinations for chicken and beef</td>
<td>• Student work is lacking and</td>
<td>• Student work is lacking and</td>
</tr>
<tr>
<td></td>
<td>• Student shows work or explains how the two solutions were</td>
<td>• Student shows partial work or provides a partial solution</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5-C-53
<table>
<thead>
<tr>
<th>Determined</th>
<th>Explanation about how the two solutions were determined</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student creates a correct graph of the relationships</td>
<td>Student creates a correct graph of the relationships</td>
<td>Student provides one possible combination of chicken and beef</td>
</tr>
<tr>
<td>Student graph is properly labeled with title, scales, axis labels and points graphed and labeled correctly.</td>
<td>One of the following might be missing from the student graph: title, x-axis scale, y-axis scale, x-axis label or y-axis label</td>
<td>Student work is lacking, and explanation is difficult to understand</td>
</tr>
<tr>
<td>Points on the graph are graphed and labeled correctly.</td>
<td>Points on the graph are graphed and labeled correctly.</td>
<td>Student attempts to create an appropriate graph of the situation, based upon their understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One of the following might be missing from the student graph: title, x-axis scale, y-axis scale, x-axis label or y-axis label</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Points (which match their previous work) may be missing from graph, or may not be labeled</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solution points are missing or graphed incorrectly.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Explanation is poor or may be absent</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student attempts to create an appropriate graph of the situation, based upon their understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student graph is lacking several of the labels: title, x-axis scale, y-axis scale, x-axis label or y-axis label</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student graph might not be scaled appropriately for the situation (or even generally)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solution points are missing or graphed incorrectly.</td>
</tr>
</tbody>
</table>
Suggestions for differentiation:
- Student may be asked to find one solution to the constraints
- Prices for each type of meat can be modified to numbers that are easier to calculate and yield whole number (or benchmark fraction) responses
- Similar to above, the total amount for spending can be reduced

Other Evidence
- Class discussions
- Class work
- Homework
- Problem of the weeks
- Student math journals
- Collaborative discussions
- Exit Tickets
- Curriculum-Based assessment opportunities:
  - Inequality Stories, Part I: This assignment will give you information about students’ understanding of how real-life contexts can be expressed in algebraic terms using inequalities.
  - Profitable Pictures: This activity will tell you how well students understand how profit lines can be used to determine an optimal value.
  - Changing What You Eat: In this assignment, students will demonstrate their understanding of how changing specific parameters in a problem affects the solution.
  - Get the Point: This investigation will give you insight into students’ abilities to think about systems of linear equations in flexible ways.
  - A Reflection on Money: This assignment will give you information about students’ comfort levels with solving systems of linear equations.
  - “How Many of Each Kind?” Revisited: This activity will tell you how well students have synthesized the ideas of the unit.
  - Producing Programming Problems: This assignment will tell you how well students understand the components of a linear programming problem.
Student Self-Assessment and Reflection

- Students will keep math portfolios where they respond to problem of the weeks and other mathematical prompts
- Students will work in an interactive classroom environment, where collaboration, discussion and feedback are everyday occurrences.

Stage 3 – Learning Plan
(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson 1: Cookies and Inequalities

How many of Each Kind?
In the unit problem, students deal with a set of constraints on ingredients, oven space, time, and cost as they try to maximize the profit from sales of two kinds of cookies. Each of the constraints can be expressed as a linear inequality in two variables, and the profit is a linear function of those same two variables. In this activity, students look for specific numeric examples that fit the constraints, and then they determine the profit for each example. They do this work without a formal introduction to linear programming, instead using their prior knowledge to follow the constraints introduced in the story. Students work in groups to explore the central unit problem. They post their findings for later reference. In a class discussion, they develop inequalities to represent the constraints and a symbolic expression for the profit.

Key Questions:
- Could the Woos make 1 dozen of each kind? What about 3 dozen plain and 5 dozen iced?
- Are you sure that this combination fits all the conditions? How do you know?
- What is a combination that does not fit all the constraints? Which constraint or constraints does it fail to satisfy?
- How did you decide whether a combination fits this constraint?
- How can you express this constraint symbolically?
- Why isn’t the profit expression included in our list of constraints?

A Simpler Cookie
By examining a simpler version of the unit problem, students begin to build an understanding of the relationship between the problem constraints and the resulting profit.

Working through a simpler problem is a valuable problem-solving strategy for complicated questions. In this activity, students will find numeric values for the numbers of plain and iced cookies that fit the constraint on preparation time:

$$0.1P + 0.15I \leq 15$$
They will compute the resulting profit for each pair of values, where the profit is a function of $P$ and $I$:

\[
\text{Profit} = 1.5P + 2I
\]

Having only one constraint makes this problem much simpler than the original.

*Investigating Inequalities* (homework)

In this activity, students investigate what manipulations of an inequality will preserve its truth.

Beginning in the Year 1 unit *The Overland Trail*, students developed procedures for changing equations into equivalent forms, often for the purpose of solving them. Now they will begin to determine procedures that change inequalities—like those that arise throughout this unit—into equivalent forms. In this activity, they will start with numeric inequalities like $-2 \leq 7$ and explore what happens to the truth of these statements when they add, subtract, multiply, or divide both sides by the same number. They will also confront the specific case of multiplying or dividing both sides of an inequality by the same negative number. In addition, they will use one-dimensional linear graphs to represent and justify their results.

Key Questions:
- What manipulations of an inequality will preserve its truth?
- What happens if you multiply both sides of $4 > 3$ by $-2$?
- What happens if you multiply both sides of an inequality by a negative number?
- Why does multiplying by a negative number reverse the inequality?
- If you have an inequality that is not true, is there any way to adjust the inequality sign to make it be true?

*My Simplest Inequality*

This activity emphasizes the notion of an equivalent inequality in one or two variables. Students establish equivalence by confirming that specific examples that make the original inequality true also make the new inequality true. They also check whether examples that do not work in the original inequality also do not work in the new one. They use these procedures to solve one-variable inequalities and then to simplify two-variable inequalities.

Students work on the two parts of this activity in groups and share their results in a class discussion.

Key Questions:
- What do you call two equations with the same solution or solutions?
- How can you write the inequality in simplest form?

*Simplifying Cookies* (homework)

In this final activity in *Cookies and Inequalities*, students practice finding equivalent inequalities, this time using the unit problem’s constraints.

Throughout the unit, students will look at many combinations of cookies that the Woos might bake. By creating equivalent inequalities, they can make this work simpler. Students will also be graphing inequalities, which will be easier if they can
flexibly find equivalent inequalities. The activity highlights the one issue that makes manipulating inequalities different from manipulating equations: the effect of multiplying or dividing by a negative number.

Key Questions:
- Why is it helpful to look at equivalent inequalities for the unit problem’s constraints?
- What are the advantages of working with integer coefficients?
- What is the advantage of solving for one variable in terms of the other?

Lesson 2: Picturing Cookies

Picturing Cookies—Part I

Students are introduced to the graphical representation of a linear inequality. A linear inequality, such as the unit problem’s oven-space constraint $P + I \leq 140$, divides the coordinate plane into two half planes with a boundary formed by the corresponding linear equation, in this case $P + I = 140$. One half plane contains ordered pairs that satisfy the inequality. In this activity, students build the half plane by testing and coloring points that either satisfy or do not satisfy the inequalities defined by the constraints in the unit problem. Graphing linear inequalities will soon lead students to the notion of a feasible region.

Students work in groups to test and plot many points on a common graph, using one color for points that fit the condition and another for points that fail to fit the condition. They compare their results with those of other groups.

Key Questions:
- How might you obtain a geometric picture of these constraints?
- What does it mean to graph an inequality in two variables?
- What color should (5, 10) be? What color should (40, 100) be?
- What is the maximum value you will need on each axis?
- Are there any restrictions for the dozens of plain cookies when you are focused only on how much icing the Woos have?
- What do the different colors represent?
- Why is $P + I = 140$ the boundary?
- Why is the graph of $P + I = 140$ a straight line?
- If the only restriction on the Woos was the amount of icing they had, how many dozens of plain and of iced cookies could they make?

Inequality Stories (homework)

This activity offers students more experience relating inequalities to real-world situations.

The IMP curriculum, beginning in *The Overland Trail*, has stressed the representational nature of symbolic algebra. Students have written expressions and equations that summarize patterns and relationships in real-world contexts. When these expressions and equations have been manipulated, the manipulations have been connected to these contexts. Further, the curriculum has fostered algebraic thinking by asking students to create contexts from expressions and equations. In this activity, students extend this work to inequalities.

Students work on the activity individually and share results in a class discussion.
POW 4: A Hat of a Different Color
Allow one week for students to complete outside of class.
This POW’s focus is on logical thinking and developing convincing arguments. Students must organize the given information and devise a chain of reasoning that leads to a convincing answer.

This activity is introduced in class, with enough time for students to sort out the problem being posed. Students might benefit from some class time to explore the problem in groups. Approximately one week later, students present their findings.

Healthy Animals (homework)
In this activity, students derive constraints and create graphs to represent another real-world context.

Students derive a set of constraints and create graphs for a new context. Several of these constraints are of the form \( ax + by \geq c \), so their half planes will be above (to the right) of their boundaries. In addition, as in many linear programming contexts, the natural domains of the variables in this problem (in other words, the values the variables can take on in the given context) are nonnegative. Students will write new inequalities to represent this.

Students work on this activity individually. A class discussion then will raise several important issues about finding the appropriate half plane for an inequality and adding implicit constraints.

Key Questions:
- What differences do you notice in your work?
- What variables shall we use? What do they represent?
- How can you tell which side of the line you want?
- Is (-1, 11) a possible choice for Curtis’s pet’s diet? Does it fit the constraints?
- How can you impose this nonnegativity condition on the problem?

Picturing Cookies—Part II
In this activity, students construct the feasible region for the unit problem by coloring, one at a time, the set of points that fits each constraint.

The set of inequalities that represent the constraints for the unit problem can be graphed on a single set of axes. The region of the plane that is the intersection of the half planes for the individual constraints is the feasible region for the problem. All of the points in the feasible region fit all of the constraints simultaneously.

Students work in groups to create a single graph of the unit problem’s feasible region. You will probably want all students to make their own graphs so they can refer to them later and include them in their portfolios.

Key Questions:
- Does (52.3, 28.7) fit the constraints? Could these be the numbers of dozens of each kind of cookie?
- Where did that inequality come from?
- What does that line represent?
- Why do we want the points on this side of the line rather than on the other side?
• Are points on the line part of the region?
• What does the final graph tell you about the cookie problem?
• How does seeing this combined graph compare to reading the verbal description of the situation?
• What does it mean that the oven-space line misses the feasible region?

What’s My Inequality?
As in Inequality Stories, this activity emphasizes the connections among representations of relationships and continues to foster algebraic thinking.

Doing the opposite of what they have been doing so far in this unit, students will start with graphical representations of equations and inequalities and develop the corresponding algebraic statements. In the process, they will focus on the role of the equation as a boundary, the distinction between strict and nonstrict inequalities, and determining the direction of an inequality.

Students work on the activity individually, then share their work in groups and with the class.

Key Question:
• Does anyone have a different way to find the equation?

Feasible Diets
This activity is a follow-up to the introduction of feasible regions in Picturing Cookies—Part II. It uses the context presented in Healthy Animals.

Students find the intersection of the half planes they defined using inequalities in Healthy Animals to find the set of points that satisfy all the inequalities simultaneously. This feasible region is also defined by the implicit constraint that the solution is nonnegative.

Students work in groups to create their own graphs showing the feasible region for this context.

Key Question:
• Should \((-1, 11)\) be in the feasible region? Is it a possible solution?

Picturing Pictures
This final activity in Picturing Cookies presents a new context in which students can bring to bear all of the tools they have developed in the unit.

In this activity, students will write inequalities to express the problem constraints; graph these constraint inequalities, including the implicit nonnegativity constraints, on a single set of axes to define the feasible region; write an equation for the profit (which has not been discussed since early in the unit); and find the profit for several points in the feasible region.

Students work on the activity individually and share their results in groups and with the class.

Key Questions:
• What does the graph tell you?
Lesson 3: Using the Feasible Region
Profitable Pictures
This activity continues students’ work from Picturing Pictures, explicitly addressing the maximizing of profit.

In a linear programming problem with two variables, the set of constraints may be written as a system of linear inequalities. In this problem, that system is:

\[
5p + 15w \leq 180 \\
p + w \leq 16 \\
p \geq 0 \\
w \geq 0
\]

The graphical solution to this system defines a feasible region, a collection of points that form a polygon in the plane. The profit is also a linear function of \( p \) and \( w \).

\[
\text{Profit} = 40p + 100w
\]

The central ideas of this activity are (1) that the set of points that produce a particular profit lie on the same line and (2) that the maximum profit is found by sliding this profit line up (increasing the profit) until it reaches the edge of the feasible region.

Students work on the activity and compare results in groups and as part of an extensive class discussion.

Key Questions:
- What do you notice about combinations that produce a given profit?
- What happens as the profit increases?
- What do you notice about the different sets of points?
- Why do the points for a given profit lie on a straight line?
- What do you notice about the set of profit lines?
- What does it mean for lines to be parallel?
- What is Hassan’s maximum possible profit? How can you be sure?
- How can you confirm that you have the right coordinates for the intersection point?
- If it’s not clear from the graph, how can you decide which point maximizes profit?

Curtis and Hassan Make Choices (homework)
In this activity, students will find sets of inputs that produce specific outputs for two functions:
\[
\text{Cost} = 2A + 3B \quad \text{(based on Healthy Animals and Healthy Diets)} \\
\text{Profit} = 50p + 175w \quad \text{(based on Picturing Pictures and Profitable Pictures)}
\]
The conclusion they should draw in each case is that the points that produce a particular cost (or profit) are collinear.

Students work on this activity individually, likely for homework during the classwork on Profitable Pictures. They review their work as part of the discussion of that larger activity.

Finding Linear Graphs
This activity focuses students’ attention on methods for graphing linear equations.
Beginning in *The Overland Trail*, students have been devising methods for solving equations. Given the focus on linear equations in this unit—and, in particular, the need to find the intersection points that form the vertices of feasible regions in order to maximize profit—this activity gives students the opportunity to share the methods they have come to understand and use for solving linear equations.

Students work on the activity individually and share their results in groups and with the class.

**Key Question:**
- How do you come up with specific points to plot?

**You Are What You Eat (homework)**
A new linear programming situation allows students to further develop their abilities to identify constraints, write inequalities, graph feasible regions, and optimize a value based on a feasible region.

In this activity, students are asked to minimize the amount of cereal needed to meet two constraints (other than those involving nonnegativity): one to ensure enough protein, the other to limit carbohydrates. One of the constraints ends up having no influence on the answer.

Students work on the activity individually and share results in groups and with the class.

**Key Questions:**
- What combination of cereals with the least total amount will satisfy Mr. Hernandez?
- Why doesn’t the carbohydrates constraint play a role in the solution?
- Did anything like this happen in the unit problem?

**Hassan’s a Hit!**
Students return to the scenario in *Picturing Pictures* and *Profitable Pictures* to search for the maximum profit using a different profit function.

Students often believe that once they have graphed the feasible region, they can maximize profit by finding the point in the region that is farthest from the origin. Through this activity, they will learn that changing the profit function can change the point that gives the maximum profit, even when the constraints remain the same, and that they must draw the profit lines to determine the maximum profit.

Students work on this activity in groups and share discoveries in a class discussion.

**Key Question:**
- Shouldn’t the point giving the maximum profit be the point in the region farthest from the origin?

**Changing What You Eat (homework)**
Students consider two variations on the problem posed in *You Are What You Eat*.

The two variations offered here change the constraints in two different ways, resulting in two new feasible regions. In each case, students are to minimize consumption. In one example, one of the constraint lines is parallel to the
consumption function, so a set of points, rather than a single point, will minimize
the consumption value.

Students work on the activity individually. In a class discussion, they review the
collection of linear programming situations that they have explored so far and
observe commonalities in their solution methods.

Key Questions:
- Will any $2\frac{1}{2}$-ounce combination work?
- Why can’t the twins simply eat $1\frac{2}{3}$ ounces of Fruit-Nuts?

**Rock ‘n’ Rap**
This activity presents students with a linear programming situation that has more
than two constraints.

Students continue to develop linear programming tools for solving real-world
situations. In this activity, they will write constraint inequalities, graph them to find
the feasible region, explore a profit function, find the maximum profit, and explain
their methods for doing so. They will review that the point of intersection of two
lines is the same as the common solution to the two equations.

Students do this activity independently, applying the concepts and techniques they
have developed so far, and share their work in groups and a class discussion. After
students explore a** Rock ‘n’ Rap Variation**, the class will return to this activity and
investigate how to solve the problem on the graphing calculator.

Key Questions:
- What are the constraints in the problem?
- What point maximizes profit?
- How does the profit line change as the profit increases?
- How do you find the coordinates of this point of intersection?
- Is $5\frac{5}{6}$ a reasonable number of CDs?

**A Rock ‘n’ Rap Variation**
In this follow-up to Rock ‘n’ Rap, the constraints remain the same but the profit
function changes.

Students recognize that the change in the profit function will change the slope of
its line. As this line is slid up to increase the profit, the last point to touch
the feasible region will be different from the one that solved the original problem.

Students work in groups on the activity, building on their discoveries in Rock ‘n’
Rap.

Key Question:
- How does changing the profit for each kind of CD affect the
overall profit?

**Getting On Good Terms** (homework)
This activity reviews the symbolic manipulations used to find equivalent
equations and inequalities.

This activity focuses on the process of solving for one variable in terms of another,
which students must be able to do to graph certain equations on the calculator.
Equivalent equations have the same solutions and the same graph.
Students work individually on the activity and share results in their groups and with the class.

**Going Out for Lunch**
In this, the final activity in Using the Feasible Region, students focus on writing and solving a system of equations, two important aspects of finding solutions to linear programming problems.

Students write two equations to express the conditions on two variables in a new context. Then they use informal methods, such as guess-and-check and graphing, to find the solution for this system.

Students work on this activity individually and discuss their solutions in groups and with the class.

**Key Questions:**
- How could you solve this problem without using equations?
- Is this the only solution? Why do you think so?
- How would you express this problem algebraically?

**Lesson 4: Points of Intersection**

**Get the Point**
In this activity, students develop algebraic methods for solving pairs of linear equations. The goal is not for them to devise any particular method, but to find an algebraic procedure that works and makes sense to them. They may even find that they like one method for some systems and another method for others.

The solution to a system of two linear equations in two unknowns, if a unique solution exists, is the set of values for the variables that solve both equations simultaneously. It is the point in the plane at which the two equations’ graphs intersect. Up to now, students have used a graphical approach or guess-and-check to find such solutions. In this activity, the focus is on developing symbolic methods. The unit emphasizes two related symbolic approaches. Substitution involves solving one equation for one variable in terms of the other, substituting this expression into the second equation, and solving that equation for the remaining variable. The second approach is to solve each equation for the same variable and set the resulting expressions equal to each other. Both methods emphasize the meaning of the solution described above.

Students work in groups to solve five pairs of equations, check their results, and then describe the methods they used in general terms. Their work is followed by a class discussion of their methods. In addition, each group presents its work to the class.

**Key Questions:**
- What is true of all points on the line $y = 3x$? How would you get the $y$-coordinate from the $x$-coordinate for a point on this line?
- How could you use this same idea in Question 1c, and maybe in Question 1d?

**Only One Variable** (homework)
This activity connects students’ work on Get the Point with methods for solving equations in one variable, which were developed beginning in the Year 1 unit The Overland Trail.
This activity reviews concepts for finding equivalent equations. It connects to students’ work, beginning back in The Overland Trail, to develop ways to produce a new equation equivalent to a given equation. These techniques involved the distributive law and methods such as adding (or subtracting) the same term to both sides of an equation or multiplying (or dividing) both sides of an equation by the same nonzero term.

As students continue to consider linear programming situations, their facility with manipulating algebraic expressions is essential. In addition, this ability must be second nature for students to efficiently use the graphing calculator.

Students work individually on this activity and discuss their findings in class.

*Set It Up* (homework)
Students write a pair of linear equations to represent a situation and solve the system using methods developed in previous activities.

This activity combines several aspects of students’ work so far in this unit: writing equations to represent real-world situations, solving equations using a graph, and solving equations symbolically. The activity also fosters algebraic thinking by asking students to make up a pair of equations that has a given solution.

Students work on the activity individually as they complete their work on *Get the Point*.

**Key Questions:**
- How did you find a system of equations with this solution?
- Is there an equation you could use that has $7x + 9y$ on the left side?
- How can you use graphs to explain why there are different systems with this solution?

*A Reflection on Money*
This activity continues work with real-world problems yielding systems of linear equations and methods for solving these systems. It asks students to use both graphical and symbolic approaches and then to reflect on the advantages and disadvantages of each.

Students work on the activity individually and share their work in groups and with the class.

*POW 6: Shuttling Around*
Allow students one week to complete outside of class
This versatile problem can be solved in many ways: logically, visually, and algebraically. The various solutions allow students to become aware of connections among the methods, such as between a visual approach and an algebraic approach.

*More Linear Systems* (homework)
This activity introduces students to the range of possible outcomes when solving a system of linear equations. Up to now, students have found a single point that
solves each system of equations they have encountered. Such a system, called an independent system, consists of two lines that intersect at a single point.

This activity includes two examples illustrating other results students could obtain when solving a linear system. The two equations might produce lines that are parallel. Such an inconsistent system has no solutions. Or, the two equations might produce the same line. A dependent system such as this has infinitely many solutions: any points that solve one equation, and only those points, will solve the other.

Students solve the questions in this activity individually. The two systems of equations that produce “weird” results will be the focus of the class discussion, which will bring out that a pair of linear equations may have no common solution and introduce the terms inconsistent equations, dependent equations, and independent equations.

Key Questions:
- What is happening in terms of the graphs of these equations?
- What usually happens when finding the common solution to a pair of linear equations?
- What about the pair of equations $2x + 3y = 1$ and $6y = 2−4x$? Are there any number pairs that fit both equations?

**Lesson 5: Cookies and the University**  
**How Many of Each Kind? Revisited**

This activity restates the unit problem, first presented in *How Many of Each Kind?* Students apply what they have learned over the course of the unit to solve the unit problem and then write a written report of their work.

In solving the unit problem, students bring to bear all of the skills and techniques they have acquired during the unit. They use constraint inequalities to define the feasible region for the situation. They find the coordinates of the vertices of the feasible region. They plot the profit function and then slide this line up to identify the optimal solution.

Students work in groups and individually to solve the problem. Each student then prepares a written report. The class discusses their findings.

Key Questions:
- What does each line on the graph represent?
- How do you find the feasible region?
- What expression describes profit?
- How do you know that (75, 50) is the best choice?
- How can you be sure where the “parallel family of lines” leaves the region? For example, how do you know it isn’t at the point (30, 80)?

**A Charity Rock** (homework)

In this activity, students review and practice writing and solving systems of equations.

Students reinforce their abilities to solve a system of equations, including recognizing inconsistent and dependent systems, and to define a system from the conditions in a problem context, including carefully defining variables.
Students work on this activity individually and share their results in groups and with the class.

**Back on the Trail**
This activity connects the work students have been doing in this unit to earlier work in the curriculum from the Year 1 unit *The Overland Trail*.

This activity contains two-equation/two-unknown problems drawn from the unit *The Overland Trail*, in which students first studied linear functions and found solutions to linear systems in informal ways. Now that students have formalized graphical and symbolic methods for solving linear systems, they can bring those skills to bear on these familiar tasks. The first problem will likely seem easier to them because one of the equations expresses one variable directly in terms of the other.

Students work individually on the two situations in this activity. The class discussion includes a look back at the two types of problems that students have been tackling in this unit.

**Inventing Problems** (homework)
This activity concludes *Cookies and the University* by asking students to create their own two-equation/two-unknown problems.

Real-world contexts have driven the mathematical activities throughout the IMP curriculum. In addition to learning how to solve equations and systems of equations, students have been learning how to set up equations by expressing conditions in real-world contexts symbolically. In this activity and the rest of the unit, students create their own contexts for problems involving systems of equations and then set up and solve them.

Students create problems individually and then work in groups to solve one another’s problems. Each group presents its “best” problem and solution to the class.

**Big State U**
Through their work in this activity, students synthesize their ideas about how to solve complex problems with several constraints, preparing them to write one of their own.

This activity presents students with one final two-variable linear programming situation to set up and solve.

Students work on this final linear programming problem in groups and present their results to the class.

**Lesson 5: Creating Problems**
**Ideas for Linear Programming Problems** (homework)
Students review several problem situations from the unit to clarify the main characteristics of linear programming problems.

The linear programming problems encountered by students in this unit share three features. They each have
• two variables
• a set of conditions, represented as a system of linear inequalities, that constrains the solutions
• a function to be maximized or minimized

By reviewing these common features, students will prepare for the final activity of the unit: creating their own linear programming problems.

Working individually, students review three of the problem contexts from the unit. They then work in groups to generate ideas for their own linear programming problems.

Producing Programming Problem
In this culminating activity, students apply all they have learned about linear programming to invent situations of their own.

As in Inventing Problems, students are asked to step back and create their own contexts for problems—in this case, linear programming problems—and then set up and solve them. They must include all the steps they have learned that are essential to understanding, translating, and solving their problems, and they must present them to their classmates in a clear and coherent way.

Working in groups, students use their work on Ideas for Programming Problems to write and solve linear programming problems. They then prepare and give formal presentations to their classmates. They will also develop criteria and use them to assess the presentations.

Just for Clarity’s Sake (homework)
This activity gives students one final opportunity to reinforce their understanding of methods for solving systems of linear equations.

In Part II, students once again encounter an inconsistent system of equations, or one with no solutions. In the discussion, students explore symbolic cues for identifying inconsistent systems. An optional extension offers an example of a dependent system, in which any solution to one equation is also a solution to the other.

Students work individually on the activity as they complete their work on Producing Programming Problems and as they begin to compile their unit portfolios. They share ideas in groups and in a class discussion.

Key Questions:
• How could you use equivalent equations to verify that these equations are clearly inconsistent?

Producing Programming Problems Write-Up (homework)
In this activity, students prepare write-ups of their group’s linear programming problems. In their write-ups, students must include all the steps essential to understanding, translating, and solving their group’s linear programming problems. Students work individually to write a statement of the group’s problem, the solution, and a proof that the solution is the best possible.
Resources and Teaching Tips

- Interactive Mathematics Program site: http://impmoodle.its-about-time.com/
- Graphing calculators
- Graph paper, rulers, colored pencils
- Online graphing calculator: http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html

Differentiation

Reinforcement Opportunities:

- **Who Am I?** (reinforcement or extension): This logic problem is a good follow-up to the discussion of **POW 4: A Hat of a Different Color**.
- **Find My Region** (reinforcement): You might use this activity, which provides a lighthearted setting in which students practice finding equations for straight-line graphs and inequalities for half planes, after discussing **What’s My Inequality?**
- **Rap Is Hot!** (reinforcement): You can assign this variation on **Rock ‘n’ Rap** any time after the discussion of that activity, perhaps after discussion of **A Rock ‘n’ Rap Variation**. You may want to wait until after students have examined **Rock ‘n’ Rap** on the graphing calculator.

Extension Opportunities:

- **Who Am I?** (reinforcement or extension): This logic problem is a good follow-up to the discussion of **POW 4: A Hat of a Different Color**.
- **Algebra Pictures** (extension): You might use this activity after discussing **Picturing Cookies—Part II**. It extends ideas in the unit by including nonlinear as well as linear inequalities.
- **More Cereal Variations** (extension): This activity asks students to create some additional variations to the cereal activities, **You Are What You Eat** and **Changing What You Eat**. If time allows, you may want to use **More Cereal Variations** in class after the discussion of these activities.
- **How Low Can You Get?** (extension): This activity is appropriate for use beginning late in **Using the Feasible Region**, after students have had some experience examining the effects of changing parameters in a problem, as in **Changing What You Eat** and **A Rock ‘n’ Rap Variation**.
- **Shuttling Variations** (extension): This activity presents two major generalizations of the problem in **POW 6: Shuttling Around**. You may want to assign some or all of this activity as part of the POW itself.
- **And Then There Were Three** (extension): You might have students work on this activity after **Get the Point** and after they have had experience making up two-equations/two-unknown problems in **Inventing Problems**.
- **An Age-Old Algebra Problem** (extension): This activity is a follow-up to the previous supplemental activity, **And Then There Were Three**.

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design
• **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.

• **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.

• **21st Century Learning** – the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow’s workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth. (AASL, 2007)

**Universal Design:** Throughout the unit, students are expected to approach algebraic concepts through visual, written and oral formats. Students are given situations in verbal or written formats, and expected to translate these situations into graphs and equations. Conversely, students are expected to “bring life” to tables and graphs by creating a situation that would create the given table or graph. The use of graphing calculators is integral to the work in this unit.

**21st Century Learning:** This curriculum focuses very strongly upon the use of 21st century skills in mathematics. Students engage in a problem-based curriculum, and are expected to think critically about the problems presented, and to develop strategies to solve these problems. Throughout the problem-solving process, students must be able to build on previous knowledge to build new information and continue to make decisions based on the mathematics at hand. This process is done largely in cooperative groupings. Students have access to technology, and are given multiple opportunities to extend their work into new situations to create generalizations.

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**Technology Integration**

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

• **8th Grade Technology Literacy** - the ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information to improve learning in all subject areas and to acquire lifelong knowledge and skills in the 21st Century (SETDA, 2003).

Students engage daily with graphing calculators as a tool to solve problems. Even with the use of graphing calculators, students must have a solid understanding of creating equations that represent a given situation. Graphing calculators then allow students to create tables and graphs efficiently so that they can identify feasible regions and possible solutions to problems.
Content Connections
Content Standards integrated within instructional strategies

There are many opportunities to relate concepts in biology to those in this unit. Students are often asked to use representations of data to discuss findings and make conclusions about their work. This unit is also highly connected to the engineering, technology and application of science standards set.

Sample Activity Ideas:

NGSS Related Life Science Standards:

HS-LS1-1. Construct an explanation based on evidence for how the structure of DNA determines the structure of proteins which carry out the essential functions of life through systems of specialized cells.

HS-LS1-2. Develop and use a model to illustrate the hierarchical organization of interacting systems that provide specific functions within multicellular organisms.

HS-LS1-3. Plan and conduct an investigation to provide evidence that feedback mechanisms maintain homeostasis.

HS-LS1-4. Use a model to illustrate the role of cellular division (mitosis) and differentiation in producing and maintaining complex organisms.

HS-LS1-5. Use a model to illustrate how photosynthesis transforms light energy into stored chemical energy.

HS-LS1-6. Construct and revise an explanation based on evidence for how carbon, hydrogen, and oxygen from sugar molecules may combine with other elements to form amino acids and/or other large carbon-based molecules.

HS-LS1-7. Use a model to illustrate that cellular respiration is a chemical process whereby the bonds of food molecules and oxygen molecules are broken and the bonds in new compounds are formed resulting in a net transfer of energy.