Appendix A:

Mathematics Unit

Delaware Model Unit Gallery Template

This unit has been created as an exemplary model for teachers in (re)design of course curricula. An exemplary model unit has undergone a rigorous peer review and jurying process to ensure alignment to selected Delaware Content Standards.

Unit Title: **Comparing Numbers**

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Content Area: Mathematics

Grade Level(s): Grade 6

Summary of Unit

In this unit, students will develop number sense around fractions, decimals, ratios and rates. Using manipulatives and, later, visual models and symbols, students will work to develop flexibility when working with different representations, and will develop skills for solving real-world problems as they become fluent with using the representations interchangeably. Of particular focus in this unit are the concepts of equivalence, fractions, and ratios.

Stage 1 – Desired Results

What students will know, do, and understand

Delaware Content Standards

CC.6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, Candidate C received nearly three votes."*

CC.6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b not 0, and use rate language in the context of a ration relationship.

CC.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

CC.6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum to two whole number 1-100 with a common factor as a multiple of a sum of two whole number with no common factor. *For example, express 36 + 8 as 4(9+2).*

CC.6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.6 Attend to precision
MP.7 Look for and make use of structure
MP.8 Look for and express regularity in repeated reasoning

Big Idea(s)

- Flexibility with understanding and comparing values
- Multiple representations can be used equivalently, and we should use the representation that best helps to solve the given problem
- While fractions and ratios may look the same, they are not necessarily
- Using models and pictures can help us understand and solve problems
- Fractions, ratios and rates all help us solve problems

Unit Enduring Understanding(s)

- There are multiple ways to represent numbers.
- We can convert from one form to another in order to solve problems (or to solve problems more efficiently).
- Equivalence implies that two representations have the same value; they may or may not look similar.
- There are an infinite number of equivalent fraction and ratio statements available.
- Scaling is used to find equivalent fractions and ratios; they are multiplicative, not additive.

Unit Essential Questions(s)

- How many ways can we compare numbers?
- How do ratio statements help us understand and make comparisons?
- How can fraction strips and other models help us understand patterns for finding equivalent fractions?
- How can fraction strips and other models help us solve problems?
- What is equivalence—and what does it mean for fractions and ratios?

Knowledge and Skills

Students will know…

- There are multiple ways to represent numbers
- There are infinite equivalent fractions possible

Vocabulary:

- Fraction
- Numerator
- Denominator

- Equivalent fractions
- Number line
- Ratio
- Rate
- Unit rate

Students will be able to…

- Model fractions, decimals, ratios and rates
- Make comparisons between representations
- Use number lines to help with making comparisons
- Find equivalent fractions
- Demonstrate understanding of the Fundamental Law of Fractions
- Use ratios to compare quantities
- Distinguish between fractions representing numbers and ratios representing comparisons
- Distinguish between a difference, which is an additive comparison, and a ratio, which is a multiplicative comparison
- Apply a variety of scaling strategies to solve problems using ratios and unit rates

Stage 2 – Assessment Evidence

Evidence that will be collected to determine whether or not Desired Results are achieved

Suggested Performance/Transfer Task(s)

Adapted from: http://schools.nyc.gov/NR/rdonlyres/DA04B2E8-94CE-4DE4-B902-CEE331D651FB/0/NYCDOEG6MathRatioReasoning_Final.pdf

Mark was mixing blue paint and yellow paint in the ratio of 2:3 to make green paint. He wants to make 45 liters of green paint. He began to make a table to help him think about the problem, but is unsure of what to do next.

Liters of Blue Paint	Liters of Yellow Paint	Liters of Green Paint
2	3	

a. Explain how to continue to place values into the table.

b. Write an explanation in words to Mark about how he can use find out how many liters of blue paint and yellow paint he will need to make 45 liters of paint.

c. What fraction of his mixture must be blue paint? Draw a model and explain how you know this is correct.

Formative Assessment Questions

- How many liters of green paint should there be in the first row?
- How many liters of each color would you need to mix to make 10 liters of green paint?
- Will there be a time when you'll add more blue than yellow to mix the green paint?
- What happens if you don't follow the ratio when you mix the paint?

Differentiation Opportunities

Intervention:

- Allow students to work in partners or in groups.
- Allow students to use manipulatives to model the problem.

Extension:

- Have students create their own ratio problem to share with others.

Rubric(s)

4	*All of the following must be present:*
	• Student recognizes that the green paint is calculated by finding the total blue + yellow, and that additional rows of data can be completed using proportional reasoning. (Student does not need to refer to it as 'proportional reasoning.')
	  ○ Student provides a strong explanation about filling in the table values.
	• Student finds that Mark will need 18 liters of blue and 27 liters of yellow paint to create 45 liters of green paint.
	  ○ Student provides a strong explanation for how to determine the number of liters of blue and yellow paint.
	• Student correctly identifies that blue is 2/5 of the total mixture.
	  ○ Student provides a sound explanation for their reasoning.

3	*Based upon the components of a 4, 1-2 of the explanations may be inadequate or basic. Mathematical responses must be accurate.* • Student recognizes that the green paint is calculated by finding the total blue + yellow, and that additional rows of data can be completed using proportional reasoning. (Student does not need to refer to it as 'proportional reasoning.') ○ Student provides a strong explanation about filling in the table

	values.
	• Student finds that Mark will need 18 liters of blue and 27 liters of yellow paint to create 45 liters of green paint.
	  ○ Student provides a strong explanation for how to determine the number of liters of blue and yellow paint.
	• Student correctly identifies that blue is 2/5 of the total mixture.
	  ○ Student provides a sound explanation for their reasoning.

2	*Student incorrectly responds to one of the three mathematical portions, those correct mathematical responses have reasonable explanations.*
	• Student recognizes that the green paint is calculated by finding the total blue + yellow, and that additional rows of data can be completed using proportional reasoning. (Student does not need to refer to it as 'proportional reasoning.')
	  ○ Student provides a strong explanation about filling in the table values.
	• Student finds that Mark will need 18 liters of blue and 27 liters of yellow paint to create 45 liters of green paint.
	  ○ Student provides a strong explanation for how to determine the number of liters of blue and yellow paint.
	• Student correctly identifies that blue is 2/5 of the total mixture.
	  ○ Student provides a sound explanation for their reasoning.

1	• Student might answer one of the mathematical parts correctly, and is able to provide an explanation. Or • Student provides reasonable explanations but does not give any mathematical responses.
0	None of the parts is answered correctly

Other Evidence

Formative Assessments:

- Daily warm ups
- Observations
- Class work
- Unit problems
- Math journals
- ACE questions (homework assignments)
- Exit Tickets
- Class discussions
- Group discussions
- Portfolios

Summative Assessments:

- Quizzes and tests
- Performance Task

Student Self-Assessment and Reflection

- Math journals
- Self-corrections
- Class and group discussions

Stage 3 – Learning Plan

(Design learning activities to align with Stage 1 and Stage 2 expectations)

Key learning events needed to achieve unit goals

Lesson 1.1: Fundraising

- Focus Question: What are two ways to compare a $500 fundraising goal to a $200 fundraising goal?
- Launch the lesson by activating prior knowledge about students' experiences with fundraising, and ideas about comparing numbers. Discuss the idea of a fundraising goal to make sure a common language is developed.
- Tell the story of the fundraising campaign given in Problem 1.1.
- Make sure student understand their role in the problem: their goal is to evaluate each given claim, and be prepared to discuss whether each is true or not.
- Allow students time to work collaboratively on parts A and B, circulating to groups. Use formative assessment questions to make sure groups have sound strategies for evaluating the claims. Look for groups who have novel approaches to evaluation. When finished, discuss each claim, and have a few groups share their own comparisons.
- Have groups work on part C. Use formative assessment questions to make sure groups have sound strategies for answering the question.
- Summarize the lesson. The goal is to help students make sense of multiplicative comparisons. Students should realize that comparing by subtracting gives different results than using part:whole fractions or ratios. Begin discussing the concept of "for every" to help lay the groundwork for ratios in future lessons, comparing these types of statements to fraction statements. During the summary, encourage students to self-reflect upon their work and make notes/changes in their notes.

Lesson 1.2: Fundraising Thermometers

- Focus Question: How does a "for every" statement show a ratio comparison?
- Launch the lesson by watching the animation video provided with the text. Students should notice that, while the fundraising goal thermometers look the same, the total amount is different, implying that the fractional parts are each worth different amounts. Ensure that students understand the thermometer model.
- Have students work individually, then with partners, to answer part A. Use formative assessment questions to make sure groups have sound strategies for determining the values at the unlabeled marks. Review these as a class.

- Have groups work on parts B-D. Use formative assessment questions to make sure groups have sound strategies for answering the questions.
- Summarize the lesson. The primary goals are tow get students to develop strategies for partitioning, and to use partitioning to generate/evaluate ratio statements using the phrase "for every." Be sure to include popular strategies in the discussion (e.g., the one-tenth and one-half strategies and how they relate to each other), and begin to lay the foundation for equivalence. As students review ratios, be sure to discuss conventions (the first number in the ratio describes the first quantity) as well. During the summary, encourage students to self-reflect upon their work and make notes/changes in their notes.

Lesson 1.3: On the Line *(two days)*

- Focus Question: When you fold fraction strips, what relationship do you see emerge that show how the numerator and denominator change to make equivalent fractions?
- Launch the lesson by connecting to the 6^{th} grade fundraising goal progress. The introduction to this problem should peak students' interest to the new challenge. Make sure students understand that todays' problem involved developing a model for measuring the fundraising progress over time. Discuss the term "equivalent fractions.
- Allow partners to work on parts A and B. Use formative assessment questions to make sure groups have sound strategies for folding their paper and answering the questions.
- Discuss partners' strategies for folding the fractions. Spend time with those in part B, as this thinking will help develop the concept of equivalent fractions.
- Summarize for parts A and B. Begin building the concept of equivalent fractions by looking at commonalities among the fraction strips, and the concept of fractions as values on a number line. During the summary, encourage students to self-reflect upon their work and make notes/changes in their notes.
- Have groups work on parts C-E, making sure that each student creates his or her own individual number line in the process. Use formative assessment questions to make sure groups have sound strategies for answering the questions.
- If time permits, have students create a clothesline number line as a class.
- Summarize the lesson. The focus should be on explicit strategies for finding equivalent fractions, as well as using number lines as a way to interpret fractional values. During the summary, encourage students to self-reflect upon their work and make notes/changes in their notes.

Lesson 1.4: Measuring Progress

- Focus Question: How can fraction strips help you to find part of a number?
- Launch the problem by revisiting the fraction strip model. Students will be able to use these to measure progress in this problem. Make sure students understand they are to focus on determining daily progress for just the 6^{th} graders.

- Allow students to work in groups to answer questions A-E. Use formative assessment questions to make sure groups have sound strategies for answering the questions.
- Summarize the lesson. Students should be comfortable with part:whole relationships and using partitioning to represent these relationships. The summary should also focus upon the concept of equivalence, especially on the relationships rather than the quantities at hand. Ensure that students are able to communicate the meanings of the numerator and denominator in a fraction, and are able to compare fractions with common denominators. During the summary, encourage students to self-reflect upon their work and make notes/changes in their notes.

Check for understanding:

- If the sixth graders and seventh graders each collected $250 out of their goals of $300 and $450, what fractions of each of their goals have they collected?

Lesson 1.5: Comparing Fundraising Goals

- Focus Question: What does it mean for two fractions to be equivalent? What does it mean for two ratios to be equivalent?
- Launch the lesson with the video provided with the text. Ensure that students understand the examples in the video. Introduce the Day 10 fundraising thermometers, comparing and contrasting to previous representations. Review the colon notation if necessary.
- Have students work in groups to complete parts A-E. Use formative assessment questions to make sure groups have sound strategies for answering the questions. If groups are struggling, it could be useful to guide them toward creating a new set of fraction strips for these thermometers.
- Summarize the lesson. Focus on strategies used to make sense of different-sized thermometers, and how part:whole relationships are useful when looking at the money raised by each group. Reinforce the beginnings of proportional reasoning. During the summary, encourage students to self-reflect upon their work and make notes/changes in their notes.
- Have students complete the four mathematical reflection questions in their math journals.

Resources and Teaching Tips

- Student text
- Number lines, fractions strips, physical manipulatives
- Equivalent Fractions online manipulatives: http://illuminations.nctm.org/Activity.aspx?id=3510
- Equivalent Fractions game: http://www.funbrain.com/fract/
- Khan Academy Fractions: https://www.khanacademy.org/math/arithmetic/fractions/Equivalent_fractions/v/equivalent-fractions
- Khan Academy Ratios: https://www.khanacademy.org/math/pre-algebra/rates-and-ratios/ratios_and_proportions/v/introduction-to-ratios--new-hd-version

- Equivalent Ratios game: http://www.mathplayground.com/ASB_RatioBlaster.html

Differentiation

- Students will work in heterogeneous collaborative groupings to provide additional support
- Manipulatives will be available for students to use as needed
- Homework assignments will be differentiated based upon student needs
- Scaffolded questioning strategies will be used to elicit student thinking and construction of knowledge
- Additional practice problems may be available for students who need additional exposure to the concepts
- Extension problems will be available for students who demonstrate understanding

Design Principles for Unit Development

At least one of the design principles below is embedded within unit design

- **International Education** - the ability to appreciate the richness of our own cultural heritage and that of other cultures in to provide cross-cultural communicative competence.
- **Universal Design for Learning** - the ability to provide multiple means of representation, expression and engagement to give learners various ways to acquire and demonstrate knowledge.
- **21st Century Learning –** the ability of to use skills, resources, & tools to meet the demands of the global community and tomorrow's workplace. (1) Inquire, think critically, and gain knowledge, (2) Draw conclusions make informed decisions, apply knowledge to new situations, and create new knowledge, (3) Share knowledge and participate ethically and productively as members of our democratic society, (4) Pursue personal and aesthetic growth.(AASL,2007)

Universal Design for Learning: Students will have opportunities to build their understanding through different ways of representing the relationships in the problems. Students will also convey their learning through manipulatives, written and oral responses, and will have opportunities to complete extension problems.

21st Century Learning: Student will use critical thinking skills while developing understanding of equivalence with fractions and ratios. They will work to develop their own thinking, while questioning and providing feedback to others. Students will work collaboratively to build and apply this understanding to multiple real-world problems.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

- Students will learn how to use calculators to solve problems involving equivalence
- Additional online resources for support:
  - Equivalent Fractions online manipulatives: http://illuminations.nctm.org/Activity.aspx?id=3510
  - Equivalent Fractions game: http://www.funbrain.com/fract/
  - Equivalent Ratios game: http://www.mathplayground.com/ASB_RatioBlaster.html

Content Connections

Content Standards integrated within instructional strategies

ELA connection: Students will be speaking, listening and writing about mathematics on a daily basis. Students will be expected to provide responses, defend their responses, and have discussions about mathematics. These learning experiences will also be reflected in their written responses to problems (including explanations for why their solutions work), as well as their math journal entries daily.