SpringBoard Mathematics © 2014
Common Core Edition

Course 2 - Unit Sampler

The Pathway to Advanced Placement and College Readiness
SpringBoard provides a comprehensive and systematic approach to preparing ALL stud
demands of rigorous AP courses, college classes, and other postsecondary experiences
prepares students through sequential, scaffolded development of the prerequisite skill
knowledge needed for success in AP Calculus and Statistics.

In each unit of study, explicit AP Connections are outlined in the Planning the Unit page
teacher editions and are reinforced as they appear in student activities. Through ongoing
rigorous mathematics content and experience with the thinking processes needed to a
and explain complex math problems, students exit SpringBoard equipped with the kind
order thinking skills, knowledge, and behaviors necessary to be successful in AP classes.

For More Information on the SpringBoard Program visit www.Collegeboard.org/Spring
Unit 3: Ratio and Proportion

Planning the Unit

- **Instructional Focus and Pacing:** Identifies essential skills and knowledge leading to success on interim Embedded Assessments
- **AP/College Readiness Connections:** Prepares students for the rigor of AP courses and provides teachers with important scaffolding insights

Unit 3 Overview

- **Academic Vocabulary and Math Terms:** Defined, taught, and analyzed in context
- **Essential Questions:** Lesson guideposts ensure classroom work is focused and
- **Getting Ready:** Formative evaluation of students’ readiness for unit topics; identifies requisite skill areas in need of remediation

Activity 9: Proportional Reasoning

Lesson 9.1: Equations Representing Proportional Relationships

- **Learning Targets/Common Core State Standards:** Identified for each activity
- **Embedded Assessment**
- **Plan—Teach—Access—Adapt:** Lesson format consistent with AP instruction.
- **Differentiating Instruction:** Lesson-specific strategies to adapt product, process, and content for a variety of learners

Lesson 9.2: Constants of Proportionality

- **Math Tip:** Lesson-specific strategies printed in student text to assist with problem-solving.
- **Teacher to Teacher:** Lesson delivery considerations, written by teachers for teachers.
- **Mathematical Practice Standards:** Integration of standards to support student learning and higher-order thinking

Activity 9 Practice

Embedded Assessment #1: Ratios, Proportions, and Proportional Reasoning

- **Performance-Based Task:** Weighing in on Diamonds
- **Scoring Guide:** Criteria-based rubric assesses content understanding, solution approach, representation, and communication

Additional materials in the back of this Sampler include...

- Vertical Unit Connections
To the Teacher

Welcome to SpringBoard Mathematics, a highly engaging, student-centered instructional program. This revised edition of SpringBoard is based on the standards defined by the Common Core State Standards for Mathematics for each course. The program may be used as a core curriculum that will provide the instructional content that students need to be prepared for future mathematical courses. SpringBoard courses do the following:

- Expect students to practice applying mathematical ways of thinking to real-world issues and challenges.
- Require students to develop a depth of understanding and ability to apply mathematics to novel situations—as college students and employees regularly are called to do.
- Emphasize mathematical modeling and reasoning—using mathematics and statistics to analyze empirical situations, understand and make inferences about the situations, and improve decision making about how to solve problems and justify those solutions.

**Shifting Mathematics Instruction**

With an increased emphasis on better preparing students to understand and master mathematical concepts, mathematics instruction has become a major focus of attention. Efforts at improvement center around the following points:

**Greater Focus on the Content of the Standards:**

- Learn more about less by spending more time on fewer concepts.
- Significantly narrow and deepen the scope of how time and energy are spent in the classroom.
- Focus on the essential learning that helps students develop strong foundational knowledge and deep conceptual understanding to enable them to transfer skills and knowledge across concepts and grades.

**Coherence to Link Major Topics:**

- Connect learning within a grade and build knowledge across grades.
- Spiral learning so that students can build new understanding on the foundations built in previous years.
- Focus on learning progressions so that teachers can continue counting on students' deep conceptual understanding of core content and build on it.

**Rigor with Balance:**

- Develop fluency in procedural skills—computation, application, understanding.
- Promote depth and mastery by connecting concepts, practice, and independent application.
career, or both. Students who are prepared for college or career will be able to do the following:

- **Build on content knowledge**: Students will have a base knowledge of number and quantity, algebra, functions, geometry, and statistics and probability on which to extend their learning.
- **Use mathematical models**: Students will be able to use a variety of mathematical representations to model what they know and to justify how they are using their knowledge.
- **Communicate mathematics**: Students will communicate verbally and in writing to explain their discoveries and understanding of mathematics and how it works theoretically and in the real world.
- **Collaborate with others**: Students will participate in discourse focused on discovery and problem solving, evaluate the contributions of others, and collaborate to present and defend viable solutions.
- **Use technology**: Students will use appropriate technology to enhance their understanding of mathematics and to gain greater precision in areas where technology is appropriate.

The implications of these student expectations are that students will need to develop greater depth of knowledge, higher-level thinking skills, and effective communication skills. What they need less of will be memorization, drills and worksheets, and “one size fits all” content.

**SpringBoard’s Role in Preparing Students for College and Career Success**

With this revised edition, the SpringBoard program provides a roadmap for attaining the knowledge and skills students require for success in Advanced Placement courses, in college-level work, and in careers. Based on the Common Core State Standards for Mathematics and current research on best instructional practices, SpringBoard uses a “back-mapping” instructional design that starts with the end in mind, namely, the skills and knowledge students need to use mathematics effectively and to demonstrate that ability through performance on various assessments.

The mathematics instruction follows a balanced approach in which concepts are presented based on the most effective instructional methods: **directed** for basic mathematics principles, including examples and practice; **guided** for concepts that need a combination of direct instruction and investigatory learning; and **investigative** activities that allow students to explore and discover mathematics concepts through a contextual setting.
Multiple lessons per activity.
• Worked-out examples as needed to help students learn and apply concepts.
• Frequent Check Your Understanding questions to help students assimilate and apply knowledge.
• Mathematical practices called out so students are reminded to apply them as they respond to problems and applications.
• Lesson Practice problems to provide the opportunity to practice new learning and to build fluency.
• Activity Practice provides additional practice problems for each lesson in the activity.
• Embedded Assessments give students new contexts for applying the concepts learned in the unit and give you the opportunity for regular formative assessment.

Integration of Mathematical Practices
Through its instructional design, the SpringBoard mathematics program requires students to integrate effective mathematical practices into their learning. With its process of questioning students within a lesson and asking them to think through concepts and applications, SpringBoard reinforces the actions and practices that help students build knowledge and skills. SpringBoard requires students to:
• Make sense of and connect mathematics concepts to everyday life through problem contexts.
• Model with mathematics to solve problems, justify solutions and their reasonableness and communicate mathematical ideas.
• Use appropriate tools, such as number lines, protractors, technology, or paper and pencil, strategically to help solve problems.
• Communicate abstract and quantitative reasoning both orally and in writing through viable arguments and critiques.
• Analyze mathematical relationships through structure and repeated reasoning to connect ideas.
• Attend to precision in both written and oral communication of mathematical ideas.

Engaging and Interactive Online Edition
With this new edition, SpringBoard introduces an all-new interactive online experience for both students and teachers. In addition to providing all content online, the new SpringBoard Digital program:
• Allows access at any time.
• Discerns the device you’re using and adjusts content to fit screens—from desktops to laptops to tablets.
New Assessment Options

The SpringBoard program now provides the option of using the ExamView test generator program for all grades. Teachers will have multiple options for choosing premade tests or making their own. Options include:

- Unit tests aligned to standards and the content in each unit.
- Test banks allowing teachers to choose items and create tests for multiple needs, including benchmark tests and quarter or semester tests.
- Expanded test item types, including short response and interactive simulations and manipulatives.

What Sets SpringBoard Apart from Other Mathematics Programs?

Three key things set SpringBoard apart:

1. The expectation that students can do rigorous work with the right preparation.
2. Learning materials that reflect both rigor and the expectations about what students should know and be able to do.
3. Extensive teacher support through professional development and coaching services.

Unique features of SpringBoard include:

- **Rigorous, standards-based instruction**: Instructional content organized around the Common Core State Standards for Mathematics to provide coherent topics that build knowledge and skills throughout each course and across grade levels.

- **Mathematical practices**: Integration of the Standards for Mathematical Practice that support student learning and higher level thinking.

- **Research-based instruction**: Back-mapped instructional design gives students a learning target and scaffolds activities to develop students' knowledge and skills and prepare them to demonstrate their learning on an Embedded Assessment.

- **Student-centered, interactive, collaborative activities and lessons**: Each course is organized into short, interactive activities that are further divided into focused lessons. Lessons engage students and aid learning by having students participate in class discussions, solve problems and justify solutions, and demonstrate learning through multiple means of evaluation.

- **Integrated teaching and learning strategies**: Suggested Learning Strategies in each lesson help students use methodical approaches to learning new content, helping students take control of their own learning by identifying which strategies work best for them. Teachers also use
To the Teacher  continued

- **Professional development:** Unparalleled professional development builds teacher capacity to deliver challenging curriculum to meet the needs of all students while honoring the creativity and intelligence teachers bring to the classroom. Face-to-face training is supported by an online system featuring resources that include an interactive professional learning Community that allows peer-to-peer sharing and sustains successful teaching.

**The Pathway to Advanced Placement and College Readiness**

SpringBoard provides a comprehensive and systematic approach to preparing ALL students for the demands of rigorous AP courses, college classes, and other postsecondary experiences. SpringBoard prepares students through sequential, scaffolded development of the prerequisite skills and knowledge needed for success in AP Calculus and Statistics. In each unit of study, explicit AP Connections are outlined in the Planning the Unit pages of the teacher editions and are reinforced as they appear in student activities. Through ongoing exposure to rigorous mathematics content and experience with the thinking processes needed to analyze, solve, and explain complex math problems, students exit SpringBoard equipped with the kind of higher-order thinking skills, knowledge, and behaviors necessary to be successful in AP classes and beyond.

**From Pre-AP to AP and Beyond**

Beginning in middle school, students are introduced to concepts and skills that are fundamental to success in AP mathematics and statistics courses.

**Grade 6** students learn to:

- Model functions in numerical, symbolic (equation), table, and graphical forms.
- Communicate mathematics in writing and verbally, justifying answers and clearly labeling charts and graphs.
- Explore and represent data in a variety of forms.
- Use multiple representations to communicate their mathematical understanding.

**Grade 7** students continue to:

- Acquire an algebraic and graphical understanding of functions.
- Write, solve, and graph linear equations; recognize and verbalize patterns; and model slope as a rate of change.
- Communicate clearly to explain methods of problem solving and to interpret results.
- Investigate concepts presented visually and verbally.
**Algebra 1** students:
- Gain an understanding of the properties of real numbers.
- Formalize the language of functions.
- Explore the behavior of functions numerically, graphically, analytically, and verbally.
- Use technology to discover relationships, test conjectures, and solve problems.
- Write expressions, equations, and inequalities from physical models.
- Communicate mathematics understanding formally and informally.

**Geometry** students:
- Read, analyze, and solve right triangle and trigonometric functions within contextual situations.
- Develop area formulas necessary for determining volumes of rotational solids, solids with known cross sections, and area beneath a curve.
- Explain work clearly so that the reasoning process can be followed throughout the solution.

**Algebra 2** students:
- Develop the algebra of functions through operations, composition, and inverses.
- Read and analyze contextual situations involving exponential and logarithmic functions.
- Work with functions graphically, numerically, analytically, and verbally.
- Learn optimization problems.
- Compare the relative rate of change of linear and exponential functions.
- Learn the concept of infinite sum as a limit of partial sums.
- Work with statistics in numerical summaries, calculations using the normal curve, and the modeling of data.

**Precalculus** students:
- Gain an introductory understanding of convergence and divergence.
- Collect, analyze, and draw conclusions from data.
- Solve problems in contextual situations dealing with polynomial, rational, logarithmic, and trigonometric functions.
- Model motion using parametric equations and vectors.
- Develop an intuitive understanding of limits and continuity.
- Justify their reasoning and understanding verbally, in writing, and with models.
- Use technology to explore and support conjectures.

**The SpringBoard Mathematics Classroom**
safe to explore ideas and learn effective communication skills. Collaborative
groups allow learning to be active as students engage in discussions, make
conjectures, question, and discover new ideas as they fulfill tasks within the
group. Group structure should encourage all members to work together to
complete a given task while ensuring that each student is held accountable for
equal participation in the assignment.

Debriefing/Reflections
Frequently in a mathematics classroom, students and teachers should engage in debriefings. The purpose of debriefing is to allow students to reflect on
their learning, correct misconceptions, identify thinking processes used during
an activity, summarize information, and process what they have learned.
Debriefing can be accomplished in a variety of ways, including whole-class
discussion, small-group discussion, group presentation, and individual writing.

Interactive Word Wall
The class Word Wall facilitates vocabulary development and provides a
reference during class and group discussions. Creating and maintaining a Word
Wall is an ongoing activity. It should be an instructional tool, not just a display.
Designate a specific space (such as a bulletin board or blank wall space) in the
classroom for the Word Wall. Words may be written on index cards, sentence
strips, or blank paper by you or by your students.

- You or your students may add new words to the Word Wall.
- Spend time revisiting words on the Word Wall whenever possible.
- Emphasize the categorization of words to help students see the logic in
  language.
- Invite students to generate a list of potential categories to sort words
  (alphabetical order, mathematical concepts, problem solving techniques,
  etc.).
- Make the words into manipulatives by writing them on cards so that they
  can be shifted, added, and/or eliminated.
- Encourage students to use words from the Wall correctly in their class and
  group discussions, and specifically on the Embedded Assessments.

Math Notebook
Keeping a Math Notebook helps students learn and explore new vocabulary
while also summarizing notes about math concepts and ideas. It is an
intentional tool for students to expand their understanding of mathematics
terms and concepts. The Math Notebook may be any type of notebook, or it
may be an online tool for students who have regular access to SpringBoard
Mathematics Online. As students are introduced to new vocabulary, they can
explore terms and concepts and make notes about other math terms that are
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RESOURCES
Formulas
Learning Strategies
Glossary
Academic Vocabulary Graphic Organizers
In this unit students develop an understanding of and apply proportional relationships as they study ratios, unit rates, equations and the constant of proportionality. Students study percent and a wide variety of applications such as tax, commission, mark-up, discount, and percent increase/decrease and error. They study and apply scale drawings and solve related problems.

**Vocabulary Development**

The key terms for this unit can be found on the Unit Opener page. These terms are divided into Academic Vocabulary and Math Terms. Academic Vocabulary includes terms that have additional meaning outside of math. These terms are listed separately to help students transition from their current understanding of a term to its meaning as a mathematics term. To help students learn new vocabulary:

- Have students discuss meaning and use graphic organizers to record their understanding of new words.
- Remind students to place their graphic organizers in their math notebooks and revisit their notes as their understanding of vocabulary grows.
- As needed, pronounce new words and place pronunciation guides and definitions on the class Word Wall.

**Embedded Assessments**

Embedded Assessments allow students to do the following:

- Demonstrate their understanding of new concepts.
- Integrate previous and new knowledge by solving real-world problems presented in new settings.

They also provide formative information to help you adjust instruction to meet your students’ learning needs.

Prior to beginning instruction, have students unpack the first Embedded Assessment in the unit to identify the skills and knowledge necessary for successful completion.

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**CollegeBoard**

**Algebra / AP / College**

This unit develops student proportion by:

- Using real-world contexts to learn and develop concepts.
- Encouraging students to reason about the reasonableness of results and relative accuracy.
- Providing opportunities for students to work in groups to analyze data and communicate their findings.
- Asking students to apply mathematics to real-world situations.

**Unpacking the Embedded Assessment**

The following are the key skills that will need to know for each assessment.

**Embedded Assessment**

- Solve problems involving ratios, proportions, and unit reasoning.
- Convert between measurements using rates and using proportional relationships.
- Represent constant rates using proportions of the form \( y = kx \).
- Determine the constant rate from a table, graph, or equation.

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http://www.springboardonline.com/flipbooks/springboard-middle-school-math/springboar...
Planning the Unit

Embedded Assessment 3

**Percents and Proportions, Socializing and Selling**
- Find the percent of a number
- Find the percent that one number is of another
- Given the percent and the whole, find the part
- Solve problems about sales tax, tips, and commissions
- Solve problems about percent increase, percent decrease, markups, and discounts
- Solve problems about interest and percent error

**Suggested Pacing**
The following table provides suggestions for pacing using a 45-minute class period. Space is left for you to write your own pacing guidelines based on your experiences in using the materials.

<table>
<thead>
<tr>
<th></th>
<th>45-Minute Period</th>
<th>Your Comments on</th>
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<tbody>
<tr>
<td>Unit Overview/Getting Ready</td>
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<td></td>
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<td>Activity 8</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Activity 9</td>
<td>4</td>
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<td>Activity 11</td>
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<td>Activity 12</td>
<td>4</td>
<td></td>
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<tr>
<td>Embedded Assessment 3</td>
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<td></td>
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<tr>
<td><strong>Total 45-Minute Periods</strong></td>
<td><strong>22</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Additional Resources**

Additional resources that you may find helpful for your instruction include the

http://www.springboardonline.com/flipbooks/springboard-middle-school-math/springboar...
Ratio and Proportion

Unit Overview
In this unit, you will use pictures, graphs, tables, and verbal descriptions to study unit rates, rate of change, and proportions. You will solve problems involving scale, percentage, and proportional relationships.

Key Terms
As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary
- tip

Math Terms:
- ratio
- rate
- unit rate
- proportion
- cross products
- conversion factor
- constant of proportionality
- constant ratio
- constant rate of change
- relative size
- scale drawing
- percent
- percent equation
- discount
- markup
- interest
- percent error

Developing Math Language
As this unit progresses, use support among peers to help students develop background knowledge need to...
UNIT 3
Getting Ready

Use some or all of these exercises for formative evaluation of students’ readiness for Unit 3 topics.

Prerequisite Skills
- Ratios, Tables and Graphs (Items 1, 2, 3) 6.RP.A.3, 6.RP.A.3a
- Expressions and Equations (Items 4, 5) 8.EE.C.7
- Fractions, Decimals and Percents (Items 6, 7, 8) 7.NS.A.2, 5.NF.B.4

Answer Key
1. \( \frac{7}{10} \); 7 to 10
2. See below table.
3. Janese’s Toe Touches

<table>
<thead>
<tr>
<th>Time in Seconds</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
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<td>0</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td>35</td>
<td>42</td>
</tr>
</tbody>
</table>

4. a. \( \frac{106.25}{x} \) b. \$3.67g c. \( 3n - 5 \)
5. a. \( x = \frac{3}{2} \) b. \( x = 4 \)
6. | % | Decimal | Fraction |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>0.25</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>50%</td>
<td>0.5</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>15%</td>
<td>0.15</td>
<td>( \frac{3}{20} )</td>
</tr>
</tbody>
</table>
7. a. \( 25\% \) b. \( 40\% \)
8. Answers will vary. To find \( \frac{1}{3} \) of 60, multiply \( \frac{1}{3} \times 60 \), which equals 20. To find 25% of 60, change 25% to a decimal .25 and multiply .25 \times 60, which is 15. So, \( \frac{1}{3} \) of 60 is larger, because 20 > 15.

UNIT 3
Getting Ready

Write your answers on notebook paper. Show your work.
1. Janese can complete 7 toe touches in 10 seconds. Write a ratio of Janese’s toe touches to seconds in three ways.
2. Complete the following table representing Janese’s toe touches.

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toe Touches</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

3. Use the grid below to graph Janese’s toe touches. Label the horizontal and vertical axes. Provide a scale on the horizontal and vertical axes.

4. Write an algebraic expression for each of the following.
   a. The cost of each ticket, if \( x \) tickets cost \( \frac{106.25}{x} \)
   b. The cost of \( g \) gallons of gas if each gallon costs \$3.67
   c. Five less than 3 times a number

5. Solve each of
   a. \( 2x + 5 = 8 \)
   b. \( 16 + 3x = -2 \)
6. Copy and complete the table showing equivalent values:

<table>
<thead>
<tr>
<th>%</th>
<th>Decimal</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>0.25</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>0.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7. What percent
   a. \( \frac{1}{2} \)
   b. \( \frac{3}{4} \)
8. Explain how to solve the following values:
   a. \( \frac{1}{2} \)
   b. \( \frac{3}{4} \)

Getting Ready Practice:
For students who may need additional instruction on one or more of the prerequisite skills for this unit,
Proportional Reasoning
Scrutinizing Coins
Lesson 9-1 Equations Representing Proportional Relationships

Learning Targets:
- Given representations of proportional relationships, represent constant rates of change with equations of the form $y = kx$.
- Determine the meaning of points on a graph of a proportional relationship.
- Solve problems involving proportional relationships.

Suggested Learning Strategies: Shared Reading, Marking the Text, Summarizing, Use Manipulatives, Look for a Pattern, Predict and Confirm, Discussion Groups

Ratios and proportions are used to solve all kinds of problems in the real world. For example, ratios and proportions are used in cooking to double recipes, by travelers to find distances on maps, and by architects to make scale models.

Work with your group to explore the proportional relationship between the number of pennies in a stack and their heights in millimeters. You will need a centimeter ruler and 25 pennies. As you work with your group, you may hear math terms or other words that are unfamiliar. Record words that are frequently used in your math notebook. Ask for clarification of their meaning and make notes to help you remember how they are used.

1. Without using your pennies or ruler, predict the height of a stack of 150 pennies, and explain why you made this prediction. Be sure to include units in your prediction.
   I thought 1 penny might be about 2 mm, so I multiplied 150 by 2 to get 300 mm.

2. Attend to precision. Explore this finding by measuring and recording the height of a stack of each number of pennies in the table below. Sample answers are given.

<table>
<thead>
<tr>
<th>Number of Pennies</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Stack (mm)</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
</tr>
</tbody>
</table>

3. Write a ratio in fraction form that relates the number of pennies to the height of a stack. Answers will depend on results given in table.
   a. 10 pennies $\frac{10}{12}$
   b. 15 pennies $\frac{15}{18}$
   c. 20 pennies $\frac{20}{24}$
   d. 25 pennies $\frac{25}{30}$

4. Write a ratio that relates the number of pennies in each stack at the right to the height of the stack.
   The ratio is 5:7. For every 9 pennies added, the stack will increase by 7 mm.

Common Core State Standards for Activity 9
7.RP.A.2 Recognize and represent proportional relationships between quantities.
ACTIVITY 9 Continued

7–8 Create Representations, Look for a Pattern Students will use proportional reasoning to complete a table using the ratio 5:7, which represents the relationship between the number of pennies in a stack and the height of the stack in millimeters. In Item 8, they write two equivalent ratios as an equation. Point out that since the ratios are equivalent, the equation is a proportion.

9–11 Debriefing In this set of items, students should understand that they do not need to measure the height of a stack of 60 pennies in order to predict height. Instead they can use the proportion they wrote in Item 8 with the height of the pennies as a variable, and then solve the proportion for the value of the variable. They will need to recall how to solve proportions from Lesson 8–3.

TEACHER TO TEACHER

In Item 9, it is important that students first make a reasonable estimate of the height of 60 pennies before they solve the proportion. This will help them find errors when writing the proportion. It is important to debrief this part of the lesson to remind students that if the height of the stack is the denominator of the left-hand side of the proportion, then the unknown height of the stack is also in the denominator of the right-hand side of the proportion. In Item 11, you want students to contrast the two ways of solving a proportion.

CONNECT TO AP

Students observe that the height of a stack of pennies is proportional to the number of pennies in the stack. They will spend a great deal of time in future mathematics courses, such as calculus, writing proportions from written descriptions of proportional relationships. They need a solid understanding of this kind of proportional reasoning as a basis of work they will do with volume in

<table>
<thead>
<tr>
<th>Equations Représen</th>
</tr>
</thead>
</table>
| 5. What do you notice about the ratio
  The ratios are equivalent or very close.
| 6. Use the ratio you found in Item 4 to complete the table below.
| Number of Pennies | 10 |
| Height of Stack (mm) | 14 |
| 7. Use your table in Item 6 to answer
  a. Write two ratios in fraction form to the height of the stacks.
  \[ \frac{10}{14}, \frac{16}{21} \]
  b. Write these ratios as an equation
  \[ \frac{10}{14} = \frac{16}{21} \]
  c. Is your equation a proportion? Yes, because the two ratios are equal.
| 8. When quantities are proportional, change.
  a. What is the rate of change of the stack?
  b. Explain what the rate of change means. For every 5 pennies added, the stack will be 4 mm higher.
| 9. Make use of structure. How can we use our understanding of the height of a stack of 60 pennies to find the height of a stack of 372 pennies?
  You could write a proportion and solve for the unknown.
  \[ \frac{10}{14} = \frac{372}{x} \]
  This means the stack will be 521 mm high.
| 10. Now suppose we wanted to find the height of a stack of 60 pennies without having 60 pennies.
  a. Determine a reasonable estimate of the height of a stack.
  b. I could write a proportion and solve for the unknown. In this case, I would have
  \[ \frac{10}{14} = \frac{372}{x} \]
  This means the stack will be 521 mm high.
| 11. Compare and contrast your method. It was more difficult to find the answer common numerators or denominators. In the case of Item 10, I had to use cross-products and solve the equation, 10x = 14 * 372.
Lesson 9-1  
Equations Representing Proportional Relationships

12. Why might the value you determined for height in Items 9 and 10 be different from the actual measured height of a stack of 60 pennies or 372 pennies?
   The ratio 6:7 gives an estimate only. Actual pennies may be worn or not stacked evenly or may have a number of other conditions that could influence the actual height.

13. Write and solve a proportion to determine the number of pennies, $x$, in a stack that is 100 mm high. Use numbers, words, or both to explain your method.
   \[
   \frac{5}{7} = \frac{x}{100 \text{ mm}}
   \]
   Write the proportion as an equation using cross-products, $7x = 5 \times 100$. Next, solve the equation for $x$ by dividing 500 by 7. The number of pennies in a 100 mm stack is about 71.
   
   The proportional relationship between the number of pennies in a stack and the height of the stack that you recorded in the table in Item 7 can also be represented in a graph. The graph will help you predict the height of a stack of pennies.

14. Graph the data from Item 7 onto the graph.

   ![Graph](image)

15. What does a point $(x, y)$ on the graph mean for this situation?
   A stack of $x$ pennies has a height of $y$ mm.

16. Construct viable arguments. Does it make sense to include the point $(0, 0)$ on your graph? Explain. If yes, plot $(0, 0)$ on your graph.
   Yes; it represents no pennies in a stack that is 0 mm tall.
**ACTIVITY 9 Continued**

**17-19 Visualization, Critique Reasoning.** After graphing the points, help students see that the points could be connected by a line through the origin. The graph represents a proportional relationship and can therefore be used to predict the height of a given stack of pennies. Discuss the meaning of each point \((x, y)\) to help students understand the relationship between the number of pennies and the height of the stack of pennies. Have students explain why a point without whole-number coordinates does not make sense in this situation.

**20-24 Look for a Pattern, Create Representations, Debriefing.** In this set of items, students use the graph they created in Item 14 to explore other possible points of the graph. If the other points lie along the same line, then they can be used to predict the height of a given number of pennies. Make sure you debrief these items to help students understand that this proportional relationship can also be written as an equation of the form \(y = kx\), where \(k\) is the height of one penny. In later lessons, students will describe this type of equation as a direct variation equation.

**ACTIVITY 9 continued:**

**Math Tip:**
Remember to think about whether or not you should connect the points on your graph.

17. If the points on your graph were scattered, what would they look like?
   - a line
   - a curve
   - no pattern

18. How does the graph in Item 14 show that the relationship between the height of the stack and the number of pennies is proportional?
   - The graph is a straight line through the origin.
   - The graph is a curve.
   - The graph is a horizontal line.

19. Does it make sense to include the point \((0, 0)\)? Explain.
   - No, it is not reasonable to have zero pennies.
   - Yes, it is reasonable to have zero pennies.

20. Use your graph to predict the height of a stack of 30 pennies. Explain your method.
   - I extended the pattern of the graph.
   - I used a table to find the height of 30 pennies.
   - I estimated the height by measuring on the graph.

21. What does it mean for the ratio of the height of a stack of pennies to be in the ratio of 1 penny to 1.4 pennies?
   - For every 1 penny, the height is 1.4 times the number of pennies.
   - For every 1 penny, the height is 1.4 times the height of the stack.
   - For every 1 penny, the height is 1.4 times the number of pennies.

22. Find the height of a stack of 30 pennies.
   - Use the graph. Explain your reasoning.
     - I extended the pattern of the graph.
   - Use the proportion.
     - The height of 1 penny is 1.4 times the number of pennies.

23. What equation could you write to find the height of a stack of pennies?
   - \(y = 1.4x\)
   - \(y = kx\)
   - \(y = \frac{x}{1.4}\)

24. Use your equation in Item 24 to find the height of a stack of 35 pennies. Confirm this solution using another method.
   - 48 mm; yes
   - 48 mm; no
   - 48 mm; unsure

**MINI-LESSON:** Equivalent Proportions

Students should be familiar with these proportions equivalent to...
Lesson 9-1
Equations Representing Proportional Relationships

Check Your Understanding

25. Model with mathematics. Look back at your original prediction for the height of a stack of 150 pennies.
   a. Use a proportion to revise your original prediction. Explain your reasoning.
   b. Use the equation you wrote in Item 24 to revise your original prediction. Justify your reasoning.
   c. Explain how you could use your graph to revise your original prediction.

LESSON 9-1 PRACTICE

26. Solve the proportion \( \frac{3}{5} = \frac{28}{x} \) using two different methods. Explain each method.

27. Construct viable arguments. Solve \( \frac{x}{42} = \frac{3}{7} \) using two different strategies. Explain each strategy.

28. Is the ratio 4.2:1.5 proportional to the ratio 12.6:4.5? Explain.

29. Is the ratio 35 to 10 proportional to the ratio 7 to 2? Explain.

30. At Lake Middle School, the average ratio of boys to girls in a classroom is 3:2. Use a proportion to predict the number of girls in a classroom that has 15 boys.

31. Complete the ratio table below to show ratios equivalent to 4:18.

| 48 | 160 | 20 | 8 | 90 |

32. Use the graph at the right.
   a. Predict the number of chocolate chips in nine pancakes. Explain.
   b. Predict the number of pancakes that would have 48 chocolate chips. Explain.
   c. What does the point (1, 8) mean in this situation?
   d. Which of the equations below represents this situation?
      A. \( y = 16x \)
      B. \( y = 8x \)
      C. \( y = x \)
      D. \( y = 48x \)

33. Three steps of a staircase are shown here.
   a. What is the ratio of the width of a step to its height? \( \frac{11.5}{8.5} \text{ or } \frac{11.5}{8} \)
   b. Explain why the staircase represents a constant rate of change.
   The ratio of the width to the height of each step is constant.
   c. What does the rate of change mean in the context of a staircase?
   For every width of 11.5 inches, there is a height of 8.5 inches.

LESSON 9-1 PRACTICE

26. \( \frac{4}{5} = \frac{28}{x} \); One way to solve this is to multiply the denominator by 7. This gives an answer of 35.
   Another way to solve this proportion is to set the
ACTIVITY 9 Continued

Lesson 9-2

Pace: 1 class period

**Chunking the Lesson**

#1-7
#8-14

Check Your Understanding
Lesson Practice

**TEACH**

**Bell-Ringer Activity**

Have students determine the relationship between the number of students in the class and the number of chair legs in the class. Ask them to predict how the number of chair legs would change if 4 more students were added to the class. Discuss with students how they made their predictions.

**Introduction Shared Reading:** Have students read the introductory paragraphs and highlight the important terms in cooperative learning groups. Remind students that the circumference of a circle is the length around the circle.

1-6 **Create Representations, Look for a Pattern, Think-Pair-Share**

In these items, students use tables to record the circumference and diameter of different circles (coins) to the nearest millimeter. They then write the ratios of the length of the circumference to the length of the diameter as a fraction. Point out that each ratio is equal to about 3.15, and that this is the approximate value of \( \pi \) (pi).

Have students recall the formula for the circumference \( C \) of a circle is \( C = \pi d \), where \( d \) is the diameter. While students work on Item 6, point out that the formula is an equation of the form \( y = kx \), where \( k \) is 3.15, \( y \) is \( C \), and \( x \) is \( d \). It is important for students to have the opportunity to discuss the model for circumference of a circle given the diameter before moving on to representing an equation of the form \( y = kx \) in a graph.

**Math Terms**

A **constant ratio** occurs when the ratio between two variables is constant.

**Math Tip**

To measure the circumference, wrap a piece of tape around the edge of your coin. Make a mark on the tape to show where the tape begins to overlap.

Unwrap the tape and place it along the edge of your centimeter ruler.

Find the average of a set of data items by adding the items and then dividing by the number of data items.

**Developing Math Language**

Encourage students to express ideas using the appropriate academic language.

Have students add these words to their math notebooks. As they move on...

---

**Learning Targets:**

- Determine the constant of proportionality, or verbal description of a proportional relationship, from a graph, table, two sets of data, or the context of a real-world problem.

**Suggested Learning Strategies:**

- Text, Interactive Word Wall, Notes, Revision

When two quantities are proportional the constant ratio can be found between the corresponding input values.

Work with a partner and use the relationships of a circle to explore finding the circumference. You will need a centimeter ruler, tape, a penny, or nickel, or quarter.

1. Use your string and ruler to measure to the nearest millimeter. Record the measurements below.

<table>
<thead>
<tr>
<th>Circumference (mm)</th>
<th>Penny</th>
<th>Nickel</th>
<th>Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
<td>52</td>
<td>58</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

2. Use your ruler to measure the diameter in millimeter. Record the measurements.

3. For each coin write the ratio of the circumference to the length of the diameter as a fraction.
   - Penny: \( \frac{50}{19} \approx 2.63 \)
   - Nickel: \( \frac{52}{20} = 2.60 \)
   - Quarter: \( \frac{58}{21} \approx 2.76 \)

4. Because the ratios are very close to proportional relationship. Calculate the ratios. About 3.15 mm

5. Suppose you had a coin with a diameter that you are not sure it will be. Expect its circumference to be a little over 10 mm; \( 30(3.15) \approx 94.5 \).

6. Model with mathematics. Write an equation using the constant of proportionality to determine the circumference, \( y \), of a circle. Explain. \( y = 3.15 \times \text{the diameter} \)
Lesson 9-2
Constants of Proportionality

The factor \( k \) that you multiplied by in Item 6 also represents the **constant rate of change** in the situation.

7. What is the constant rate of change in the equation you wrote?  
   \[ 3.15 \]

Graphs can also be used to find a constant of proportionality in proportional relationships.

The graph below shows the number of pennies in a number of standard coin rolls.

8. Plot a point at \((0, 0)\) and connect the points with a line. What does the point \((0, 0)\) represent?  
   **The point \((0, 0)\) means that there are 0 pennies in 0 coin rolls.**

9. Create a table showing this information in your My Notes column.

10. Why do the points in the graph lie on a straight line?  
    **It shows a proportional relationship and a constant rate of change.**

11. What is the ratio of number of pennies to the number of coin rolls?  
    \[ \frac{50}{1} = 50 \]

12. Define the variables and write an equation in the form \( y = kx \) for this situation.  
    \( x \) = the number of coin rolls, \( y \) = the number of pennies; \( y = 50x \)

13. What is the constant of proportionality in this situation?  
    \[ 50 \]

14. Describe what the constant of proportionality means in this situation.  
    **The constant of proportionality means that there are 50 pennies per coin roll.**
ACTIVITY 9 Continued

Check Your Understanding
Debrief students' answers to these items to ensure that students understand how to find the constant of proportionality for different representations.

Answers
15. Find the ratio of $y : x$.
16. Find the ratio of $y : x$.
17. Put equation in the form $y = kx$; $k$ is the constant of proportionality.

ASSES
Use the lesson practice to assess your students' understanding of how to complete a table and graph for a proportional relationship.
See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 9-2 PRACTICE
18. a. 40
   b. $x =$ the number of coin rolls, $y =$ the number of nickels; $y = 40x$
   c. Sample answer:

<table>
<thead>
<tr>
<th>Number of Coin Rolls</th>
<th>Number of Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
</tr>
<tr>
<td>4</td>
<td>160</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
</tr>
</tbody>
</table>

d. Represent this information in the graph:

e. How many nickels are needed to determine your answer.

MINI-LESSON: Constant of Proportionality

Check students' answers to the Lesson
ACTIVITY 9 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 9-1
1. Complete the ratio table to show ratios equivalent to 16:10.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>8</th>
<th>36</th>
<th>9</th>
<th>72</th>
</tr>
</thead>
</table>

2. Solve the proportion \( \frac{3}{5} = \frac{21}{x} \) using two different methods. Explain each method.

3. Solve \( \frac{x}{48} = \frac{5}{6} \) using two different strategies. Explain each strategy.

4. Is the ratio 25 to 16 proportional to the ratio 5 to 4? Explain.

5. Are the ratios 2:5:3 and 5:7 proportional? Explain.


7. At the library, the average ratio of hardbound books to paperback books on a shelf is 5:3.
   a. Use a proportion to predict the number of hardbound books on a shelf that has 75 paperback books.
   b. Use a proportion to predict the number of paperback books on a shelf that has 75 hardbound books.

For Items 8–12, use the following graph to make predictions.

- Use the graph to predict the number of miles driven in 8 hours. Choose the correct answer below.
  - A. 150 miles
  - B. 175 miles
  - C. 200 miles
  - D. 250 miles

- Use the graph to predict the number of hours it would take to drive 162.5 miles. Choose the correct answer below.
  - A. 15.5 hours
  - B. 6 hours
  - C. 6.5 hours
  - D. 7 hours

10. What does the point (0, 0) mean in this situation?

11. What does the point (1, 25) mean in this situation?

12. Write an equation in \( y = kx \) form to represent this situation.
ACTIVITY 9 Continued

13. 1.5
14. \( x = \) cups of apple juice, \( y = \) cups of orange juice; \( y = 1.5x \)
15. Sample answer:

<table>
<thead>
<tr>
<th>Apple Juice (cups), ( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orange Juice (cups), ( y )</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

16. Represent this information in the graph below.

17. 18 cups
18. 5 1/2 cups
19. (0, 0) means that if 0 cups of apple juice are used then 0 cups of orange juice should be used.
20. (1, 1.5) means that for every 1 cup of apple juice used 1.5 cups of orange juice should be used.
21. 1.6
22. \( y = 1.6x \)
23. $176
24. 1600

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the eBook Teacher Resources for additional practice problems.

Lesson 9-2

For Items 13–20, use the following information.

A fruit punch uses 1.5 cups of orange juice for every cup of apple juice.

13. What is the constant of proportionality used to find the number of cups of orange juice needed for any amount of apple juice?
14. Define the variables and write an equation that can be used to show this relationship.
15. Create a table of this information.

<table>
<thead>
<tr>
<th>Apple Juice (cups), ( x )</th>
<th>Orange Juice (cups), ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

16. Represent this information in the graph below.

17. How many cups of orange juice are needed for 12 cups of apple juice?
Write your answers on notebook paper. Show your work.
You may have had diamonds in your mouth before. Many dentists' drills are embedded with diamonds. In fact, 18% of your body is made up of carbon, and diamonds are also made of compressed carbon. That must mean you are priceless!

For Items 1–8, use the following information.
Diamonds are weighed in units called carats. Carat weight is based on the diamond's weight in milligrams. The table at the right shows the relationship between carats and milligrams.

1. Write an equation to convert a diamond's weight in carats to its weight in milligrams. Be sure to define your variables.

2. What is the constant of proportionality represented in the table at the right?

3. Complete the last row of the table by using the constant of proportionality.

4. Use your equation to find the weight in milligrams of the Tiffany Yellow Diamond, which weighs 287.42 carats.

5. Create a graph of the information in the table.

6. Explain the meaning of the point (0, 0) on your graph.

7. Use your graph to determine the weight in milligrams of a diamond weighing 8 carats.

8. Give the ordered pair for the point on the graph that shows how many milligrams a 1-carat diamond weighs.

Solve.

9. The Cullinan is the largest rough gem-quality diamond ever found. It was 3,106.75 carats. It weighed about 0.62 kg uncut. Recall that 1 kg is equal to 2.2 pounds. What was the uncut Cullinan weight in pounds?

10. How many pounds would a 0.5 kg diamond weigh?

11. The ratio of a diamond's hardness to its specific gravity is 10:3.515, and the ratio of the hardness to specific gravity for a ruby is 9:4.05. Are these ratios in proportion? Explain your answer.

Common Core State Standards for Embedded Assessment 1

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and...
### Embedded Assessment 1

**Teacher to Teacher**

You may wish to read through the scoring guide with students and discuss the differences in the expectations at each level. Check that students understand the terms used.

**Unpacking Embedded Assessment 2**

Once students have completed this Embedded Assessment, turn to Embedded Assessment 2 and unpack it with them. Use a graphic organizer to help students understand the concepts they will need to know to be successful on Embedded Assessment 2.

### Ratios, Proportions

For Items 12–13, use the following information:
The largest diamond is thought to be 10 billion trillion trillion carats away from Earth. One light-year is about 5.8 trillion kilometers far. Light travels through space in 12.

- Use a proportion to determine how far Earth is from the diamond.

13. Write an equation in $y = kx$ form to check your answer from

### Scoring Guide

<table>
<thead>
<tr>
<th>Scoring Guide</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13)</td>
<td>- Clear and accurate understanding of ratios, unit rates, and solving proportions.</td>
<td>- An understanding of ratios, unit rates, and solving proportions that usually results in correct answers.</td>
<td>- An understanding of ratios, unit rates, and solving proportions may not result in correct answers.</td>
</tr>
<tr>
<td>Problem Solving (Items 4, 7, 9, 10, 12)</td>
<td>- Accurate interpretation of the solution of a proportion to solve a problem.</td>
<td>- Interpretation of the solution of a proportion to solve a problem.</td>
<td>- Difficulty in interpretation solution of a proportion to solve a problem.</td>
</tr>
<tr>
<td>Mathematical Modeling / Representations (Items 1, 5, 6, 7, 8, 13)</td>
<td>- Accurate representation of a problem situation with a proportional equation or a graph.</td>
<td>- A mostly correct representation of a problem situation with a proportional equation or a graph.</td>
<td>- Difficulty in accuracy of representation of a problem situation with a proportional equation or a graph.</td>
</tr>
<tr>
<td>Reasoning and Communication (Items 6, 11)</td>
<td>- Precise use of appropriate math terms and language to explain proportional relationships.</td>
<td>- An adequate explanation of solutions using proportional relationships.</td>
<td>- A misleading explanation using proper relationships.</td>
</tr>
</tbody>
</table>

### Common Core State Standards for Embedded Assessments

- 7.RP.A.2c: Represent proportional relationships by equations. For example, if total cost $C$ is $8 for $5 worth of tomatoes, then you can write an equation as $C = 1.60t$.
## SpringBoard Middle School Mathematics
### Vertical Unit Connections

<table>
<thead>
<tr>
<th>Course 1</th>
<th>Course 2</th>
<th>Course 3</th>
</tr>
</thead>
<tbody>
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<td>Personal Financial Literacy</td>
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*http://www.springboardonline.com/flipbooks/springboard-middle-school-math/springboar... 4/17/2014*
To the Teacher

Instructional Units

UNIT 1 NUMBER CONCEPTS
Planning the Unit
Unit 1 Overview
Getting Ready

Activity 1 Whole Numbers and Decimals—Science, Shopping, and Society
Lesson 1-1 Comparing and Ordering Whole Numbers and Decimals
Lesson 1-2 Adding and Subtracting Decimals
Lesson 1-3 Multiplying Decimals
Lesson 1-4 Dividing Whole Numbers
Lesson 1-5 Dividing Decimals
Activity 1 Practice

Embedded Assessment 1 Comparing and Computing with Whole Numbers and Decimals—For the Birds

Activity 2 Prime Factorization and Exponents—The Primes of Your Life
Lesson 2-1 Prime Factorization
Lesson 2-2 Exponents
Activity 2 Practice

Activity 3 Greatest Common Factor and Least Common Multiple—Parties and Pups
Lesson 3-1 Greatest Common Factor
Lesson 3-2 Least Common Multiple
Activity 3 Practice

Embedded Assessment 2 Prime Factorization, Exponents, GCF, and LCM—Winter Sports

Activity 4 Fractions and Mixed Numbers—The Choice Is Yours
Lesson 4-1 Meaning of Fractions
Lesson 4-2 Comparing and Ordering Fractions
Lesson 4-3 Mixed Numbers
Lesson 4-4 Comparing and Ordering Mixed Numbers
Activity 4 Practice

Activity 5 Multiplying Fractions and Mixed Numbers—Skateboarding Fun
Lesson 5-1 Multiplying by Fractions
Contents continued

**Embedded Assessment 3** Multiplying and Dividing Fractions and Mixed Numbers—
*Juan's Bookcase*

**UNIT 2** **INTEGERS**
Planning the Unit
Unit 2 Overview
Getting Ready

**Activity 7** **Introduction to Integers—Get the Point?**
Lesson 7-1 Integers and the Number Line
Lesson 7-2 Comparing and Ordering Integers
Activity 7 Practice

**Activity 8** **Adding and Subtracting Integers—What's the Temperature?**
Lesson 8-1 Using Models to Add Integers
Lesson 8-2 Using Rules to Add Integers
Lesson 8-3 Subtracting Integers
Activity 8 Practice

**Embedded Assessment 1** Integer Sums and Differences—*Hot and Cold*

**Activity 9** **The Coordinate Plane—Map It Out!**
Lesson 9-1 Integers in the Coordinate Plane
Lesson 9-2 Distance and Reflections in the Coordinate Plane
Activity 9 Practice

**Activity 10** **Multiplying and Dividing Integers—Temperature Ups and Downs**
Lesson 10-1 Multiplying Integers
Lesson 10-2 Dividing Integers
Activity 10 Practice

**Embedded Assessment 2** Coordinate Plane and Multiplying and Dividing Integers—
*Scavenger Hunt*

**UNIT 3** **EXPRESSIONS AND EQUATIONS**
Planning the Unit
Unit 3 Overview
Getting Ready

**Activity 11** **Expressions—A Fairly Ordered Operation**
Lesson 11-1 Order of Operations
Lesson 11-2 Evaluating Algebraic Expressions
Embedded Assessment 1 Order of Operations and Expressions—The Cost of After-School Activities

Activity 13 Solving Addition and Subtraction Equations—Music to My Ears
Lesson 13-1 Modeling and Solving Addition Equations
Lesson 13-2 Solving Addition Equations
Lesson 13-3 Modeling and Solving Subtraction Equations
Lesson 13-4 Solving Subtraction Equations
Activity 13 Practice

Activity 14 Solving Multiplication and Division Equations—Trash Talk
Lesson 14-1 Modeling and Solving Multiplication Equations
Lesson 14-2 Solving Multiplication Equations
Lesson 14-3 Solving Division Equations
Activity 14 Practice

Activity 15 Expressions and Equations—Up in the Air
Lesson 15-1 Representing Situations with Inequalities
Lesson 15-2 Solving One-Step Inequalities
Activity 15 Practice

Activity 16 Expressions and Equations—Moving Right Along
Lesson 16-1 Representing Relationships
Lesson 16-2 Dependent and Independent Variables
Activity 16 Practice

Embedded Assessment 2 Expressions and Equations—The School Book Fair

UNIT 4 RATIOS
Planning the Unit
Unit 4 Overview
Getting Ready

Activity 17 Understanding Ratios—All About Pets
Lesson 17-1 Understanding Ratios
Lesson 17-2 Ratios in Proportional Relationships
Activity 17 Practice

Activity 18 Reasoning with Ratios—A Picture Is Worth
Lesson 18-1 Solve Problems Using Ratios
Lesson 18-2 Converting Between Measurements Using Ratios
Activity 18 Practice
Activity 20  Using Models to Understand Percents—A “Cent” for Your Thoughts
Lesson 20-1 Using Models to Understand Percents
Lesson 20-2 Percents, Fractions, and Decimals
Lesson 20-3 More Percents, Decimals, and Fractions
Activity 20 Practice

Activity 21  Applying Percents—Feel the Beat
Lesson 21-1 Using Models to Understand Percents
Lesson 21-2 Find the Part Given a Percent and a Whole
Lesson 21-3 Find the Whole Given a Part and the Percent
Activity 21 Practice

Embedded Assessment 2  Understanding and Applying Percents—An Ice Cream Treat

UNIT 5  GEOMETRIC CONCEPTS
Planning the Unit
Unit 5 Overview
Getting Ready

Activity 22  Angles and Triangles—Triangle Trivia
Lesson 22-1 Properties of Triangles and Side Lengths
Lesson 22-2 Properties of Triangles and Angle Measures
Activity 22 Practice

Activity 23  Area and Perimeter of Polygons—Play Area
Lesson 23-1 Recalling Quadrilaterals
Lesson 23-2 Perimeter and Area of Composite Figures
Lesson 23-3 Area of Triangles, Trapezoids, and Polygons
Activity 23 Practice

Activity 24  Polygons on the Coordinate Plane—Wall Art
Lesson 24-1 Defining Polygons on the Coordinate Plane
Lesson 24-2 Area of Polygons on a Coordinate Plane
Activity 24 Practice

Embedded Assessment 1  Geometric Concepts—Astronomy Logo

Activity 25  Nets and Surface Area—All Boxed Up
Lesson 25-1 Nets and Surface Area of Cubes
Lesson 25-2 Nets and Surface Area of Prisms
UNIT 6  DATA ANALYSIS
Planning the Unit
Unit 6 Overview
Getting Ready

Activity 27  Summarizing Data Graphically—Making a Survey
Lesson 27-1 Survey Questions and Variability
Lesson 27-2 Types of Variables and Graphs
Lesson 27-3 Shapes of Distributions
Activity 27 Practice

Activity 28  Measures of Center—Bull’s Eye
Lesson 28-1 Mean and Outliers
Lesson 28-2 Median
Lesson 28-3 Summarize the Center of a Distribution
Activity 28 Practice

Embedded Assessment 1  Types of Variables and Measures of Center—Dribble, Shoot, Score!

Activity 29  Measures of Variability—Making the Grade
Lesson 29-1 Range
Lesson 29-2 Mean Absolute Deviation
Lesson 29-3 Interquartile Range (IQR)
Activity 29 Practice

Activity 30  Summarizing Numerical Data Graphically—Batter Up!
Lesson 30-1 Box Plots
Lesson 30-2 Histograms
Lesson 30-3 More on Histograms
Activity 30 Practice

Embedded Assessment 2  Measures of Variability and Numerical Graphs—“Take a Snapshot” Revisited

UNIT 7  PERSONAL FINANCIAL LITERACY
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Unit 7 Overview
Getting Ready

Activity 31  Using Financial Services—You Can Bank on It
Lesson 31-1 Understanding Bank Accounts
Lesson 31-2 Using Credit
To the Teacher

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UNIT 1  NUMERICAL RELATIONSHIPS
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Unit 1 Overview
Getting Ready

Activity 1  Investigating Patterns—Laws and Order
Lesson 1-1 Analyzing Sequences
Lesson 1-2 Analyzing More Sequences
Lesson 1-3 Increasing and Decreasing Sequences
Activity 1 Practice

Activity 2  Operations with Fractions—And the Beat Goes On
Lesson 2-1 Adding and Subtracting Fractions
Lesson 2-2 Multiplying and Dividing Fractions
Activity 2 Practice

Embedded Assessment 1  Patterns and Quantitative Reasoning—Game On

Activity 3  Powers and Roots—Squares and Cubes
Lesson 3-1 Area, Squares, and Square Roots
Lesson 3-2 Volume, Cubes, and Cube Roots
Lesson 3-3 Exponents, Roots, and Order of Operations
Activity 3 Practice

Activity 4  Rational Numbers—Know When to Fold 'Em
Lesson 4-1 Modeling Fractions
Lesson 4-2 Rational Number Representations
Lesson 4-3 Comparing Rational Numbers
Activity 4 Practice

Activity 5  Rational and Irrational Numbers—Where Am I?
Lesson 5-1 Estimating Irrational Numbers
Lesson 5-2 Comparing Rational and Irrational Numbers
Activity 5 Practice

Embedded Assessment 2  Representing Rational and Irrational Numbers—Weather or Not?

Activity 6  Properties of Exponents—That's a Lot of Cats
Lesson 6-1 Multiply and Divide with Exponents
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Activity 8 Operations with Scientific Notation—How Big is That Planet?
Lesson 8-1 Multiply and Divide in Scientific Notation
Lesson 8-2 Add and Subtract in Scientific Notation
Activity 8 Practice

Embedded Assessment 3 Exponents and Scientific Notation—Contagious Mathematics

UNIT 2 EQUATIONS
Planning the Unit
Unit 2 Overview
Getting Ready

Activity 9 Writing Expressions—Pebbles in the Sand
Lesson 9-1 Representing Patterns
Lesson 9-2 Using Patterns to Write and Evaluate Expressions
Activity 9 Practice

Activity 10 Solving Equations—Cups and Cubes
Lesson 10-1 Solving Linear Equations with Models
Lesson 10-2 Solving Linear Equations Algebraically
Activity 10 Practice

Embedded Assessment 1 Expressions and Equations—What a Good Idea!

Activity 11 Exploring Slope—High Mountain Ratio
Lesson 11-1 Linear Equations and Slope
Lesson 11-2 More on Linear Equations and Slope
Activity 11 Practice

Activity 12 Slope-Intercept Form—Leaky Bottle
Lesson 12-1 Identifying Slope Using Tables and Graphs
Lesson 12-2 Comparing Slopes of Different Lines
Lesson 12-3 Linear Relationships Using Slope-Intercept Form
Activity 12 Practice

Activity 13 Proportional Relationships—Vary Interesting
Lesson 13-1 Linear Proportional Relationships
Lesson 13-2 Directly Proportional Relationships
Activity 13 Practice

Embedded Assessment 2 Linear Equations and Rates of Change—Who Is That?

Activity 14 Graphing Systems of Linear Equations—System of Trees
Embedded Assessment 3 Solving Systems of Linear Equations—
Supply and Demand

UNIT 3 GEOMETRY
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Unit 3 Overview
Getting Ready

Activity 16 Angle-Pair Relationships—The Winning Angle
Lesson 16-1 Complementary and Supplementary Angles
Lesson 16-2 Angles Formed by Parallel Lines
Activity 16 Practice

Activity 17 Angles of Triangles and Quadrilaterals—The Parallel Chute
Lesson 17-1 Angles in a Triangle
Lesson 17-2 Exterior Angles and Angles in Quadrilaterals
Activity 17 Practice

Embedded Assessment 1 Angle Measures—Light and Glass

Activity 18 Introduction to Transformations—Move It!
Lesson 18-1 What is a Transformation?
Lesson 18-2 Translations and Coordinates
Lesson 18-3 Reflections and Coordinates
Lesson 18-4 Rotations and Coordinates
Activity 18 Practice

Activity 19 Rigid Transformations and Compositions—All the Right Moves
Lesson 19-1 Properties of Transformations
Lesson 19-2 Composition of Transformations
Activity 19 Practice

Embedded Assessment 2 Rigid Transformations—In Transformations We Trust

Activity 20 Similar Triangles—Mirrors and Shadows
Lesson 20-1 Exploring Similarity
Lesson 20-2 Properties and Conditions of Similar Triangles
Activity 20 Practice

Activity 21 Dilations—Alice’s Adventures in Shrinking and Growing
Lesson 21-1 Stretching and Shrinking Geometric Figures
Lesson 21-2 Effects of Scale Factor
Activity 21 Practice
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Lesson 23-2 The Pythagorean Theorem and the Coordinate Plane
Activity 23 Practice

Activity 24 Converse of the Pythagorean Theorem—Paper Clip Chains
Lesson 24-1 The Converse of the Pythagorean Theorem
Lesson 24-2 Pythagorean Triples
Activity 24 Practice

Embedded Assessment 4 The Pythagorean Theorem—Camp Euclid

Activity 25 Surface Area—Greenhouse Gardens
Lesson 25-1 Lateral and Surface Areas of Prisms
Lesson 25-2 Lateral and Surface Areas of Cylinders
Activity 25 Practice

Activity 26 Volume of Solids—Castles in the Sand
Lesson 26-1 Volumes of Prisms and Pyramids
Lesson 26-2 Volumes of Cylinders, Cones, and Spheres
Lesson 26-3 Volumes of Composite Solids
Activity 26 Practice

Embedded Assessment 5 Surface Area and Volume—Air Dancing

UNIT 4 FUNCTIONS
Planning the Unit
Unit 4 Overview
Getting Ready

Activity 27 Introduction to Functions—It’s All Related
Lesson 27-1 What is a Function?
Lesson 27-2 Mapping Inputs and Outputs
Lesson 27-3 Identifying Functions
Lesson 27-4 Graphs of Functions
Activity 27 Practice

Activity 28 Comparing Functions—Which Car Wins?
Lesson 28-1 Representing Functions
Lesson 28-2 Analyzing Functions
Activity 28 Practice

Activity 29 Constructing Functions—Hold On to Your Hats
Lesson 29-1 Construct a Function
Lesson 29-2 Rate of Change and Initial Value
Activity 29 Practice
Activity 31 Linear and Non-Linear Functions—Measure Up
Lesson 31-1 Bean Experiment
Lesson 31-2 Bean Experiment Continued
Lesson 31-3 Scale Experiment
Activity 31 Practice

Embedded Assessment 2 Scatter Plots and Trend Lines—Geographically Speak

UNIT 5 PROBABILITY AND STATISTICS
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Unit 5 Overview
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Activity 32 Scatter Plots and Association—Cracker Snacker
Lesson 32-1 Scatter Plots
Lesson 32-2 Association
Activity 32 Practice

Activity 33 Bivariate Data—Sue Swandive
Lesson 33-1 Collecting Data
Lesson 33-2 Trend Lines
Lesson 33-3 The Competition!
Activity 33 Practice

Embedded Assessment 1 Scatter Plots, Associations, and Trends—U.S. Census

Activity 34 Median-Median Line—Homework Help Line
Lesson 34-1 Finding the Median-Median Line
Lesson 34-2 Using the Median-Median Line
Activity 34 Practice

Activity 35 Two-Way Tables and Association—Student Opinions
Lesson 35-1 Two-Way Tables
Lesson 35-2 Investigating Association
Activity 35 Practice

Embedded Assessment 2 Median-Median Line and Two-Way Tables—Mokher's Measurements

UNIT 6 PERSONAL FINANCIAL LITERACY
Planning the Unit
Unit 6 Overview
Getting Ready
## SpringBoard Learning Strategies

### READING STRATEGIES

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<th>DEFINITION</th>
<th>PURPOSE</th>
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</thead>
<tbody>
<tr>
<td>Activating Prior Knowledge</td>
<td>Recalling what is known about a concept and using that information to make a connection to a new concept</td>
<td>Helps students establish connections between what they already know and what knowledge is related</td>
</tr>
<tr>
<td>Chunking the Activity</td>
<td>Grouping a set of items/questions for specific purposes</td>
<td>Provides an opportunity to dissect information and assess student understanding before moving on to a new concept</td>
</tr>
<tr>
<td>Close Reading</td>
<td>Reading text word for word, sentence by sentence, and line by line to make a detailed analysis of meaning</td>
<td>Assists in developing a comprehensive understanding of the text</td>
</tr>
<tr>
<td>Graphic Organizer</td>
<td>Arranging information into maps and charts</td>
<td>Builds comprehension and discussion by representing ideas in a visual form</td>
</tr>
<tr>
<td>Interactive Word Wall</td>
<td>Visually displaying vocabulary words to serve as a classroom reference of words and groups of words as they are introduced, used, and mastered over the course of a year</td>
<td>Provides a visual reference and builds word knowledge</td>
</tr>
<tr>
<td>KWL Chart (Know, Want to Know, Learn)</td>
<td>Activating prior knowledge by identifying what students know, determining what they want to learn, and having them reflect on what they learned</td>
<td>Assists in organizing information, reflecting on learning to build knowledge and increase comprehension</td>
</tr>
<tr>
<td>Marking the Text</td>
<td>Highlighting, underlining, and marking text to focus on key information to help understand the text or solve the problem</td>
<td>Helps the reader identify important information in the text and makes the interpretation of ideas and concepts more precise</td>
</tr>
<tr>
<td>Predict and Confirm</td>
<td>Making conjectures about what results will develop in an activity; confirming or modifying the conjectures based on outcomes</td>
<td>Stimulates thinking by making predictions about the outcomes and correcting predictions based on evidence from the outcomes</td>
</tr>
<tr>
<td>Levels of Questions</td>
<td>Developing literal, interpretive, and universal questions about the text while reading the text</td>
<td>Focuses reading, helps in integrating information into the text by seeking and applying multiple interpretations and making connections</td>
</tr>
<tr>
<td>Paraphrasing</td>
<td>Restating in your own words the essential information in a text or problem description</td>
<td>Assists with comprehending and summarizing information, and problem solving</td>
</tr>
<tr>
<td>Role Play</td>
<td>Assuming the role of a character in a scenario</td>
<td>Helps interpret and visualize how a problem might be solved from different perspectives</td>
</tr>
<tr>
<td>Shared Reading</td>
<td>Reading the text aloud (usually by the teacher) as students follow along silently, or reading a text aloud by the teacher and students</td>
<td>Helps auditory learners do and analyze challenging texts</td>
</tr>
<tr>
<td>Summarizing</td>
<td>Giving a brief statement of the main points in a text</td>
<td>Assists with comprehending and summarizing information</td>
</tr>
<tr>
<td>Think Aloud</td>
<td>Talking through a difficult text or problem</td>
<td>Helps in comprehending and discussing complex texts and ideas</td>
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### SpringBoard Learning Strategies

#### COLLABORATIVE STRATEGIES

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<th>DEFINITION</th>
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<tbody>
<tr>
<td>Critique Reasoning</td>
<td>Through collaborative discussion, respond to the arguments of others; question the use of mathematical terminology, assumptions, and conjectures to improve understanding and to justify and communicate conclusions</td>
<td>Helps students learn from each other as they make connections and uncertainties in concept learning. Mathematical conjectures are verbalized and ideas expressed that make sense to peers.</td>
</tr>
<tr>
<td>Debriefing T</td>
<td>Discussing the understanding of a concept to lead to consensus on its meaning</td>
<td>Helps clarify misconceptions and ensure understanding of content.</td>
</tr>
<tr>
<td>Discussion Groups</td>
<td>Working within groups to discuss content, to create problem solutions, and to explain and justify a solution</td>
<td>Aids understanding through sharing of ideas, interpretation of problem scenarios.</td>
</tr>
<tr>
<td>Group Presentation</td>
<td>Presenting information as a collaborative group</td>
<td>Allows opportunities for collaborative solutions and responsibility for delivering ideas to an audience.</td>
</tr>
<tr>
<td>Jigsaw</td>
<td>Reading different texts or passages, students become &quot;experts&quot; and then move to a new group to share their information; after sharing, students go back to the original group to share new knowledge</td>
<td>Provides opportunities to present information that facilitates understanding of passage (or multiple texts) without having each student learn all the information.</td>
</tr>
<tr>
<td>Sharing and Responding</td>
<td>Communicating with another person or a small group of peers who respond to a piece of writing or proposed problem solution</td>
<td>Gives students the opportunity to improve their work with peers, and/or to receive appropriate feedback on their own ideas.</td>
</tr>
<tr>
<td>Think-Pair-Share</td>
<td>Thinking through a problem alone, pairing with a partner to share ideas, and concluding by sharing results with the class</td>
<td>Enables the development of ideas and are then tested with a larger group.</td>
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</tbody>
</table>

### WRITING STRATEGIES

<table>
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<tbody>
<tr>
<td>Drafting</td>
<td>Writing a text in an initial form</td>
<td>Assists in getting first form and ready for revision.</td>
</tr>
<tr>
<td>Note Taking</td>
<td>Creating a record of information while reading a text or listening to a speaker</td>
<td>Helps in organizing ideas and information.</td>
</tr>
<tr>
<td>Prewriting</td>
<td>Brainstorming, either alone or in groups, and refining thoughts and organizing ideas prior to writing</td>
<td>Provides a tool for beginning process and determining writing.</td>
</tr>
<tr>
<td>Quickwrite</td>
<td>Writing for a short, specific amount of time about a designated topic</td>
<td>Helps generate ideas.</td>
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## SpringBoard Learning Strategies
### PROBLEM-SOLVING STRATEGIES

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<th>Strategy</th>
<th>Description</th>
<th>Benefits</th>
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<tbody>
<tr>
<td>Construct an Argument</td>
<td>Use mathematical reasoning to present assumptions about mathematical situations, support</td>
<td>Helps develop the process of mathematical information reasoning skills, and enhance the</td>
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<td>conjectures with mathematically relevant and accurate data, and provide a logical progression</td>
<td>communication skills in such conjectures and conclusions</td>
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<td>of ideas leading to a conclusion that makes sense</td>
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<tr>
<td>Create a Plan</td>
<td>Analyzing the tasks in a problem and creating a process for completing the tasks by finding</td>
<td>Assists in breaking tasks into parts and identifying the steps to complete the entire task</td>
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<td>information needed for the tasks, interpreting data, choosing how to solve a problem,</td>
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<td>communicating the results, and verifying accuracy</td>
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<tr>
<td>Create Representations</td>
<td>Creating pictures, tables, graphs, lists, equations, models, and/or verbal expressions to</td>
<td>Helps organize information and answer questions or show a problem</td>
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<td>interpret text or data</td>
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<tr>
<td>Guess and Check</td>
<td>Guessing the solution to a problem, and then checking that the guess fits the information in</td>
<td>Allows exploration of different solutions to a problem; guess and check when other strategies are obvious</td>
</tr>
<tr>
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<td>the problem and is an accurate solution</td>
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</tr>
<tr>
<td>Identify a Subtask</td>
<td>Breaking a problem into smaller pieces whose outcomes lead to a solution</td>
<td>Helps to organize the pieces of a problem and reach a conclusion</td>
</tr>
<tr>
<td>Look for a Pattern</td>
<td>Observing information or creating visual representations to find a trend</td>
<td>Helps to identify patterns to make predictions</td>
</tr>
<tr>
<td>Simplify the Problem</td>
<td>Using “friendlier” numbers to solve a problem</td>
<td>Provides insight into the problem and identifies the strategies needed to solve</td>
</tr>
<tr>
<td>Work Backward</td>
<td>Tracing a possible answer back through the solution process to the starting point</td>
<td>Provides another way to check the correctness of answers for accuracy</td>
</tr>
<tr>
<td>Use Manipulatives</td>
<td>Using objects to examine relationships between the information given</td>
<td>Provides a visual representation that supports comprehension of a problem</td>
</tr>
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</table>