CCGPS Frameworks 3rd Unit 2

Mathematics

Third Grade Unit Two
Operations and Algebraic Thinking: The Relationship Between Multiplication and Division

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

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Unit 2
The Relationship Between Multiplication and Division

TABLE OF CONTENTS (* indicates new task)

OVERVIEW

In this unit, students will:

- begin to understand the concepts of multiplication and division
- learn the basic facts of multiplication and their related division facts
- apply properties of operations (commutative, associative, and distributive) as strategies to multiply and divide
- understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
- fluently multiply and divide within 100, using strategies such as the patterns and relationships between multiplication and division
- understand multiplication and division as inverse operations
- solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.
- represent and interpret data

"Multiplication and division are commonly taught separately. However, it is very important to combine the two shortly after multiplication has been introduced. This will help the students to see the connection between the two." (Van de Walle and Lovin, Teaching Student-Centered Mathematics 3-5, p. 60)

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision during this cluster are: operation, multiply, divide, factor, product, quotient, strategies, and properties-rules about how numbers work.

COMMON MISCONCEPTIONS

Some common misconceptions that students may have are thinking a symbol (? or ☐) is always the place for the answer. This is especially true when the problem is written as 15 ÷ 3 =? or 15 = ☐ x 3. Studer
of models is essential in helping students eliminate this understanding.

Another key misconception is that the use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation. Presenting students with

multiple situations in which they select the symbol and explain what it represents will counter this misconception.

	Unknown Product	**Group Size Unknown** ("How many in each group? Division)	**Number of Groups Unknown** ("How many groups?" Division)
	3 × 6 = ?	**3 × ? = 18, and 18 ÷ 3 = ?**	**? × 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are6 inches long. How many pieces of string will you have?
Arrays [1], Area [2]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3times as long?	A red hat costs $18 and that is3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Adapted from Common Core State Standards Glossary, pg 89

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations. The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

COMMON MULTIPLICATION AND DIVISION SITUATIONS

STANDARDS FOR MATHEMATICAL CONTENT

Represent and solve problems involving multiplication and division.

MCC.3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.

MCC.3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

MCC.3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

MCC. 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.
Multiply and divide within 100

MCC.3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Represent and interpret data.

MCC.3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

MCC.3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.** Students make sense of problems involving multiplication and division.

2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning by connecting arrays with multiplication problems.

3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding mental math strategies focusing on multiplication and division.

4. **Model with mathematics.** Students are asked to use tiles to model various understandings of multiplication by creating arrays or groups. They record their thinking using words, pictures, and numbers to further explain their reasoning.

5. **Use appropriate tools strategically.** Students use graph paper to find all the possible rectangles for a given product.

6. **Attend to precision.** Students will learn to use terms such as multiply, divide, factor, and product with increasing precision.

7. **Look for and make use of structure.** Students use the distributive property of multiplication as a strategy to multiply.

8. **Look for and express regularity in repeated reasoning.** Students use the distributive property of multiplication to solve for products they do not know.

Mathematical Practices 1 and 6 should be evident in EVERY lesson!

ENDURING UNDERSTANDINGS

- Multiplication and division are inverses; they undo each other.
- Multiplication and division can be modeled with arrays.
- Multiplication is commutative, but division is not.
- There are two common situations where division may be used.
    - Partition (or fair-sharing) - given the total amount and the number of equal groups, determine how many/much in each group
    - Measurement (or repeated subtraction) - given the total amount and the amount in a group, determine how many groups of the same size can be created.
- As the divisor increases, the quotient decreases; as the divisor decreases, the quotient increases.
- There is a relationship between the divisor, the dividend, the quotient, and any remainder.
- Multiplication facts can be deduced from patterns.
- The associative property of multiplication can be used to simplify computation.
- The distributive property of multiplication allows us to find partial products and then find their sum.
- Patterns are evident when multiplying a number by ten or a multiple of ten.

ESSENTIAL QUESTIONS Choose a few questions based on the needs of your students.

- How are multiplication and division alike and different?
- How are multiplication and division related?
- How are subtraction and division related?
- How can I model multiplication by ten?
- How can multiplication and division be used to solve real world problems?
- How can multiplication be represented?
- How can multiplication products be displayed on a multiplication chart?
- How can the same array represent both multiplication and division?
- How can we determine numbers that are missing on a times table chart by knowing multiplication patterns?
- How can we divide larger numbers?
- How can we model division?
- How can we practice multiplication facts in a meaningful way that will help us remember them?
- How can we use arrays to help develop an understanding of the commutative property?
- How can we use patterns to solve problems?
- How can we write a mathematical sentence to represent division models we have made?

- How can you interpret the product by making equal groups?
- How can you use multiplication facts to solve unknown factor problems?
- How can you use what you know about multiplication to help you write your own multiplication problem?
- How can you write a mathematical sentence to represent a multiplication model we have made?
- How do estimation, multiplication, and division help us solve problems in everyday life?
- How do I decide what increment scale to use for a bar graph?
- How do you interpret data in a graph?
- How do you know which type of graph is appropriate for your data?
- How does an array represent the meaning of multiplication?
- How does drawing an array help us think about different ways to decompose a number?
- How does the order of the digits in a multiplication problem affect the product?
- How does understanding the distributive property help us multiply large numbers?
- How is division an unknown factor problem?
- How is multiplication and division used to solve a problem?
- How is multiplying by ten related to place value?
- How is place value related to multiples of ten?
- How is the associative property of multiplication used in solving a problem?
- How is the commutative property of multiplication evident in an array model?
- In what ways can division be represented?
- What are different ways to multiply two factors to get the same product?
- What are strategies for leaning multiplication facts?
- What are the strategies for learning multiplication?
- What happens to a number when it is multiplied by ten?
- What is the relationship between the factors and the product?
- What patterns of multiplication can we discover by studying a times table chart?
- What strategies can be used to find factors or products?
- What strategies can be used to solve multiplication problems?
- What strategies can be used to solve real world division problems?
- What strategies can help you solve real world multiplication problems?
- What strategy did you find most efficient when dividing?
- When can you use multiplication or division in real life?
- Why are there different types of graphs?

CONCEPTS/SKILLS TO MAINTAIN

In Grade 2, instructional time focused on four critical areas:

- Furthering their understanding for the base-ten system. Students worked with counting in fives, tens and multiples of hundreds, tens and ones. Students also recognize that the digits in each place of a number represent the amounts of thousands, hundreds, tens, or ones.

- Using their understanding of addition to develop fluency within 100. They solve problems within 1,000 by using models of addition and subtraction.
- Recognizing the need for units of measure (centimeter and inch) and understand how to use rulers and other measurement tools to get linear measurement.
- Developing an understanding of shapes by analyzing and describing them based on their sides and angles.

Specifically, it is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- odd and even numbers
- skip counting by twos, threes, fives, and tens
- determining reasonableness using estimation
- addition and subtraction as inverse operations
- commutative, associative, and identity properties of addition
- basic addition facts
- making tens in a variety of ways
- basic subtraction facts
- place value for ones, tens, hundreds, thousands, and tenths
- modeling numbers using base 10 blocks and on grid paper
- using addition to find the total number of objects in a rectangular array

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are **for teacher reference only** and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers. Common Core Mathematics Glossary

- array
- array
- associative property of multiplication
- bar graph
- commutative property of multiplication
- distributive property
- dividend
- division

- divisor
- equal groups
- equations
- factor
- fourths
- groups of
- halves
- identity property of multiplication
- inch
- line plot
- measurement division (or repeated subtraction)
- multiplicand
- multiplication
- multiplier
- partial products
- partition division
- partitioned equally
- picture graph
- product
- quotient
- remainder
- scale
- strategy
- unknown
- whole numbers

STRATEGIES FOR TEACHING AND LEARNING

(Adapted from Common Core Resources, NC Dept. of Public Instruction)

Represent and solve problems involving multiplication and division.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This strategy is a foundation for multiplication because students should make a connection between repeated addition and multiplication.

Students need to experience problem-solving involving equal groups (whole unknown or size of group is unknown) and multiplicative comparison (unknown product, group size unknown or number of groups unknown) as shown in the table in the unit overview. No attempt should be made to teach the abstract structure of these problems.

Encourage students to solve these problems in different ways to show the same idea and be able to explain their thinking verbally and in written expression. Allowing students to present several different strategies provides the opportunity for them to compare strategies.

Sets of counters, number lines to skip count and relate to multiplication and arrays/area models will aid students in solving problems involving multiplication and division. Allow students to model problems using these tools. They should represent the model used as a drawing or equation to find the solution.

This shows multiplication using grouping with 3 groups of 5 objects and can be written as 3×5.

Provide a variety of contexts and tasks so that students will have more opportunity to develop and use thinking strategies to support and reinforce learning of basic multiplication and division facts.

Have students create multiplication problem situations in which they interpret the product of whole numbers as the total number of objects in a group and write as an expression. Also, have students create division-problem situations in which they interpret the quotient of whole numbers as the number of shares.

Students can use known multiplication facts to determine the unknown fact in a multiplication or division problem. Have them write a multiplication or division equation and the related multiplication or division equation. For example, to determine the unknown whole number in 27 ÷ ☐ = 3, student
27. They should ask themselves questions such as, "How many 3s are in 27 ?" or "3 times what number is 27?" Have them justify their thinking with models or drawings.

Students need to apply properties of operations (commutative, associative and distributive) as strategies to multiply and divide. Applying the concept involved is more important than students knowing the name of the property. Understanding the commutative property of multiplication is developed through the use of models as basic multiplication facts are learned. For example, the result of multiplying 3 x 5 (15) is the same as the result of multiplying 5 x 3 (15).

To find the product of three numbers, students can use what they know about the product of two of the factors and multiply this by the third factor. For example, to multiply 5 x 7 x 2, students know that 5 x 2 is 10. Then, they can use mental math to find the product of 10 x 7 (70). Allow students to use their own strategies and share with the class when applying the associative property of multiplication.

Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty. For example, students can split a 6 x 9 array into 6 groups of 5 and 6 groups of 4; then, add the sums of the groups.

The 6 groups of 5 is 30 and the 6 groups of 4 is 24. Students can write 6 x 9 as 6 x 5 + 6 x 4. Students' understanding of the part/whole relationships is critical in understanding the connection between multiplication and division.

Multiply and divide within 100

Students need to understand the part/whole relationships in order to understand the connection between multiplication and division. They need to develop efficient strategies that lead to the big ideas of multiplication and division. These big ideas include understanding the properties of operations, such as the commutative and associative properties of multiplication and the distributive property. The naming of the property is not necessary at this stage of learning.

In Grade 2, students found the total number of objects using rectangular arrays, such as a 5 x 5, and wrote equations to represent the sum. This is called unitizing, and it requires students to count groups, not just objects. They see the whole as a number of groups of a number of objects. This strategy is a foundation for multiplication in that students should make a connection between repeated addition and multiplication.

As students create arrays for multiplication using objects or drawing on graph paper, they may discover that three groups of four and four groups of three yield the same results. They should observe that the arrays stay the same, although how they are viewed changes. Provide numerous situations for students to develop this understanding.

To develop an understanding of the distributive property, students need decompose the whole into groups. Arrays can be used to develop this understanding. To find the product of 3 × 9, students can decompose 9 into the sum of 4 and 5 and find 3 × (4 + 5).

The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers in Grade 5.

Once students have an understanding of multiplication using efficient strategies, they should make the connection to division. Using various strategies to solve different contextual problems that use the same two one-digit whole numbers requiring multiplication allows for students to commit to memory all products of two one-digit numbers.

Represent and interpret data.

Representation of a data set is extended from picture graphs and bar graphs with single-unit scales to scaled picture graphs and scaled bar graphs. Intervals for the graphs should relate to multiplication and division with 100 (product is 100 or less and numbers used in division are 100 or less). In picture graphs, use values for the icons in which students are having difficulty with multiplication facts. For example, ☺ represents 7 people. If there are three ☺, students should use known facts to determine that the three icons represents 21 people. The intervals on the vertical scale in bar graphs should not exceed 100.

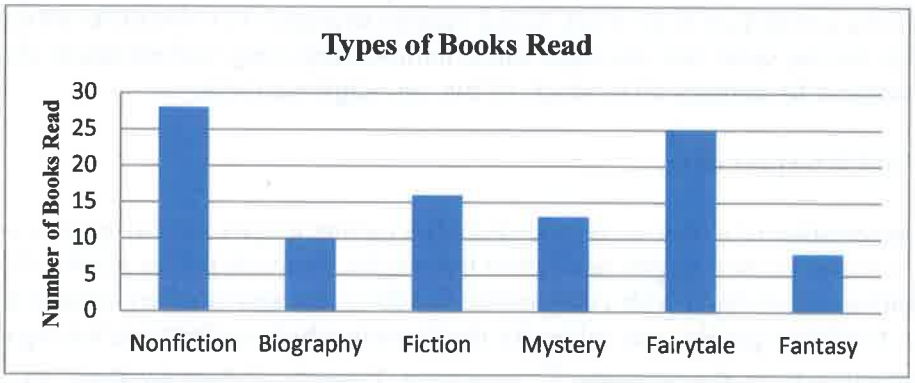
Students are to draw picture graphs in which a symbol or picture represents more than one object. Bar graphs are drawn with intervals greater than one. Ask questions that require students to compare quantities and use mathematical concepts and skills. Use symbols on picture graphs that student can easily represent half of, or know how many half of the symbol represents. Students are to measure lengths using rulers marked with halves and fourths of an inch and record the data on a line plot. The horizontal scale of the line plot is marked off in whole numbers, halves or fourths. Students can create rulers with appropriate markings and use the ruler to create the line plots.

Although intervals on a bar graph are not in single units, students count each square as one. To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip- counting when determining the value of a bar since the scale is not in single units.

Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?

Number of Books Read	
Nancy	
Juan	
 = 5 books	

Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Analyze and Interpret data:

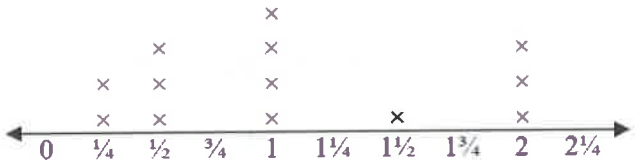
- How many more nonfiction books where read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library which would be the best genre? Why?

Students in second grade measured length in whole units using both metric and U.S. customary systems. It is important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.

This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch.

Example:
Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼? ½? etc. …

Objects on My Desk



Measurements in Inches

**For additional assistance see the Unit Webinar at georgiastandards.org.

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- use mental math to multiply and divide
- use estimation to determine reasonableness of products and quotients computed
- be able to read, interpret, solve, and compose simple word problems dealing with multiplication and division
- understand how to use inverse operations to verify accuracy of computation
- understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
- fluently multiply and divide within 100, using strategies such as the patterns and relationships between multiplication and division
- apply properties of operations (commutative, associative, and distributive) as strategies to multiply and divide
- represent and interpret data

TASKS

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in Unit 2. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrated the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

Scaffolding Task	Tasks that build up to the learning task.
Constructing Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
CTE Classroom Tasks	Designed to demonstrate how the Common Core and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.
***3-Act Task**	A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org and the K-5 CCGPS Mathematics Wiki.

Task Name	Task Type *Grouping Strategy*	Skills	Standard(s)
One Hundred Hungry Ants!	Scaffolding Task *Individual/Partners*	Multiplication and Arrays	MCC.3.OA.1
Arrays on the Farm	Scaffolding Task *Small Group/Partners*	Multiplication and Arrays	MCC.3.OA.5
What's My Product?	Scaffolding Task *Individual/Partners*	Multiplication	MCC.3.OA.1
The Doorbell Rang	Scaffolding Task *Individual/Partners*	Division	MCC.3.OA.2
Family Reunion	Constructing Task *Individual/Partners*	Multiplication and Division	MCC.3.OA.5 MCC.3.0A.6
Skittles Cupcake Combos	Constructing Task *Individual/Partners*	Division	MCC.3.OA.3
Seating Arrangements	Constructing Task *Individual/Partners*	Multiplication and Arrays	MCC.3.OA.5
Stuck on Division	Scaffolding Task *Individual/Partners*	Division	MCC.3.OA2
*Apple Solar Farm	3-Act Task *Whole Group*	Multiples of Ten, Multiplying 1 digit numbers by multiples of 10	MCC.3.NBT.3
Base Ten Multiplication	Practice Task *Partners*	Multiplication	MCC.3.OA.1
Multiples of Tens	Scaffolding Task *Partners/Small Group*	Multiples of Ten	MCC.3.NBT.3
How Many Tens?	Constructing Task *Small Group*	Multiplying 1 digit numbers by multiples of 10	MCC.3.NBT.3
What Comes First, the Chicken or the Egg?	Constructing Task *Individual/Partners*	Division	MCC.3.OA.4
Sharing Pumpkin Seeds	Constructing Task *Individual/Partners*	Division	MCC.3.OA.2 MCC.3.OA.3
*Egg Tower	3-Act Task *Whole Group*	Multiplication and Arrays	MCC.3.OA.1
Array-nging Fact Families	Practice Task *Individual/Partners*	Multiplication and Division	MCC.3.OA.5 MCC.3.OA.6
Finding Factors	Constructing/Practice Task *Small Group/Partners*	Multiplication and Division	MCC.3.OA.5 MCC.3.OA.6 MCC.3.OA.7

Shake, Rattle, and Roll Revisited	Practice Task *Individual/Partners*	Multiplication	MCC.3.OA.1 MCC.3.OA.2
Use What You Know	Practice Task *Individual/Partner Task*	Unknown Factors	MCC.3.OA.6
Multiplication Chart Mastery	Practice Task *Individual/Small Group*	Multiplication Chart	MCC.OA.5 MCC.OA.6 MCC.OA.7
Making the "Hard" Facts Easy	Constructing Task *Small Group/Partners*	Distributive Property of Multiplication	MCC.3.OA.5 MCC.3.OA.7
Find the Unknown Number	Practice Task *Individual/Partners*	Unknown Factors	MCC.3.OA.5 MCC.3.OA.6 MCC.3.OA.7
Making Up Multiplication	Constructing Task *Individual/Partners*	Multiplication Stories	MCC.3.OA.5 MCC.3.OA.6
Field Day Blunder	Constructing Task *Partners*	Multiplication	MCC.3.OA.3 MCC.3.OA.4
Leap Frog	Constructing Task *Individual/Partners*	Line Plot Graph	MCC.3.MD.4
Our Favorite Candy	Constructing Task *Individual/Partners*	Pictograph and Bar Graph	MCC.3.MD.3
My Special Day	**Culminating Task** ***Individual***	**Multiplication, Division, and Data**	MCC.3.OA.5 MCC.3.OA.6 MCC.3.OA.7
Ice Cream Scoops	**Culminating Task** ***Individual***	**Multiplication and Division**	MCC.3.OA.1 MCC.3.OA.2 MCC.3.OA.3 MCC.3.OA.4 MCC.3.MD.3

SCAFFOLDING TASK: One Hundred Hungry Ants!

In this task, students will determine the factors of a product by creating equal groups of counters/colored tiles.

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This standard interprets products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

Example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? (5 groups of 3, 5 x 3 = 15)

Describe another situation where there would be 5 groups of 3 or 5 x 3.

COMMON MISCONCEPTIONS

A major conceptual hurdle in working with multiplicative structure is understanding groups of items as single entities while also understanding that a group contains a given number of objects (Clark & Kamii, 1996). Students can solve the problem "How many apples are in 4 baskets of 8 apples each?" by counting out four sets of eight counters and then counting them all. To think multiplicatively about this problem as four sets of eight requires students to conceptualize each group of eight as a single item to be counted. Experiences with making and counting equal

groups, especially in contextual situations, are extremely useful. (From: Van de Walle, *Teaching Student-Centered Mathematics*, Vol. II 3-5, page 111)

ESSENTIAL QUESTIONS

- How can multiplication be represented?
- How does an array represent the meaning of multiplication?

MATERIALS

- Colored tiles or two-sided counters
- Linking cubes (100 for groups of 4)
- Something to help organize groups such as paper plates, cups, bowls, etc.
- *One Hundred Hungry Ants,* by Elinor J. Pinczes or similar story
- Math Journal

GROUPING

Individual/Partners

NUMBER TALK

In the third grade overview, the importance of giving students opportunities to mentally compute and explain computational strategies is discussed. *Number Talks* is an excellent way to do this. Between 5 and 15 minutes should be dedicated daily to students sharing the authority of determining whether answers are accurate. Students are expected to think through all solutions and strategies carefully (Parrish, 2010). During the *Number Talk*, the teacher is not the definitive authority. The teacher is the facilitator and is listening for and learning from the students' natural mathematical thinking. The teacher writes a problem horizontally on the board in whole group or a small setting. Students mentally solve the problem and share with the whole group HOW they derived the answer. They must justify and defend their reasoning. The teacher simply records the students thinking and poses extended questions to draw out deeper understanding for all. The effectiveness of numbers talks also has a lot to do with the routines and environment that is established. Students must be given time to think quietly without the pressure of their peers. To develop this, the teacher should establish a signal of some sort to identify that one has a strategy to share. One way to do this is to place a finger on their chest indicating that they have one strategy to share. If they have two strategies to share, they place two fingers out on their chest and so on. The following number strings involve repeated addition, which will introduce students to the concept of multiplication being addressed throughout this unit.

50 + 50	25+ 25 + 25 + 25	20 + 20 + 20 + 20 + 20
51 + 51	26 + 26 + 26 + 26	21 + 21 + 21 + 21 + 21

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task students determine factors of a product by creating equal groups of counters/colored tiles.

Part I (SMP 1, 2, 3, 4, 5, 6, 7, and 8)
Begin the lesson by reading *One Hundred Hungry Ants,* by Elinor J. Pinczes or similar story. Discuss the ways the ants reorganized themselves into equal groups. You can begin the discussion by asking the following question: When the ants were first interrupted, how did they arrange themselves? Provide groups of 4 with 100 linking cubes and let them investigate this idea. Have the students create the model with linking cubes. Write the multiplication sentence next to the model. Ask students to explain the factors, by asking "Which number represents which part of the model?" At this point the discussion will develop around groups and how many are in the groups. Continue discussing and modeling the arrangements that the ants are put into each time they are interrupted. To emphasize the idea of equal groups, you may want to ask the students, "Why did the ants not organize into groups of 3 or 6?" Allow students time to struggle with this idea and to use linking cubes to investigate.

Part II (SMP 1, 4, 5, and 6)
After students have had discussions about the ways that the ants have organized themselves, they will have an opportunity to organize ants of their own. Students will be given 20 counters and asked to arrange them in as many different equal groups as they can. Students should record their reasoning using pictures, words, and numbers, in their math journal.

FORMATIVE ASSESSMENT QUESTIONS

- How many ways were you able to organize 20 ants?
- What does your number sentence look like?
- How can you explain your picture and number sentence in words?

DIFFERENTIATION

Extension

- Allow students to use different numbers of ants. (24, 36, 42). They should explain their reasoning using pictures, words, and numbers.

Intervention

- Allow students to work in small guided groups and reduce the number of ants to 12

SCAFFOLDING TASK: Arrays on the Farm

In this task the students use arrays to solve multiplication problems. Farmers grow their crops in arrays to make them easier to look after and to harvest. Additionally, students are asked to be involved in guessing and estimating. These are both useful skills that take time to develop. This task provides some practice for these skills.

APPROXIMATE TIME: 3-4 days

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) Use arrays, area models, and manipulatives to develop understanding of properties.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 85)

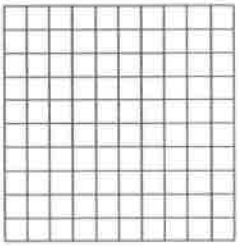
"It is not intuitively obvious that 3 x 8 is the same as 8 x 3 or that, in general, the order of the numbers makes no difference (the order or commutative property). A picture of 3 sets of 8 objects cannot immediately be seen as 8 piles of 3 objects. Eight hops of 3 land at 24, but it is not clear that 3 hops of 8 will land at the same point.

The array, by contrast, is quite powerful in illustrating the order property, as shown in Figure 3.9. Children should draw or build arrays and use them to demonstrate why each array represents two different multiplications with the same product."

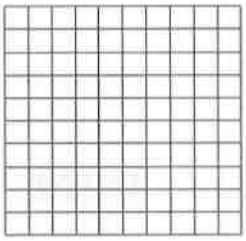
Arrays provide a quick and efficient way to count things. For example, this can be done by adding the numbers in each row together. However, it is quickest to determine the number of objects in an array by multiplying the number in the width by the number in the height using area models.

This task provides different contexts to explore multiplication concepts using arrays such as the one below. This array is an area model.

6 rows of 7
or
6 x 7 = 42



7 rows of 6
or
7 x 6 = 42



COMMON MISCONCEPTIONS

One common misconception is that students must answer their problems using multiplication number sentences. Students may continue to use repeated addition equations to represent their work. It is important for teachers to use this time to relate multiplication to the student's prior knowledge of repeated addition by introducing the multiplication sign and explaining what the two factors mean. (Taken from: Van de Walle, *Teaching Student-Centered Mathematics*, Vol. II 3-5)

ESSENTIAL QUESTIONS

- How can multiplication be represented?
- How does an array represent the meaning of multiplication?

MATERIALS

- Counter or small manipulatives
- Large pieces of paper for recording
- scissors
- Henry's Array Farm recording sheet

- Array Circles recording sheet
- Enlarged Small Station Task problems
- Station Task problems individual recording sheet

GROUPING

Partner/Small Group

NUMBER TALK

As discussed in the beginning lesson of this unit, Number Talks give students opportunities to mentally compute and explain computational strategies. The problems are ordered so that the strategy involved in solving each problem can be used to solve the next one.

2 x 4
4 x 4
2 x 5
4 x 5
4 x 7

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION (SMP 1, 2, 3, 4, 5, 6, 7, and 8)

Part I Scaffolding

1. Begin the task with the "Farmer's Problem"
   Pose the problem:

   Henry's farm produces potatoes every fall. He has 36 potato plants. How could Henry arrange his plants into equal rows and columns?

   Provide a pile of counters on an overheard, use a smart board with a shape tool (or plant image), or ask volunteers to come and show what the first row of plants might look like.

   Is there only one way to arrange Henry's potato plants?

   It's important for the students to understand what a row is so they can make sense of the problem.

   What are some predictions? How many different strategies will Henry be able to choose from? Record predictions on the board or someplace the class can see.

   2. Arrange the class into small mixed ability groups with 3 or 4 students in each. Give each group a large sheet of paper. Ask them to fold their piece of paper so it makes 4 boxes.

Strategy 2	**Strategy 4**
Strategy 1	**Strategy 3**

Allow some time for each group to see if they can come up with 4 different ways to solve the Farmer's Problem and record their method in the 4 boxes.
The teacher should circulate to prompt groups and challenge their thinking.

Ask: *Are there more efficient ways to determine how many plants there are without having to count each one?*

3. Ask the groups to cut their large sheet into the 4 boxes. Each student will take a different strategy from their group. They must be aware that they will explain the strategy and ready to do so. Gather the class in a circle or position them so each one can see the presenting student and strategy. Ask the groups to share what they think is their most interesting strategy. Place each group's strategy in the middle of the circle or area as they are being shared. Once each group has contributed, ask the students to offer strategies that no one has shared yet.

Likely strategies	Possible teacher responses
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 ….	Can you think of a quicker way to work out how many plants there are? How many plants are there in one row?
6,12,18,24,30,36	Do you know what 6 + 6 =? Can that help you solve this problem quicker?
6 x 6 = 36	What if Henry had 9 rows of plants and there were 8 plants in each row?
6 + 6 = 12; 12 + 12 = 24; 24 + 12 = 36	You used adding to work that out. How could you have used multiplication?
2 x 6 = 12; 12 + 12 + 12 = 36	If 2 x 6 = 12, what does 3 x 6 =? How could you work out 6 x 6 from this?
3 x 6 = 18 and then doubled it	Awesome, clever you, could you work out 9 rows of 6 for me?
5 x 6 = 30; and 6 more = 36	

4. The shared strategies can be put into similar groups.

Ask: *Who used a strategy like this one?* Show Henry's Array Farm Format sheet. *How do you think we could use this to solve Henry's problem?* Send groups off to experiment with Henry's Array Farm Format sheet. Again circulate in the classroom and observe what the groups are doing.

5. As a class, share the different ways that students used the array to solve Henry's Farm problem.

6. Challenge the students to use the array in the same way as another group did and pose the following problems.

7. Review predictions made before breaking up into small groups? Which predications were most reasonable?

Henry plants 2 more rows of 6 potato plants. How many rows does he have all together now? How many plants is that all together?	*Henry's sister, Katie has a farm exactly the same size with the same amount of plants. How many plants do they have all together?*

Part II Constructing

Before the students work on small group tasks, you will need to prepare the material from Henry's Array Farm recording sheet, Array Circles sheet and Station Task sheets.

The next few days will involve working on similar problems. Allow students the opportunity to choose intervention, if they are finding the problems challenging and are unable to get started independently. This is an opportunity for the teacher to go over more examples and clarify any misunderstandings.

Begin by role playing the Airplane Rides Over the Farm problem from the Station Task sheet to get the students started. Ask 6 students to pretend to be the passengers on the plane. Ask them to bring their chair to the front and sit in a row. Leave a gap in the middle to show where the aisle is. Then choose a second group of 6 to make a second row behind the first row of 6 students.

Ask students to think about:
How many passengers would there be if there were 4 rows of 6?
What about if there were 9 rows?
Encourage students to share and demonstrate their strategies.

Place the enlarged Station Task problem cards at each station with Henry's Array Farm, Circle Array sheets and counters or small manipulatives. (You may want to have multiple stations of

each problem depending on your class size. Make sure the Station Task number is large, so when students are ready to move they can identify an open station easily)
Read the Station Tasks to the class one at a time to clarify any misunderstandings.

Explain the Station Task Recording Sheet and how it is used. (As students have finished a Station Task, they complete the tracking box and continue to the next problem. Encourage students to show their math language, working knowledge and solutions on this sheet.)

Set/remind students of classroom procedures for partner work.

Assign the partners and allow students to choose which Station Task to solve first. Spend the session circulating the stations and questioning students.

How many did you think there would be to start with?
Why did you predict that to start with?
What gave you a clue to make that prediction?
Can you think of another way to use the array to solve that problem?
Can you think of a way to solve the problem without using the array?
Which way do you think is faster? Why?

Task Station 1 (have counters, Henry's Array Farm and Array Circles available)

Peanut Farm (Part 1)	Peanut Farm (Part 2)
Paul has a peanut farm with 6 plants in each row and there are 8 rows of plants. How many plants does he have in all? Prediction Solution	Paul wants to plant another field of peanut plants. He can get 24 plants at a cheap price. How many different ways can he plant these peanut plants? How many rows should he plant? How many peanut plants should be in each row? Show all the different ways Paul could plant his peanut plants in rows by making your own array on a blank piece of paper.

Task Station 2 (have counters or manipulatives available)

Onion Farm (Part 1)	Onion Farm (part 2)
Bill has an onion farm. Bill has 4 onion plants in each row and has 4 rows. Bill has a total of 3 fields the same size. How many onion plants are there on his farm? Prediction Solution	Bill's son has just bought the plants to start his own onion farm. He has purchased 36 plants and wants some help on how to arrange the plants into rows. What suggestions would you give Bill's son? Draw all the possibilities

Task Station 3

Watermelon Farm (Part 1)	Watermelon Farm (Part 2)
Wanda has a watermelon farm. In each row she has 12 watermelon plants and there are 4 rows. How many watermelon plants does she have in all? Prediction Solution	Wanda wants to double the size of her watermelon farm each year. How many plants will she have in all after? 1 year 2 years 3 years Record your strategies on an array

Task Station 4

Airplane Rides Over the Farm	Airplane Rides Over the Farm
Henry is giving airplane rides over his farm. His plane has 6 seats in each row. The seat row numbers go up to 21. How many passengers can Henry's plane hold? Prediction Solution	Henry's plane makes 2 trips over the farm each day. If the plane was full each time, how many passengers would get to see Henry's farm from the airplane? Prediction Solution

At the end of each session allow for a sharing time to discuss what students were finding interesting and challenging. Share some strategies used.

Part III
Conclusion/Reflection

On the final day of the task, ask the students to make up their own multiplication problems for their partner to solve.

Set the problem: Tell the students they are going to pretend to be tomato growers. They are to decide how many rows of tomato plants they want in each row and how many rows they will have altogether.

Now they challenge their partner to see if the partner can work out how many tomato plants they will have altogether. Ask for volunteers to share their problems and the partner share what strategy they used to solve the problem.

Challenge the students to make up 2 more problems for their partner. Suggest that they solve the problem using an array.

Conclude the lesson by talking about the types of problems we have explored and solved over the past few days. They were doing multiplication problems! Let them know there are many ways of solving these problems and that the array is just one of these ways.

FORMATIVE ASSESSMENT QUESTIONS

- What are two strategies you used to solve the problems?
- How can the same problem be represented by two different arrays?
- How does an array model show repeated addition?
- Can you think of a more efficient way to work out how many _____ there are?
- How many ____ are there in one row?
- What if _____ had 9 rows of _____ and there were 8 _____ in each row?
- You used adding to work that out. How could you have used mulitiplication?
- If 2 x 6 = 12, what does 3 x 6 =? How could you work out 6 x 6 from this?
- Awesome, clever you, could you work out 9 rows of 6 for me?

DIFFERENTIATION

Extension

- Replace Station Task amounts with greater numbers.
- Provide larger numbers and challenge the students to create as many possible arrays as they can for each farm

Intervention

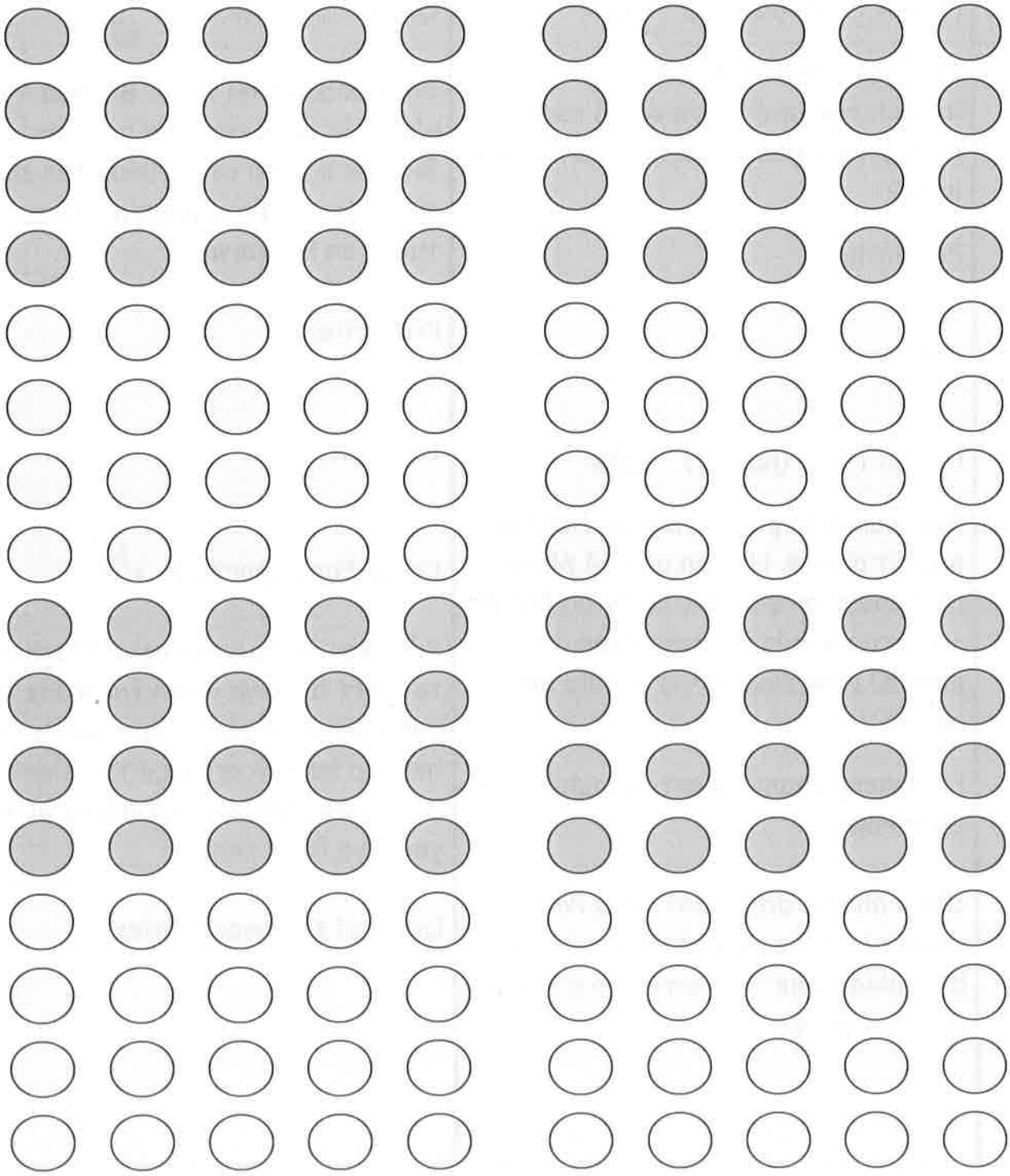
- Replace Station Tasks with lower amounts
- Provide Station Tasks with partial arrays that the student needs to complete

TECHNOLOGY CONNECTION

http://www.multiplication.com/
Practice games for multiplication facts as well as teacher resource pages with instructional ideas on how to introduce multiplication.
Note: This site contains advertising.

Make arrays and see the associated fact
http://www.haelmedia.com/OnlineActivities_txh/mc_txh3_002.html

Array of Circles



Arrays on the Farm: Task Station Cards (enlarge)

Peanut Farm (Part 1) 

Paul has a peanut farm with 6 plants in each row and there are 8 rows of plants. How many plants does he have in all?

Prediction

Solution

Peanut Farm (part 2) 

Paul wants to plant another field of peanut plants. He can get 24 plants at a cheap price. How many different ways can he plant these peanut plants? How many rows should he plant?

How many peanut plants should be in each row?

Show all the different ways Paul could plant his peanut plants in rows by making your own array on a blank piece of paper.

Onion Farm (Part 1) 

Bill has an onion farm. Bill has 4 onion plants in each row and has 4 rows. Bill has a total of 3 fields the same size. How many onion plants are there on his farm?

Prediction

Solution

Onion Farm (part 2) 

Bill's son has just bought the plants to start his own onion farm. He has purchased 36 plants and wants some help on how to arrange the plants into rows. What suggestions would you give Bill's son?

Draw all the possibilities

Watermelon Farm (Part 1) 

Wanda has a watermelon farm. In each row she has 12 watermelon plants and there are 4 rows. How many watermelon plants does she have in all?

Prediction

Solution

Watermelon Farm (Part 2) 

Wanda wants to double the size of her watermelon farm each year. How many plants will she have in all after...

1 year

2 years

3 years

Record your strategies on an array

Airplane Rides Over the Farm (Part 1) 

Henry is giving airplane rides over his farm. His plane has 6 seats in each row. The seat row numbers go up to 21. How many passengers can Henry's plane hold?

Prediction

Solution

Airplane Rides Over the Farm (Part 2) 

Henry's plane makes 2 trips over the farm each day. If the plane was full each time, how many passengers would get to see Henry's farm from the airplane?

Prediction

Solution

Name _____

Arrays on the Farm
Student Recording Sheet

I. Station Tasks

Stations	Initial when Completed
Peanut Farm	
Onion Farm	
Watermelon Farm	
Delta's New Plane	

Peanut Farm (part 1)
Paul has a peanut farm with 6 plants in each row and there are 8 rows of plants. How many plants does he have in all?

Prediction:

Show your math language and work to find the answer here

Peanut Farm (part 2)
Paul wants to plant another field of peanut plants. He can get 24 plants at a cheap price. How many different ways can he plant these peanut plants? How many rows should he plant?

How many peanut plants should be in each row?

Draw all the different ways Paul could plant his peanut plants in rows by making arrays.

Onion Farm (Part 1)

Bill has an onion farm. Bill has 4 onion plants in each row and has 4 rows. Bill has a total of 3 fields the same size. How many onion plants are there on his farm?

Prediction

Show your math language and work to find the answer here

Onion Farm (part 2)

Bill's son has just bought the plants to start his own onion farm. He has purchased 36 plants and wants some help on how to arrange the plants into rows. What suggestions would you give Bill's son?

Draw all the possible arrays Bill could use to organize his plants.

Watermelon Farm (Part 1)

Wanda has a watermelon farm. In each row she has 12 watermelon plants and there are
4 rows. How many watermelon plants does she have in all?

Prediction

Show your math language and work to find the answer here

Watermelon Farm (Part 2)

Wanda wants to double the size of her watermelon farm each year. How many plants will she have in all after.

Record your strategies on an array

1 year

2 years

3 years

Airplane Rides Over the Farm (Part 1)

Henry is giving airplane rides over his farm. His plane has 6 seats in each row. The seat row numbers go up to 21. How many passengers can Henry's plane hold?

Prediction

Show your math language and work to find the answer here

Airplane Rides Over the Farm (Part 2)

Henry's plane makes 2 trips over the farm each day. If the plane was full each time, how many passengers would get to see Henry's farm from the airplane?

Prediction

Show your math language and work to find the answer here

SCAFFOLDING TASK: What's My Product?

This task allows students to interpret products of whole numbers by creating equal groups with manipulatives.

APPROXIMATE TIME - 1 class session

STANDARDS OF MATHEMATICAL CONTENT

MCC.3.OA.1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

BACKGROUND KNOWLEDGE

This standard interprets products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

Example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? (5 groups of 3, 5 x 3 = 15)

Describe another situation where there would be 5 groups of 3 or 5 x 3.

COMMON MISCONCEPTIONS

Traditionally multiplication tables are emphasized when students begin learning about multiplication. Students are sent home with flash cards without a true understanding of what multiplication is. This way of learning multiplication can be difficult for students to understand. Naturally, students make groups and groups of groups. The creation of groups is a way to find the total of something in the most efficient way. The following activity allows students to build on their natural ability to form groups and learn multiplication without memorizing facts in isolation, but as number facts that can be related to each other in a multitude of ways (Frans van Galen and Catherine Twomey Fosnot, 2007, Context for Learning Mathematics).

ESSENTIAL QUESTIONS

- How can you interpret the product by making equal groups?
- What are different ways to multiply two factors to get the same product?

MATERIALS

- Colored tiles or two-sided counters
- Something to help organize groups such as paper plates, cups, bowls, etc.
- "What's My Product" recording sheet

GROUPING

Individual/Partners

NUMBER TALK

As discussed in the beginning lesson of this unit, Number Talks give students opportunities to mentally compute and explain computational strategies. The problems are ordered so that the strategy involved in solving each problem can be used to solve the next one.

2 x 3
4 x 3
8 x 3
3 x 6
6 x 6

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION (SMP 1, 4, 5, and 6)

Part I

Discuss with students how to group objects. Show a container of 20 counters. Discuss with students an easy way to count the total number of counters in the container. Have students arrange the counters into equal groups. As students discuss how to put the 20 counters into groups write their thinking on the board. Explain to students that in a multiplication problem one number represents the number of groups and the other number represents the number of objects in a group.

Part II

Provide students with a given set (over 50) of counters or tiles to separate into equal groups. The students will continue to rearrange tiles into different groupings that are equal. As each group is arranged, write a multiplication fact to match the arrangement. Students will record their thinking in the "What's My Product?" recording Sheet.

FORMATIVE ASSESSMENT QUESTIONS

- How many ways were you able to organize the number of counters you were given?
- Can you think of another way to organize your counters?
- How can you explain your picture and number sentence in words?

DIFFERENTIATION

Extension

- Give students a prime number of counters (29, 31, 47, etc.) and charge students with counting the number of counters through grouping. They cannot count/group by ones! Even though prime numbers are not a third grade standard, the purpose of this extension is for students to create equal groups with an amount "left over". Students can then count by the groups created and add the left over amount to reach the prime number.

Intervention

- Provide smaller numbers of counters and allow students to work with a partner.

Name_____Date_____

What's My Product?

Directions: Arrange counters into equal groups. Complete the table below with your arrangements.

Groups	# of Tiles/Counters in Each Group	Multiplication Fact	Total

SCAFFOLDING TASK: The Doorbell Rang

In this task, students will be introduced to division.

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as t number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

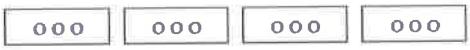
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This standard focuses on two distinct models of division. Students should begin to master multiplication facts in connection with division: partition models and measurement (repeated subtraction) models.

Partition models focus on facts. When we are trying to determine the question, "How many in each group?" a context problem for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?

Measurement (repeated subtraction) models focus on the question, "How many groups can you make?" A context problem for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?



Students often think 9 times what number is 36? It is not a separate fact but closely tied together compounding the difficulty of division notation as the unfortunate phrase "goes in to" as in, "6 goes in to 24." This phrase carries little meaning about division, especially in connection with a fair-sharing or partitioning context. The "goes in to" terminology is simply engrained in adult parlance and has not been in textbooks for years. Instead of this phrase, you can use appropriate terminology with students, such as "How many groups of 6 are in 24?" (Teacher *Student-*

Centered Mathematics Vol. II, 3-5, page 113, John A. Van de Walle and LouAnn H. Lovin, 2006).

COMMON MISCONCEPTIONS

Students may have difficulty seeing multiplication and division as inverse operations. In order to develop an understanding of this relationship, students need to have ample opportunities to explore these two operations simultaneously.

ESSENTIAL QUESTIONS

- What strategies can be used to solve real world division problems?
- In what ways can division be represented?
- How are multiplication and division alike and different?

MATERIALS

- *The Doorbell Rang* by Pat Hutchins
- 12 cookie cut-outs for each student(see attached page)
- Student "Division Patterns" recording sheet- "The Doorbell Rang"

GROUPING

Individual/Small Group Task

NUMBER TALK

As discussed in the beginning lesson of this unit, Number Talks give students opportunities to mentally compute and explain computational strategies. The problems are ordered so that the strategy involved in solving each problem can be used to solve the next one.

1 x 24
2 x 12
4 x 6
8 x 3
24 x 1

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION (SMP 1, 2, 3, 4, 5, 6, and 8)

In this task, students will analyze patterns in division.

Part I (Individual Task)

Begin this task by reading and discussing, the events in *The Doorbell Rang* by Pat Hutchins . *The Doorbell Rang* is a story about dividing a batch of cookies by a varying number of children. Give each child 12 cut out cookies to use during the story to create the different multiplication and division number sentences. Focus on how the number of cookies each child gets changes as the number of children increases.

Once the book has been read and all students have manipulated the cookies to create each situation in the story, read the book again. This time, instead of focusing on the representation, use mathematical terminology to describe what is happening in the story. For example, Mom made 12 cookies that were divided among 3 people. How many cookies did each person get? Then, write the division sentence on the board to start a discussion about what each number represents in regards to both the whole and the equal groups.

Part II (Groups of 3 or 4)

Have students continue to use their cookies to answer the questions on the student recording sheet.

Read each problem. Use your group's cookies to help find the answer. Write the division number sentence for each one.

1. Sam's mom baked 15 cookies. Sam, Charlie, and Henry shared the cookies. If the three boys shared the cookies equally, how many cookies did each boy receive?
2. Two girls want to share a batch of cookies. They baked 18 cookies. If the girls share them equally, how many cookies will each girl get?
3. Lilly wants to invite some friends over to eat cookies. She has 24 cookies and wants everyone to get 3 cookies. How many friends can she invite if she is not going to eat any cookies?
4. Alex baked 48 cookies for Halloween. He put 3 cookies in each bag. How many bags can he make?

Once all groups have completed the task, hold a class discussion and allow all groups to explain their strategies for solving each division problem.

FORMATIVE ASSESSMENT QUESTIONS

- How did you sort your cookies?
- What is a multiplication fact that could help you solve the problem?
- If you drew a picture to match this problem, what would it look like?

DIFFERENTIATION

Extension

Students may write their own division problem, using cookies as the context of the problem.

Intervention

Use plates, index cards, etc. (any visual representation) to help students form groups to count cookies into.

TECHNOLOGY CONNECTION

- http://mcq.wrdsb.on.ca/Admin/Documents/WORC/PDFs/LESSON%20PrimaryMath.pdf
- http://www.lessonplanspage.com/MathLAMultiplicationDivisionUsingTheDoorbellRang23.htm Both websites above provide teacher resources for the book *The Doorbell Rang* by Pat Hutchins.
- http://www.softschools.com/math/games/division_practice.jsp Division practice; the student or teacher can determine the parameters for the divisor, dividend, and number of problems

The Doorbell Rang

Read each problem. Use your group's cookies to help find the answer. Write the division number sentence for each one.

1. Sam's mom baked 15 cookies. Sam, Charlie, and Henry shared the cookies. If the three boys shared the cookies equally, how many cookies did each boy receive?

2. Two girls want to share a batch of cookies. They baked 18 cookies. If the girls share them equally, how many cookies will each girl get?

3. Lilly wants to invite some friends over to eat cookies. She has 24 cookies and wants everyone to get 3 cookies. How many friends can she invite if she is not going to eat any cookies?

4. Alex baked 48 cookies for Halloween. He put 3 cookies in each bag. How many bags can he make?

CONSTRUCTION TASK: Family Reunion

This task is a two part task. Part I introduces and scaffolds learning of the Associative Property. In Part II students will be constructing the associative property.

APPROXIMATE TIME: 1-2 class sessions

STANDARDS FOR MATHEMATICAL CONTENT

Understand properties of multiplication and the relationship between multiplication and division.

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 61)

Students need a good understanding of how to manipulate pattern blocks in order to solve tasks. Students should have had prior experiences with the manipulatives; they should be aware of how to use the blocks as a tool for problem solving. Emphasize the connection between multiplication and division in these tasks. Students should also have an understanding of what to do with

remainders. Sometimes remainders can be discarded, could force the number to the next highest whole, or rounded to the next whole for an approximate answer. In this activity they may experience someone being left out. Students need to understand how the remainder will be handled. In the case of seating, the students will have to force the number to the next whole.

COMMON MISCONCEPTION

Students may have difficulty seeing multiplication and division as inverse operations. In order to develop an understanding of this relationship, students need to have ample opportunities to explore these two operations simultaneously.

ESSENTIAL QUESTIONS

- How is the associative property of multiplication used in solving a problem?
- How can multiplication and division be used to solve real world problems?
- How can we use patterns to solve problems?

MATERIALS

- Item cards for Part I (printed on three different color paper or write them on three different colors of index cards)
- "Family Reunion" recording sheet
- Pattern Blocks

GROUPING

Individual/Partner Task

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION (SMP 1, 2, 3, 4, 6, 7 and 8)

This task will be in two parts: Part I introduces and scaffolds learning the Associative Property. In Part II students will be constructing the associative property.

Part I (Groups of 3)

Problem: There are 36 people at a family reunion. You have to pack coolers with the same number of food items in each cooler.

1. Start by giving each group of students a cooler card, food item card, and a package card. They are to determine, using multiplication, if they will have too much, not enough, or the right amount (36) of food. For example, 2 coolers x 5 packages x 8 hot dogs in a package = 80 hotdogs. This group has too much food.

2. Challenge the groups of three to see if they can get a different product by switching places. Ask them if it matters what position they are in? Students can record the number sentences and products either on chart paper, smart board or math notebook. Allow time for students to determine what the best equation will be for each food group. Discuss the findings and how they figured out the different amounts. Discuss the associative property and how it will affect the product when multiplying three integers.

Part II
Students will use models of tables to decide how many tables must be used to seat a given number of guests.

Students will follow the directions below from the "Family Reunion" recording sheet.

1. Help set up tables for your upcoming family reunion. Thirty-six relatives need a place at a table to sit and enjoy their food and drinks. You may use the following table styles:
    - Square tables that seat one person to a side for a total of four people at a square table.
    - Circular bistro tables that seat exactly three people.
    - Hexagonal tables that seat one person to a side for a total of 6 people.
    - Rectangular tables that seat twelve people.
    - Pentagonal tables that seat one person to a side for a total of five people.
2. Which table would you need the most? Show how you figured out how many of those tables you would need.
3. Which table would you need the least? Show how you know.
4. Choose two types of tables and draw your method for seating all 36 relatives for the family reunion. Write a number sentence to describe what you've drawn.
5. Suppose the only tables you had were pentagonal. Explain how you would seat all of your relatives.

FORMATIVE ASSESSMENT QUESTIONS

- What combinations of tables have you tried so far?
- How will you know when you find the right combination?
- Do you think there is more than one right solution for this task? Why do you think so? Do you have a way of finding out?

- How many _____ (square, circular, hexagonal, rectangular, or pentagonal) tables do you need? How do you know?

DIFFERENTIATION

Extension

- Use square tables that seat one person to a side, but this time push the tables together end to end and find out how many relatives can be seated. Continue adding tables this same way until you have enough tables to seat everyone. Enter the information in a table and describe any patterns you see. How many square tables pushed end to end would it take?

# Tables	# People Seated	Sketch	Number Pattern
1	4		(1 x 4) - 0 = 4 (1 x 2) + 2 = 4
2	6		(2 x 4) – 2 = 6 (2 x 2) + 2 = 6
3	8		(3 x 4) – 4 = 8 (3 x 2) + 2 = 8
4	10		(4 x 4) – 6 = 10 (4 x 2) + 2 = 10
5	12		(4 x 5) – 8 = 12 (5 x 2) + 2 = 12
. . .			
17	36		(17 x 2) + 2 = 36

Two possible number patterns are shown.
The first is the number of seats for the tables, minus the sides lost when tables are pushed together.
The second pattern is the number of seats along the top and bottom plus the seat at each end.

- Choose another pattern block shape and see if the same pattern holds as you push the tables together.
- Experiment to see if it will take more or less tables if a hole is left in the center or if all tables touch another table on all sides except the side where the guests will sit.
- Use a different number of relatives or allow students to make up additional types of tables (octagonal, rhomboidal, triangular, or trapezoidal).
- Rather than two types of tables, let students use three types that still yield seating for 36 people.

Intervention

- Use a smaller number of relatives, such as 12 or 20.

- Guided practice that simulates the task, done ahead of time, will enable students to develop problem solving strategies, particularly if the teacher models the strategies students are developing.

TECHNOLOGY CONNECTION

- http://www.arcytech.org/java/patterns/patterns_d.shtml Allows students to work with pattern blocks in an interactive applet and easily print their work.

Item cards for Part I

1 package	2 packages
3 packages	4 packages
5 packages	6 packages
7 packages	8 packages
9 packages	10 packages

1 cooler	2 coolers
3 coolers	4 coolers
5 coolers	6 coolers
7 coolers	8 coolers
9 coolers	10 coolers

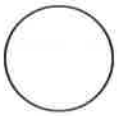
Hamburgers (8 in a package)	Hamburger Buns (4 in a package)
Hot Dogs (7 in a package)	Hog Dog Buns (3 in a package)
Bag of Individual Chips (5 in a package)	Juice Drinks (10 in a package)
Sodas (6 in a package)	Ice Pops (9 in a package)

Name _____ Date _____

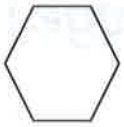
Family Reunion

1. Help set up tables for your upcoming family reunion. Thirty-six relatives need a place at a table to sit and enjoy their food and drinks. You may use the following table styles:

Square tables that seat one person to a side for a total of four people at a square table

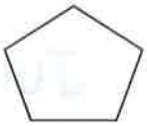


Circular bistro tables that seat exactly three people



Hexagonal tables that seat one person to a side for a total of 6 people

Rectangular tables that seat twelve people



Pentagonal tables that seat one person to a side for a total of five people

2. Of which table would you need the most? Show how you figured out how many of those tables you would need.

3. Of which table would you need the least? Show how you know.

4. Choose two types of tables and draw your method for seating all 36 relatives for the family reunion. Write a number sentence to describe what you've drawn.

5. Suppose the only had pentagonal tables that seat five people per table. Explain how you would seat all of your relatives.

CONSTRUCTING TASK: Skittles Cupcake Combos

This task assesses students' understanding of division and their ability to organize data.

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

BACKGROUND KNOWLEDGE

When students are given trivial word problems, they often just ask themselves what operation is called for; the context becomes irrelevant as they manipulate numbers, applying what they know. True context keeps students focused and interested in making sense of the math. Students begin to notice patterns and ask questions about what is going in the problem. Then students begin to defend their math to one another. The following activity allows students to build on their knowledge of grouping materials in order to divide more efficiently. (Frans van Galen and Catherine Twomey Fosnot, 2007, Context for Learning Mathematics).

COMMON MISCONCEPTION

Students may have difficulty seeing multiplication and division as inverse operations. In order to develop an understanding of this relationship, students need to have ample opportunities to explore these two operations simultaneously.

ESSENTIAL QUESTIONS

- How are multiplication and division related?

MATERIALS

- paper
- graph paper

- counters, interlocking cubes

GROUPING

Individual/Partner Task

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION (SMP 1, 2, 3, 4, 6, and 8)

Students will follow directions from the "Skittles Cupcake Combo" recording sheet.

> I love Skittles and cupcakes! I decided to bake some cupcakes. I put a bag of Skittles, 45 in all, into my batter and baked a dozen cupcakes. Each cupcake had at least three Skittles and no more than five. What are the different possible combinations of Skittles found in each of the cupcakes?

FORMATIVE ASSESSMENT QUESTIONS

- What combinations of blocks have you tried so far?
- How will you know when you find the right combination?
- Do you think there is more than one right solution for this task? Why do you think so? Do you have a way of finding out?

DIFFERENTIATION

Extension

- Replace 45 with 50, 75, 90 for students who can work with larger numbers.

Intervention

- Replace 45 with a smaller number such as 12, 24, or 36. Model this task or a similar one in a small group setting.
- Use manipulatives to create 12 groups that would add to 45.

Name _____ Date _____

Skittles Cupcake Combos

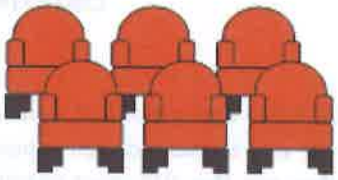
I love Skittles and cupcakes! I decided to bake some cupcakes. I put a bag of Skittles, 45 in all, into my batter and baked a dozen cupcakes. Each cupcake had at least three Skittles and no more than five. What are the different possible combinations of Skittles found in each of the cupcakes?

1. Draw pictures to show all the ways you can arrange the Skittles.

2. Label and write matching number sentences for each arrangement.

CONSTRUCTING TASK: Seating Arrangements

In this task, students will solve a word problem requiring them to make arrays using the number 24.



APPROXIMATE TIME: 1 class session

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) Use arrays, area models, and manipulatives to develop understanding of properties.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 62)

"In the beginning, students will be able to use the same models – sets and number lines – for all four operations. A model not generally used for addition but extremely important and widely used for multiplication and division is the array. An array is any arrangement of things in rows and columns, such as a rectangle of square tiles or blocks.

To make clear the connection to addition, early multiplication activities should also include writing and addition sentence for the same model."

COMMON MISCONCEPTIONS

Traditionally multiplication tables are emphasized when students begin learning about multiplication. Students are sent home with flash cards without a true understanding of what multiplication is. This way of learning multiplication can be difficult for students to understand. Naturally, students make groups and groups of groups. The creation of groups is a way to find the total of something in the most efficient way. The following activity allows students to build on their natural ability to form groups and learn multiplication without memorizing facts in isolation, but as number facts that can be related to each other in a multitude of ways (Frans van Galen and Catherine Twomey Fosnot, 2007, Context for Learning Mathematics).

ESSENTIAL QUESTIONS

- How can we use arrays to help develop an understanding of the commutative property?
- How does drawing an array help us think about different ways to decompose a number?

MATERIALS

- "Seating Arrangements" recording sheet
- Grid paper, if needed
- Manipulatives, if needed

GROUPING

Individual/Partner Task

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I (SMP 1, 4, 5, and 6)

You may want to provide grid paper or have students draw the arrays on plain copy paper. Students should develop the following arrays: 1 x 24, 2 x 12, 3 x 8, 4 x 6, 24 x 1, 12 x 2, 8 x 3, and 6 x 4. As students examine both the 4 x 6 array and the 6 x 4 array, for instance, help them understand that while both arrays have the same area, their orientation can make a difference. For example, when arranging chairs in a room, the shape of the room could dictate whether there are 6 rows of 4 chairs or 4 rows of 6 chairs. Use this to introduce the commutative property of multiplication.

Part II (SMP 1, 2, 3, 4, 6, and 7)

Students will follow the directions below from the "Seating Arrangements" recording sheet.

Your class is going to have a special presentation and your teacher has asked you to figure out a good way to place 24 chairs in your room for seating. There is only one requirement. All the chairs must be placed in an array.

1. Draw pictures to show all the ways you can arrange the chairs in an array.
2. Label and write matching number sentences for each array.
3. Choose your favorite arrangement and explain why you think it would be the best arrangement so that every student could see the presentation.

Once all students have completed the task, have a class discussion about strategies used to complete the task.

FORMATIVE ASSESSMENT QUESTIONS

- Explain how you built each array.
- With 24 blocks, can you have an array with 7 in each row? Why or why not?
- Is there a way to determine the measurements of an array for 24 without building it with blocks or drawing a diagram?
- How many different solutions do you think there are to this problem? Is there a way to check to see if you have found all possible solutions?

DIFFERENTIATION

Extension

- Using 24, or another appropriate number, have students multiply to find the number of chairs needed for 2, 3, 4, 5, and 6 third grade classrooms that use twenty-four chairs each. Ask students to develop a strategy to solve the problem. Then allow students to share their strategies.

Intervention

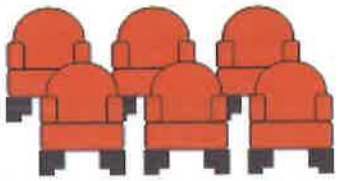
- Replace 24 with a smaller number such as 12, 18 or 20.
- Model this task or a similar one in a small group setting.

TECHNOLOGY CONNECTION

http://illuminations.nctm.org/LessonDetail.aspx?id=U109 Numerous ideas for introducing multiplication, including the array model.

Name _____ Date _____

Seating Arrangement



Your class is going to have a special presentation and your teacher has asked you to figure out a good way to place 24 chairs in your room for seating. There is only one requirement. All the chairs must be placed in an array.

1. Draw pictures to show all the ways you can arrange the chairs in an array.
2. Label and write matching number sentences for each array.
3. Choose your favorite arrangement and explain why you think it would be the best arrangement so that every student can see the presentation.

SCAFFOLDING TASK: Stuck on Division

In this task, students will experiment with a set of 12 connecting cubes to determine the division patterns when the dividend is 12.

APPROXIMATE TIME –

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

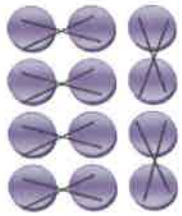
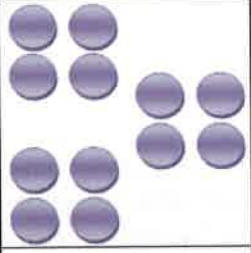
STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Students should clearly understand how to write number sentences and how to follow written directions before working independently.

One possible solution is shown below:

Division is...	Diagram	Number Sentence
Repeated subtraction "I took two circles away each time. I subtracted 2, 6 times."		12 – 2 – 2 – 2 – 2 – 2 – 2 = 0
Separating a whole into equal groups "I separated the circles into groups of 4. I have 3 groups of four."		12 ÷ 4 = 3
The opposite of multiplication "I have an array with four rows of three. That can be written as 4 x 3 = 12 or 12 ÷ 3 = 4."		4 x 3 = 12 3 x 4 = 12 12 ÷ 4 = 3 12 ÷ 3 = 4

COMMON MISCONCEPTIONS

The three ways of looking at division (separating into equal groups, repeated subtraction, and inverse of multiplication) are closely related and may be difficult for students to verbalize initially as they make connections between concrete models and their corresponding number sentences. Therefore, students need multiple experiences using a given number of cubes to model repeated subtraction, form equal groups, and explain how these two activities are alike and different. They also need to understand the inverse relationship of multiplication and division. Help students make connections to the language of mathematics and between visual and symbolic representations.

ESSENTIAL QUESTIONS

- How can we model division?
- How are multiplication and division related?
- How are subtraction and division related?
- How can we write a mathematical sentence to represent division models we have made?

MATERIALS

- 12 connecting cubes **per student**
- "Stuck on Division" task sheet
- "Stuck on Division" recording sheet
- *Divide and Ride* by Stuart J. Murphy or similar book

GROUPING

Individual/Partner Task

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will experiment with a set of 12 connecting cubes to determine the division patterns when the dividend is 12.

Part I
You may choose to open this task by reading, discussing, and modeling the events in *Divide and Ride* by Stuart J. Murphy. *Divide and Ride* is a story about dividing a group of children to ride amusement park rides. Another suitable book about division is *One Hundred Hungry Ants* by Elinor J. Pinczes. Focus on the different ways division can be described (separating into equal groups, repeated subtraction, and inverse of multiplication.)

Part II (SMP 1, 2, 3, 4, 5, 6,and 7)
Students will follow the directions below from the "Stuck on Division" task sheet.

Use 12 connecting cubes to complete this task.

1. Begin with 12 cubes and remove the same number of cubes over and over again until there are none left. Remember, you must remove the *same number* each time. Make a model of your idea with the cubes.
2. Use the first row of the "Stuck on Division" recording sheet to
    a. write about what you did
    b. draw a diagram of your model
    c. write a subtraction number sentence that describes your model
3. Find a way to separate your cubes into equal groups. How can you show the dividend, divisor, and quotient with your cubes?

4. Use the second row of the "Stuck on Division" recording sheet to
    a. write about what you did
    b. draw a diagram of your cube groups
    c. write a division number sentence
5. Now think of a multiplication fact whose product is twelve. Can you make groups of cubes that prove that division is the opposite of multiplication?
6. Use the third row of the "Stuck on Division" recording sheet to
    a. write about what you did
    b. draw a diagram of your cube groups
    c. write the fact family for your diagram
7. Compare your answers with your friends. Did everyone have the same answers? How can you tell whose solutions are correct? What different strategies did you use?

FORMATIVE ASSESSMENT QUESTIONS

- Can you explain more than one way to think about dividing a number?
- How can you write your model in a number sentence so others will understand your model?
- How can we show your model as both a division number sentence and a subtraction number sentence?

DIFFERENTIATION

Extension

Have students to complete the chart with 13 blocks. Ask students to include leftovers in their explanations, diagrams, and number sentences.

Intervention

Direct instruction in small groups can provide support for students who struggle with these concepts and can enable them to develop the ability to describe their thinking.

TECHNOLOGY CONNECTION

- http://mcq.wrdsb.on.ca/Admin/Documents/WORC/PDFs/LESSON%20PrimaryMath.pdf
- http://www.lessonplanspage.com/MathLAMultiplicationDivisionUsingTheDoorbellRang23.htm Both websites above provide teacher resources for the book *The Doorbell Rang* by Pat Hutchins.
- http://www.stuartjmurphy.com/activities/activity_ideas.php Stuart Murphy website with activity suggestions for *Divide and Ride.* (Click on level 3 and then click on the title of the book.)

Name _____ Date _____

Stuck on Division
Task Sheet

Use 12 connecting cubes to complete this task.

1. Begin with 12 cubes and remove the same number of cubes over and over again until there are none left. Remember, you must remove the *same number* each time. Make a model of your idea with the cubes.

2. Use the first row of the "Stuck on Division" recording sheet to
   a. write about what you did
   b. draw a diagram of your model
   c. write a subtraction number sentence that describes your model

3. Find a way to separate your cubes into equal groups. How can you show the dividend, divisor, and quotient with your cubes?

4. Use the second row of the "Stuck on Division" recording sheet to
   a. write about what you did
   b. draw a diagram of your cube groups
   c. write a division number sentence

5. Now think of a multiplication fact whose product is twelve. Can you make groups of cubes that prove that division is the opposite of multiplication?

6. Use the third row of the "Stuck on Division" recording sheet to
   a. write about what you did
   b. draw a diagram of your cube groups
   c. write the fact family for your diagram

7. Compare your answers with your friends. Did everyone have the same answers? How can you tell whose solutions are correct?

Name _____ Date _____

Stuck on Division
Recording Sheet

Division is...	Diagram	Number Sentence
Repeated subtraction		
Separating a whole into equal groups		
The opposite of multiplication		

*3 ACT TASK: Apple Solar Farm

Task Adapted from: http://www.101qs.com/2836

APPROXIMATE TIME: 1 class period

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students must make sense of the problem by identifying what information they need to solve it.
2. **Reason abstractly and quantitatively.** Students are asked to make an estimate (high and low).
3. **Construct viable arguments and critique the reasoning of others.** After writing down their own questions, students discuss their question with partners, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others came up with.
4. **Model with mathematics.** Once given the information, the students use that information to develop a mathematical model to solve their question.
5. **Use appropriate tools strategically.** Students write their best estimate and two more estimates – one that is too low and one that is too high to establish a range in which the solution would occur.
6. **Attend to precision.** Students use clear and precise language when discussing their strategies and sharing their own reasoning with others.
7. **Look for and make sense of structure.** Students use their understanding of properties of operations as strategies to help them multiply one-digit numbers by multiples of 10.

ESSENTIAL QUESTIONS

In order to maintain a student-inquiry-based approach to this task, it may be beneficial to wait until Act 2 to share the EQ's with your students. By doing this, students will be allowed the opportunity to be very creative with their thinking in Act 1. By sharing the EQ's in Act 2, you will be able to narrow the focus of inquiry so that the outcome results in student learning directly related to the content standards aligned with this task.

- How can multiplication be represented?
- How does an array represent the meaning of multiplication?
- What strategies can help you solve real world multiplication problems?

MATERIALS

- Act 1 picture (attached)
- Student recording sheet

GROUPING

Individual/Partner and or Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will view the picture provided and tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on **need**. When they realize they don't have the information they need, and ask for it, it will be given to them.

Background Knowledge and Common Misconceptions:
This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org and the K-5 CCGPS Mathematics Wiki.

This task allows students to work on multiplication by having them apply their understanding of
place value. In this task, students will go beyond tricks that hinder understanding such as "just
adding zeros" and explain and reason about their products. For example, for the problem 50 x 4,
students should think of this as 4 groups of 5 tens or 20 tens.

It is important to address any misconceptions that students may have with "just adding zeros".
Students need to be able to think in terms of ___ groups of ____ tens.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/) **"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."**

- Show Act 1 picture to students (Depending on students' prior knowledge, it may be beneficial to describe to students what a solar panel is.)

- Ask students what they noticed in the picture, what they wonder about, and what questions they have about what they saw in the picture. Do a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.

- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask and wish to answer: (*Main question(s) to be investigated)

- How many solar panels are in a row?
- Does every section have the same number of panels?
- *How many solar panels are pictured?

- Once students have their question, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on an empty number line. Note: As the facilitator, you may choose to allow the students to answer their own posed questions, one question that a fellow student posed, or a related question listed above. For students to be completely engaged in the inquiry-based problem solving process, it is important for them to experience ownership of the questions posed.

Important note: Although students will only investigate the main question(s) for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 2 – Student Exploration - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/) **"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."**

- During Act 2, students decide on the facts, tools, and other information needed to answer the question(s) (from Act 1). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front.

- Required Information:
  30 panels in each row
  10 rows in each section

- Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
  - What is the problem you are trying to solve?
  - What do you think affects the situation?
  - Can you explain what you've done so far?
  - What strategies are you using?
  - What assumptions are you making?
  - What tools or models may help you?
  - Why is that true?
  - Does that make sense?

Act 3 – Whole Group – Share solutions and strategies.

- Students to present their solutions and strategies and compare them.
- Reveal the solution:
  300 panels in each section.
  About 7 complete sections pictured
  About 2500 panels are pictured.

- Lead discussion to compare these, asking questions such as:

- How reasonable was your estimate?
- Which strategy was most efficient?
- Can you think of another method that might have worked?
- What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

For Act 4, reference other student-generated questions that could be used for additional classwork, projects or homework.

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?

DIFFERENTIATION

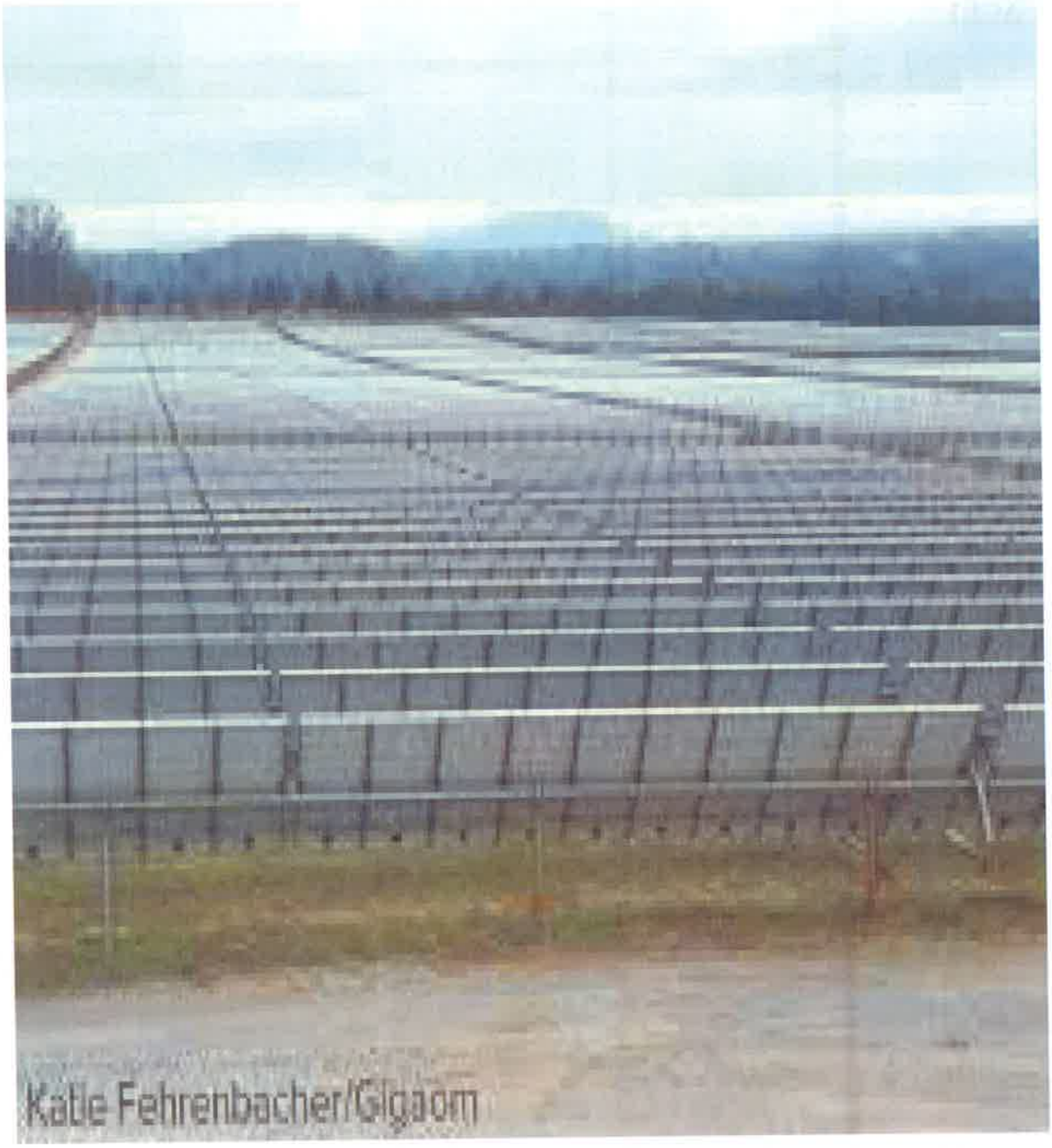
Extension

- Find additional pictures of solar panel farms, and allow students to estimate total number of panels followed by finding the total number of panels in the farm.
- Provide students with information such as, "If each row had an additional 5 (choose any number you feel fitting), how would the total number of panels be affected?

Intervention

- Provide base-ten manipulatives for the students to create the sections of the panels.

Act 1 Picture:



Task Title: _____ Name: _____

Adapted from Andrew Stadel

ACT 1

What did/do you notice?
What questions come to your mind?

Main Question: _____

What is your 1st estimate and why?

On an empty number line, record an estimate that is too low and an estimate that is too high.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimation with this information: _____

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

PRACTICE TASK: Base Ten Multiplication

(Inspired by Catherine Twomey Fosnot's, *Young Mathematicians at Work, Constructing Multiplication and Division*)

In this task students determine the factors of 100 by creating addition and/or multiplication models by placing equal number of Base Ten Blocks of a kind on index cards according to the spin of a spinner. Students then record the number sentences that their model represents.

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard interprets products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

Example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? (5 groups of 3, 5 x 3 = 15)

Describe another situation where there would be 5 groups of 3 or 5 x 3.

COMMON MISCONCEPTIONS

When students begin multiplication they are just getting used to counting. Before multiplication, 6 equaled a group of six objects. They also know that 4 equals a group of four objects. However, to think of 4 X 6 they have to think of the group of six as one unit because they need to make four sixes. The four is now used to count groups not objects. This is a hard concept to grasp for

students just learning about numbers. Students have to reorganize their thinking (Frans van Galen and Catherine Twomey Fosnot, 2007, Contexts for Learning Mathematics).
The following task will give students practice in reorganizing numbers and developing strategies that allow them to make sense of the mathematics.

ESSENTIAL QUESTIONS

- What are the strategies for learning multiplication?
- How can we practice multiplication facts in a meaningful way that will help us remember them?
- How is the commutative property of multiplication evident in an array model?

MATERIALS

- Base Ten Blocks, up to 51 cubes and 60 longs per pair (base 10 template has been provided as well)
- Spinner, numbered 1-9
- Packs of 3" X 5" index cards, from 1 to 9 cards per pack, 1 pack per pair
- Overhead Base Ten Blocks (optional)
- Math journal

GROUPING

Partners

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I: (SMP 1, 2, and 6)
Ask two volunteers to hold out their hands, palms up.

Place 2 units into each hand. Ask students how they can find the number of units in the four hands. Lead students to counting the units by twos. Record this process as the addition sentence $2 + 2 + 2 + 2 = 8$. Elicit that 2 units in each of 4 hands means that there is a total of 8 units. Point out that because the same number, 2, is added 4 times, another way of recording this is with multiplication. Write the multiplication sentence $4 \times 2 = 8$ on the board. Read it aloud as "Four groups of two equal eight." Have students to suggest ways to record 2 cubes in each of 4 hands.

Part II: (SMP 1, 2, 4, 6, and 7)

Before the task, decide which facts students need the most work with. This will be the number of cards the students will need, this can vary from group to group. Distribute the prearranged packs of index cards to the students. Instruct the students to determine how many cards are in their packs. They will spread out the cards, (the cards are the groups), then spin a spinner. The number that was spun will determine the number of unit blocks on each card. (the number of units in each group).

- *How many cards?*
- *How many* units *on each?*

The students will determine the product and record the number sentence in their math journal.

Part III (SMP 1, 2, 4, 6, and 7)

Next students will clear off their cards and put an equal number of longs in their place.

- *How many cards?*
- *How many* longs *on each?*

Students will determine the product of the longs and record their number sentence in the math journal. Students will be asked to compare the values they found for the units and for the same number of longs. What did they notice? Repeat the activity several times. (If you spin the same number as before, spin again!)

FORMATIVE ASSESSMENT QUESTIONS

- What patterns are you noticing?
- What is the relationship between the units and the longs?
- How did you determine your product?
- Could you have determined your product another way?

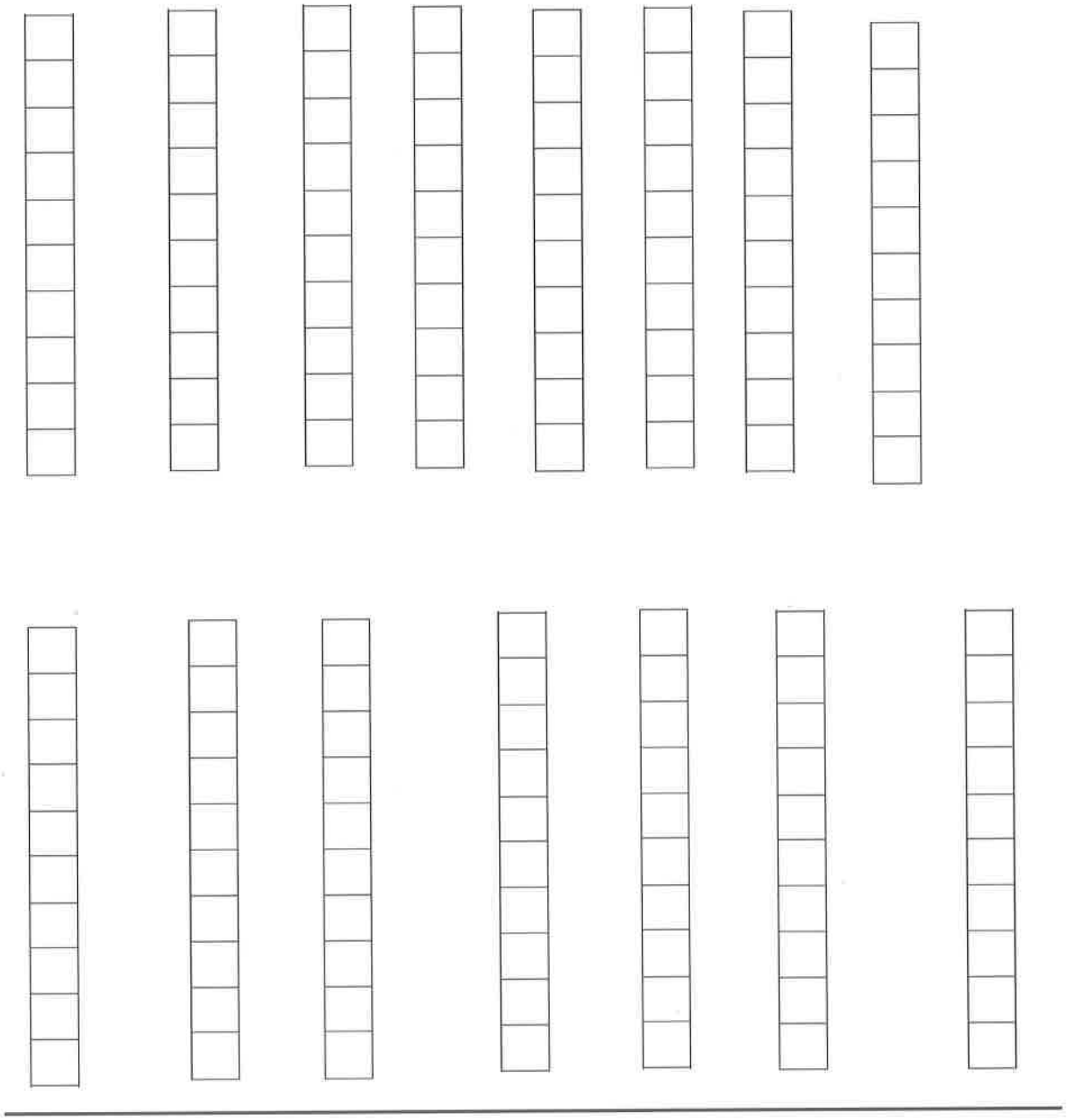
DIFFERENTIATION

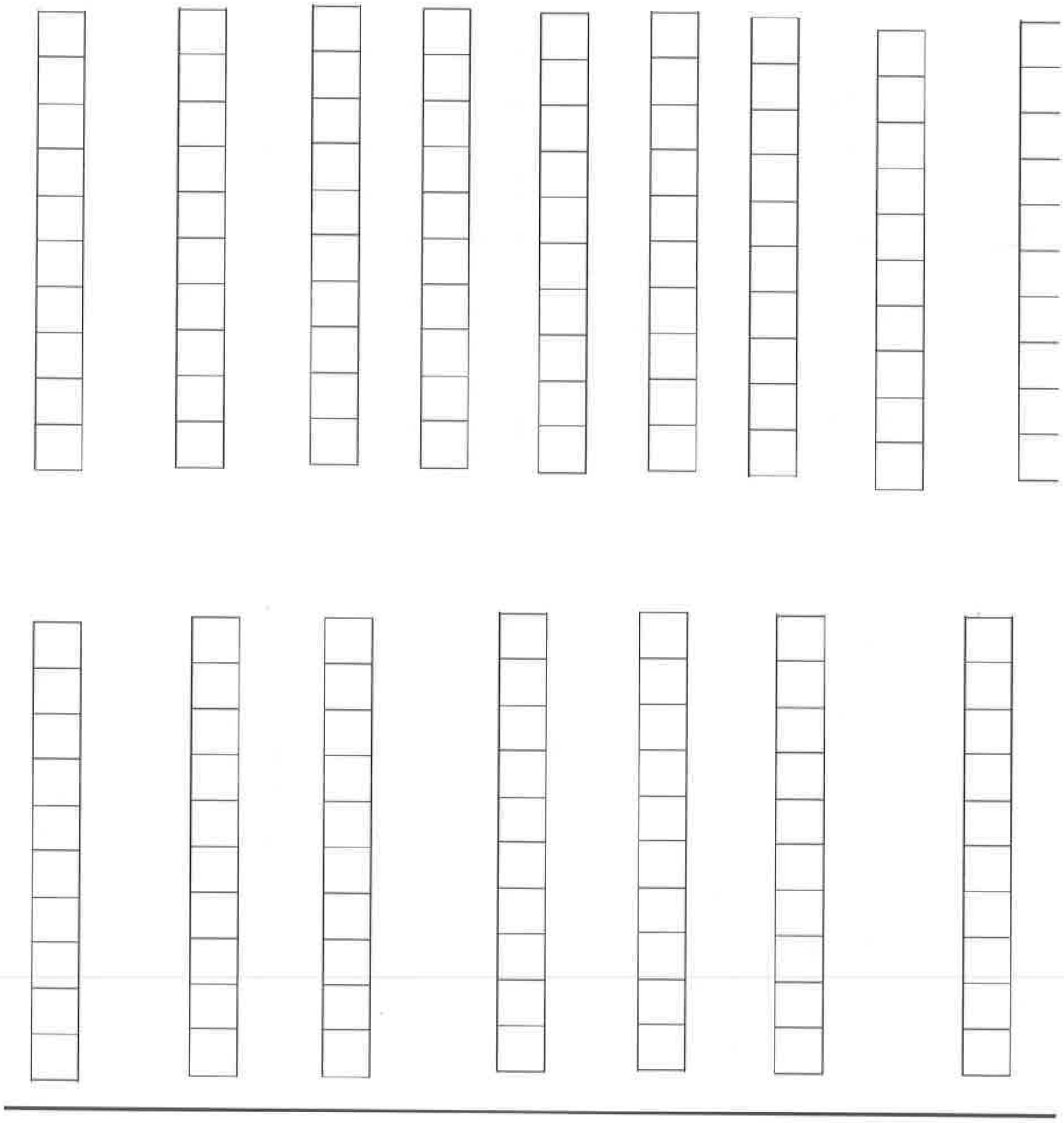
Extension

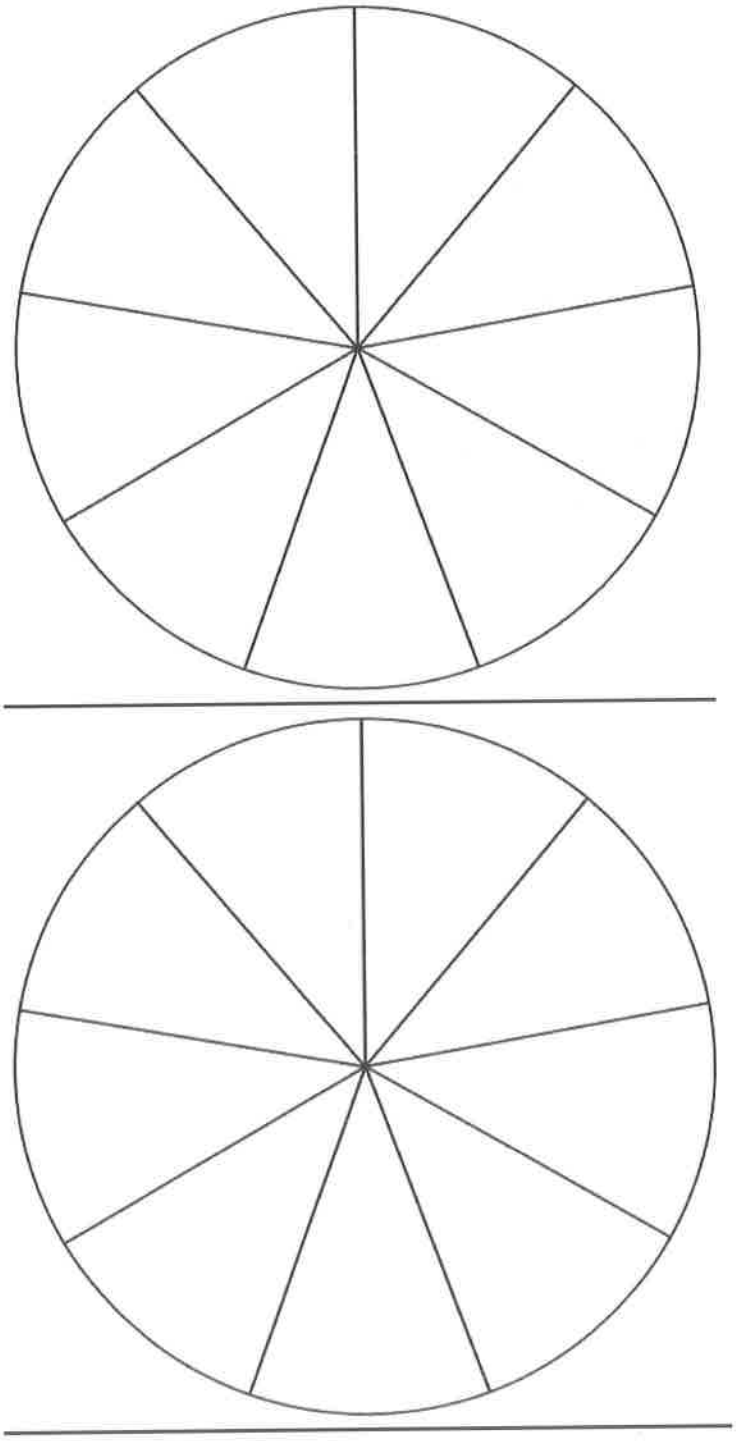
- Instead of using manipulatives to create groups, use digit cards.

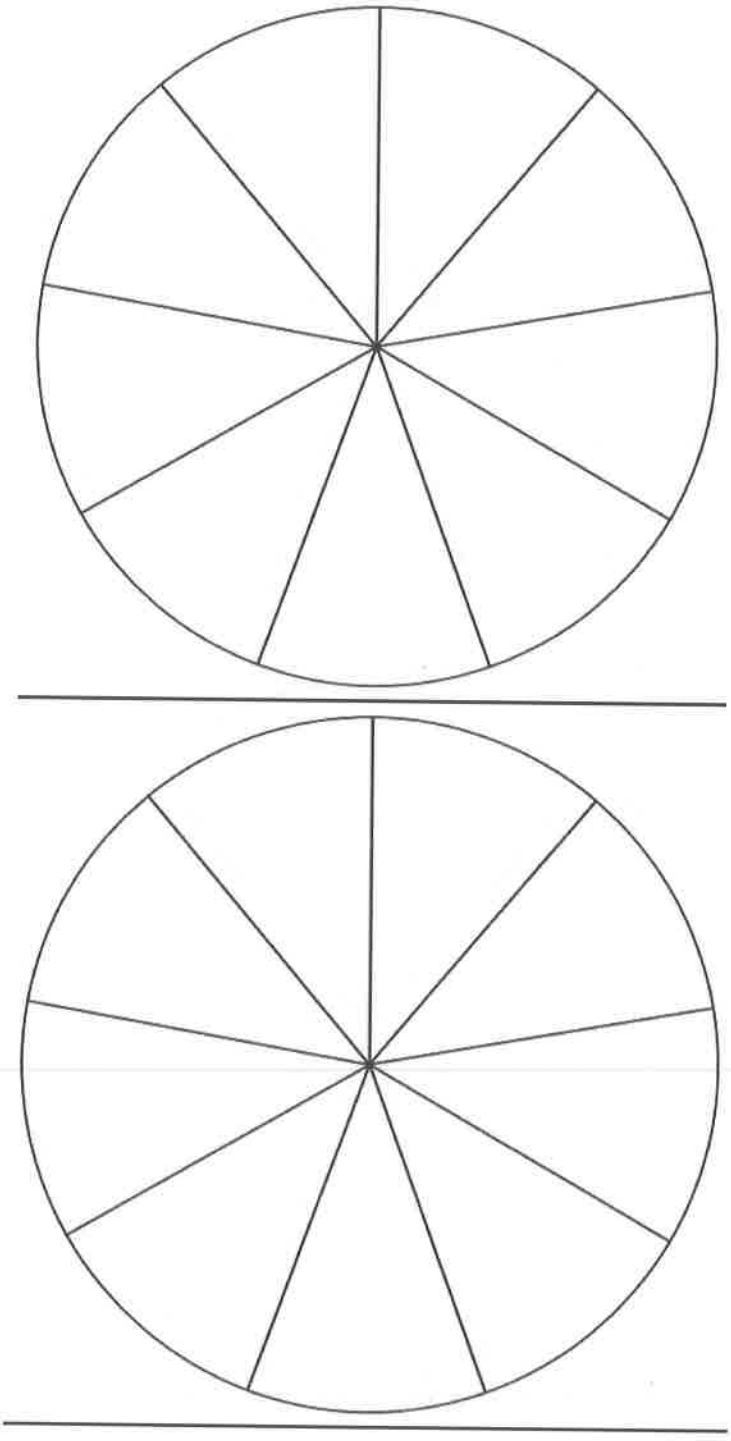
Intervention

- Use smaller numbers on the spinner.









CONSTRUCTING TASK: "MULTIPLES OF TEN"

Adapted from ETA Cuisenaire

This task allows students to apply their understanding of place value to multiplication of tens.

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

(adapted from North Carolina's DPI resources)

This task allows students to work in multiplication by having them apply their understanding of place value. In this task, students will go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens.

COMMON MISCONCEPTION

It is important to address any misconceptions that students may have with "just adding zeros". Students need to be able to think in terms of ___ groups of ____ tens.

Additionally, student misconceptions with this task may be with their lack of understanding of place value. When working with longs, students may see the group as a group of 1 instead of a group of 10. For example, when counting 5 longs the student may count them as "1, 2, 3, 4, 5" with a total of 5 objects instead of counting them as "10, 20, 30, 40, 50" or "5 groups of 10" with a total of 50. It is important for students to have this understanding of the long consisting of 10 units.

ESSENTIAL QUESTIONS

- What happens to a number when it is multiplied by ten?
- How can I model multiplication by ten?
- How is place value related to multiples of ten?

MATERIALS

- Base Ten Blocks
- Multiples of Ten Slips one set 10-100, and one set over 100 (teacher created- these are cards/slips with decade numbers on them)
- Paper bag or container (to hold multiples of ten slips)
- Large Index Cards (10) each with a decade 10-90 on it (teacher created)
- Blank index cards.

GROUPING

Students should work with a partner, or small group.

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will build snap cube structures using multiples of 10 and order the structures according to the number of cubes from which they were built. In this learning task, students should have the opportunity to:

- Count out large and small quantities of objects
- Work with multiples of 10
- Develop number sense about the relative magnitude of a number
- Work with conservation of number

Part I (SMP 1, 2, 3, and 6)

To begin, present 10 large cards displaying the decade numbers (10, 20, 30, 40, etc.) Place the 50 in the middle of a chalkboard tray, or in the middle of the floor if you are in a circle on the rug. Hand a large card to a student and ask where it belongs on the chalkboard tray. Have them explain their reasoning. Repeat this process again. When the students have decided where to

place the second card and have explained their reasoning, ask them to decide which numbers are missing and where they might go. Allow students to complete the number line. Discuss how many tens are in each number as they are placed on the number line, and the magnitude of each number.

Part II (SMP 1, 2, 4, 5, 6, and 7)

Each pair of students will:

- Draw a Multiple of Ten slip from a bag. It should be a decade number 10-90
- Provide students with Base Ten blocks. Ask them to make an array that will match their number. Ideally, one of the students will realize that they can use the 10 rods to make the arrays instead of the unit cubes.
- Have students create a T-Chart that looks similar to the one below. Students should record the number drawn and write an addition sentence to match their array. (Students have not been formally introduced to multiplication yet)

Number Chosen	Addition Sentence
40	10+10+10+10
30	10+10+10
50	10+10+10+10+10

- Students may pull several numbers to have a good comparison of their arrays.
- Once students have had many opportunities to draw from the bag and create arrays, bring them back to the carpet for a discussion.
    - How did you form your arrays?
    - Which blocks did you use?
    - Look at your number sentences. What do you notice about them?
    - How are the number chosen and the number sentence related?
- At this point you will ask students if they know how to write a multiplication number sentence that matches the addition number sentence.

Part III (SMP 1, 2, 3, 4, 5, 6, 7)

Once students complete their arrays, and have recognized the relationship between multiplying and addition, lead a discussion about the arrays using the following:

- Knowing what you know about tens, can you make an array for the number 120? Explain your thinking.
- Have each group choose one additional number from Multiples of Ten (numbers over 100).
- They will add on to the previous T-Chart with the new number. Encourage them to make an addition sentence, then a multiplication sentence.
- Discuss the patterns that they notice.

FORMATIVE ASSESSMENT QUESTIONS:

- How are you forming your arrays?
- How can you write a number sentence to match your array?
- What do you notice about the number pulled and your addition sentence?
- What happens to a number when it is multiplied by ten?

DIFFERENTIATION

Extension

- Students should investigate multiplying various decade numbers by digits 1-9 (40 x 6) by building arrays using a variety of manipulatives (tiles, connecting cubes, base ten materials, and graph paper are some suggestions).

Intervention

- These students should begin with decade numbers that match multiplication fact strategies that the student is comfortably using. For example, a student may know the x2 facts because he/she relates them to doubles facts for addition. The teacher structures problems for this student around x20, gradually allowing the student to make sense of the relationship between what he/she already knows (x2) and the new strategy (x20).

PERFORMANCE TASK: "HOW MANY TENS?"

Adapted from NZ Maths

This task further relates multiples of ten to place value by giving students the opportunity to build onto the concept that 10 tens equal one hundred, and 10 hundreds equal one thousand.

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This task is designed to develop the concept of what happens to a number when multiplied by a multiple of ten. To help build context, the teacher may read *100 Hungry Ants* by J. Pinzces. Students need to know that 10 tens make 100 and 10 hundreds make one thousand.

COMMON MISCONCEPTION

It is important to address any misconceptions that students may have with "just adding zeros". Students need to be able to think in terms of ___ groups of ____ tens.

Additionally, student misconceptions with this task may be with their understanding of place value. When working with longs, students may see the group as a group of 1 instead of a group of 10. For example, when counting 5 longs the student may count them as "1, 2, 3, 4, 5" with a total of 5 objects instead of counting them as "10, 20, 30, 40, 50" or "5 groups of 10" with a total of 50. It is important for students to have this understanding of the long consisting of 10 units.

ESSENTIAL QUESTIONS

- How can I model multiplication by ten?
- How is multiplying by ten related to place value?

MATERIALS

- Money Manipulatives
- "How many 10s" recording sheet

GROUPING

Students should work in groups of 3 to 4 members.

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I (SMP 1, 3, and 6)

Read the book, *100 Hungry Ants* by J. Pinzces to your students. During strategic points throughout the book, stop to discuss how the number 100 can be built or created from different groups of ten. (This is a perfect "anchor chart" opportunity!) Facilitate a classroom discussion about tens and hundreds to assess prior knowledge, and use this as a platform for a brief mini-lesson (if needed). Have the students work with their small groups to investigate, model (very important), and figure out the following problems:

1. What are all of the possible ways the ants could have lined up (by multiples of 10) to reach the picnic?
2. What would have been the QUICKEST way for the ants to march to the picnic so they could have gotten there before all of the food was gone?

Part II (SMP 1, 2, 6, 7, and 8)
Read the scenario to the students.

> *Problem: The Bank of Mathematics has run out of $100 bills. Alison wants to withdraw $300 in $10 dollar bills. How many $10 dollar bills does she get? Can you figure out how many $10 bills she will need with different amounts such as $600 or $900?*

Students will record answers in the "How Many Tens" table.

Repeat the same concept using the scenario below and $10 and $1 bills.

Problem: The Bank of Mathematics has run out of $100 bills. Alison wants to withdraw $256 in $10 and $1 bills. How many of each bill will she receive?

Students will record answers in "How Many Tens" table.

FORMATIVE ASSESSMENT QUESTIONS

- How did you determine how many tens to give?
- What if you didn't have any tens, what could you use?
- How does Multiplication help you determine the amount of tens needed to withdraw?

DIFFERENTIATION

Extension

- Students can determine the number of ones, then tens, then hundreds it would take to reach various student-suggested amounts. Organize the information into a student-created table, look for patterns, and explain what you see.

Intervention (SMP 4 and 5)

- Students can complete this activity with smaller amounts of money.
- Students can complete only the first part of the recording sheet.
- Students can complete the second part of the recording sheet with amounts rounded to the nearest hundred.
- Students should be given Base Ten blocks or money to help them make sense of the problem.

Recording Sheet: How Many Tens?

Problem#1: The Bank of Mathematics has run out of $100 bills. Alison wants to withdraw $300 in $10 dollar bills. How many $10 dollar bills does she get? Can you figure out how many $10 bills she will need with different amounts such as $600 or $900?

How Many Tens?

Withdraw	How many $10.00 bills?
$300.00	
$600.00	
$900.00	
$700.00	
$500.00	
$200.00	
$800.00	

Question for reflection:
How did you determine the amount of $10.00 bills needed?

Problem#2: The Bank of Mathematics has run out of $100 bills. Alison wants to withdraw $256 in $10 bills. How many of each bill will she receive?

How Many Tens and Ones?

Withdraw	$10.00	$1.00
$256.00		
$352.00		
$468.00		
$853.00		
$523.00		
$631.00		
$750.00		

Question for reflection:
How did you determine the amount of $1.00 and $10.00 bills needed?

Extension Problem:

Tickets to a concert cost $100 each. How many tickets could you buy if you have $3215?

CONSTRUCTING TASK: What Comes First, the Chicken or the Egg?

(adapted from Teaching Children Mathematics, Volume 15, Number 3, October 2008, p. 160)

In this task, students will use estimation skills to multiply by groups of ten.

STANDARDS OF MATHEMATICAL CONTENT

MCC. 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This task provides students with an opportunity to develop and discuss strategies for multiplying and/or dividing larger numbers. Various operations can be used to solve this problem. Possible strategies students may use to solve this type of problem include, using base 10 blocks, using their knowledge of multiplication and inverse operations, repeated addition, or using repeated subtraction. Third grade is students' first exposure to larger number division/multiplication and it is important to allow students time to make sense of this operation, so that they will continue to be successful with division in later grades.

If students choose to do division within this task, the conversation of remainders will naturally arise. Although remainders are not mentioned in the standards, students will encounter them naturally when dividing and they should be addressed as they come up. These conversations will lead nicely into the fourth grade standard OA3 when the remainder must be interpreted.

COMMON MISCONCEPTION

Students may have difficulty seeing multiplication and division as inverse operations. In order to develop an understanding of this relationship, students need to have ample opportunities to explore these two operations simultaneously.

ESSENTIAL QUESTIONS

- How can multiplication and division be used to solve real world problems?
- How can we use patterns to solve problems?

MATERIALS

- "What Comes First?" recording sheet
- drawing paper
- interlocking cubes or other manipulative if necessary

GROUPING

Individual/Partner Task

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Begin this task by reviewing their understanding of estimation from Unit 1. Discuss the word "approximate" and how it is used in estimating. Students should understand that rounding is not the only form of estimation.

Part II (SMP 1, 2, 3 6, and 7)
Students will follow the directions on the "What Comes First" recording sheet. This should be solved using pictures, numbers, and words.

> If most hens lay about 4 eggs each week, how many eggs does the average hen lay in one month? How many chickens would be needed to produce 50 eggs in one month? How many chickens would be needed to produce 70 eggs in one month? If 30 eggs were produced in one month, approximately how many chickens were needed to produce them?

Once all students have completed the task, hold a class discussion in which all students/groups will share the strategies used in solving the problem.

FORMATIVE ASSESSMENT QUESTIONS

- How could you use patterns to help you solve this?
- How would a table be useful in solving this problem?
- How might you use multiplication/division to solve this problem?
- Could you write a number sentence to explain your picture/table?

- How can you use estimation to help you solve this problem?

DIFFERENTIATION

Extension

- Students could determine how many eggs a hen will lay in one year. (2 hens, 3 hens, etc.)

Intervention

- Decrease the number of eggs needed each month to numbers that 4 will divide into evenly such as (24, 36, 48)
- Provide manipulatives for students to use while solving the task.

Name _____ **Date** _____

What Comes First

Show your thinking using words, pictures, and numbers in solving each of the problems below.

If most hens lay about 4 eggs each week, how many eggs does the average hen lay in one month?

How many chickens would be needed to produce 50 eggs in one month?

How many chickens would be needed to produce 70 eggs in one month?

If 30 eggs were produced in one month, approximately how many chickens were needed to produce them?

CONSTRUCTING TASK: Sharing Pumpkin Seeds

In this task, students will decide how to share pumpkin seeds fairly with a group of children.

STANDARDS OF MATNEMATICAL CONTENT

MCC.3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

MCC.3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This task provides students with an opportunity to develop and discuss strategies for dividing a two- or three-digit number by a one-digit number. Possible strategies students may use to solve this type of problem include, using base 10 blocks, using their knowledge of multiplication and inverse operations, or using repeated subtraction. Third grade is students' first exposure to larger number division and it is important to allow students time to make sense of this operation, so that students will continue to be successful with division in later grades.

This task can be paired with the following science standard: S3L1b. Identify features of green plants that allow them to live and thrive in different regions of Georgia. There are many children's books about pumpkins and pumpkin seeds, any one of them could be used as an introduction to this task. One book that deals directly with the number of seeds in a pumpkin is *How Many Seeds in a Pumpkin?* by Margaret McNamara, Illustrated by G. Brian Karas.

COMMON MISCONCEPTION

Students may have difficulty seeing multiplication and division as inverse operations. In order to develop an understanding of this relationship, students need to have ample opportunities to explore these two operations simultaneously.

ESSENTIAL QUESTIONS

- How can we divide larger numbers?
- What strategy did you find most efficient when dividing?

MATERIALS

- "Sharing Pumpkin Seeds" recording sheet
- Base 10 blocks or other materials for counting available for students who wish to use them
- *How Many Seeds in a Pumpkin?* by Margaret McNamara or similar book

GROUPING

Individual/Partner Task

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION (SMP 1, 2, 3, 4, 6, and 7)

Students will solve the two sharing problems on the "Sharing Pumpkin Seeds" recording sheet.

Problem 1

Ben and his 3 friends toasted 80 pumpkin seeds from their pumpkin. How many seeds will each child get if they share the pumpkin seeds fairly?

Clearly explain your thinking using words, numbers, and/or pictures.

Students may approach the problem 80 ÷4 in a variety of ways. Some students may build on their understanding of multiplication as the inverse of division to solve the problem.

Example 1

I know 4 x 2 = 8, so 4 x 20 = 80. If I add 4 groups of 20, I know there are a total of 80. Therefore, each child will get 20 pumpkin seeds.

Other students may build on their understanding of division as repeated subtraction.

Example 2

4 x 10 = 40 *80-40 = 40*
Each child got 10 pumpkin seeds.

4 x 10 = 40 *40– 40 = 0*
Each child got 10 more pumpkin seeds.

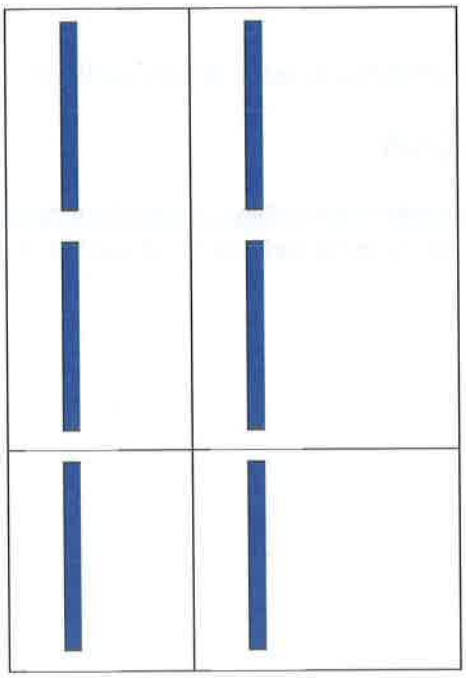
Each child received a total of 10 + 10 pumpkin seeds or 20 pumpkin seeds.

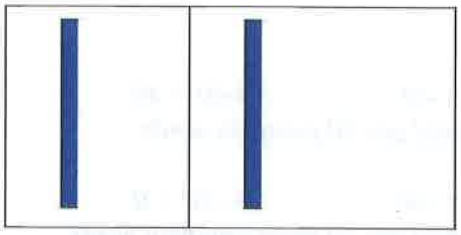
Some students may choose to use base 10 blocks to represent the division problem.

Example 3

First I took out blocks equal to 80.

Then I started sharing the ten strips among four groups.





Have students compare strategies used to solve each problem. Encourage them to look for similarities and differences in their approaches to the problem and to discuss the efficiency of each. Ask students to present their findings to the class.

FORMATIVE ASSESSMENT QUESTIONS

- What is your plan to solve this problem?
- How do you know your answer is correct?
- How does this help you answer the question in the problem?

DIFFERENTIATION

Extension

- Students can add their own division word problems within the context of sharing pumpkin seeds. These problems can be completed by a partner.

Intervention

- Give students manipulatives to assist in solving the problems.

TECHNOLOGY CONNECTION

http://mason.gmu.edu/~mmankus/whole/base10/asmdb10.htm#div A site for teachers and parents provides information on using base 10 blocks to solve division problems with an area model.

Name _____ Date _____

Sharing Pumpkin Seeds

Ben and his 3 friends toasted 80 pumpkin seeds from their pumpkin. How many seeds will each child get if they share the pumpkin seeds fairly? Clearly explain your thinking using words, numbers, and/or pictures.

Sarah and her 5 friends toasted 96 pumpkin seeds from their pumpkin. How many seeds will each child get if they share the pumpkin seeds fairly? Clearly explain your thinking using words, numbers, and/or pictures.

*3 ACT TASK: Egg Tower

APPROXIMATE TIME: 1 class period

STANDARDS FOR MATHEMATICAL CONTENT

MCC3.OA.1 Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 x 7.

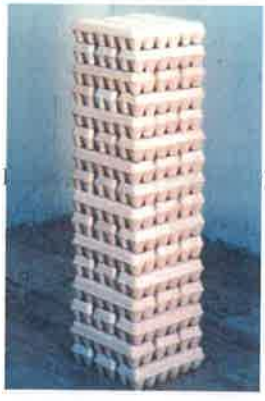
STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students must make sense of the problem by identifying what information they need to solve it.
2. **Reason abstractly and quantitatively.** Students were asked to make an estimate (high and low).
3. **Construct viable arguments and critique the reasoning of others.** After writing down their own questions, students discuss their question with partners, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others came up with.
4. **Model with mathematics.** Once given the information, the students use that information to develop a mathematical model to solve their question.
5. **Use appropriate tools strategically.** Students write their best estimate and two more estimates – one that is too low and one that is too high to establish a range in which the solution would occur.
6. **Attend to precision.** Students use clear and precise language when discussing their strategies and sharing their own reasoning with others.
7. **Look for and make sense of structure.** Students use their understanding of properties of operations as strategies to help develop an understanding of multiplicative structure.

ESSENTIAL QUESTIONS

In order to maintain a student-inquiry-based approach to this task, it may be beneficial to wait until Act 2 to share the EQ's with your students. By doing this, students will be allowed the opportunity to be very creative with their thinking in Act 1. By sharing the EQ's in Act 2, you will be able to narrow the focus of inquiry so that the outcome results in student learning directly related to the content standards aligned with this task.

- What strategies can help you solve real world multiplication problems?
- When can you use multiplication in real life?



MATERIALS

- Act 1 picture (attached)
- Student recording sheet

GROUPING

Individual/Partner and or Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will view the picture provided and tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on **need**. When they realize they don't have the information they need, and ask for it, it will be given to them.

Background Knowledge and Common Misconceptions:
This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org and the K-5 CCGPS Mathematics Wiki.

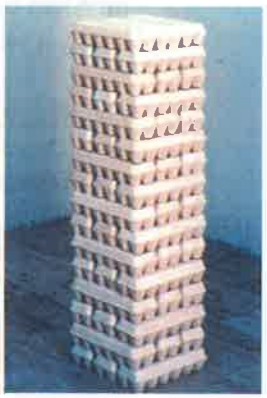
This standard interprets products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with
the same number of objects in each group. Multiplication requires students to think in terms of
groups of things rather than individual things. Students learn that the multiplication symbol 'x'
means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

A major conceptual hurdle in working with multiplicative structure is understanding groups of
items as single entities while also understanding that a group contains a given number of objects

(Clark & Kamii, 1996). Students can solve the problem "How many apples are in 4 baskets of 8 apples each?" by counting out four sets of eight counters and then counting them all. To think multiplicatively about this problem as four sets of eight requires students to conceptualize each group of eight as a single item to be counted. Experiences with making and counting equal groups, especially in contextual situations, are extremely useful. (From: Van de Walle, Teaching Student-Centered Mathematics, Vol. II 3-5, page 111)

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/) **"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."**



- Show Act 1 picture to students.
- Ask students what they noticed in the picture, what they wonder about, and what questions they have about what they saw in the picture. Do a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.
- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask and wish to answer: (*Main question(s) to be investigated)

- How many egg cartons are there altogether?
- *How many eggs are in the stack?
- How many eggs are in one layer of the tower?

- Once students have their question, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on an empty number line. Note: As the facilitator, you may choose to allow the students to answer their own posed questions, one question that a fellow student posed, or a related question listed above. For students to be completely engaged in the inquiry-based problem solving process, it is important for them to experience ownership of the questions posed.

Important note: Although students will only investigate the main question(s) for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 2 – Student Exploration - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/) **"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."**

- During Act 2, students decide on the facts, tools, and other information needed to answer the question(s)(from Act 1). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front.

- Required Information:
  12 eggs per carton
  3 cartons per layer
  16 layers of cartons

- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
  - What is the problem you are trying to solve?
  - What do you think affects the situation?
  - Can you explain what you've done so far?
  - What strategies are you using?
  - What assumptions are you making?
  - What tools or models may help you?
  - Why is that true?
  - Does that make sense?

Act 3 – Whole Group – Share solutions and strategies.

- Students to present their solutions and strategies and compare them.
- Reveal the solution.
  - 16 layers of cartons
  - 3 cartons in each layer
  - 12 eggs in each carton
- Lead discussion to compare these, asking questions such as:
  - How reasonable was your estimate?

- Which strategy was most efficient?
- Can you think of another method that might have worked?
- What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

For Act 4, share ideas below or reference other student-generated questions that could be used for additional classwork, projects or homework.

Examples (also see Extensions):

- Each carton costs $2.00. How much does the whole tower cost?

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

- Allow students to build their own egg carton towers (using drawings or manipulatives), and determine the total number of eggs in the tower.
- Provide students with a number of cartons (ex. 63 cartons), and require students to design a tower using that many cartons. Students should answer how many eggs are in their tower as well as how many layers (groups) of cartons they have and how many in each layer (group). Similarly, you could provide students with a number of eggs and require them to design a tower that holds that many eggs.

Intervention

- Use visual representations of the groups of eggs in each carton.

Act 1 Picture:

Task Title: _____ Name: _____

Adapted from Andrew Stadel

ACT 1

What did/do you notice?
What questions come to your mind?

Main Question: _____

What is your 1st estimate and why?
On an empty number line, record an estimate that is too low and an estimate that is too high.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimation with this information: _____

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

PRACTICE TASK: Array-nging Our Fact Families

In this task, students will make models on grid paper of arrays that show both multiplication and division number sentences. This task makes important connections between multiplication and division. Students will become familiar with division as the inverse operation of multiplication as they learn that the numbers in a multiplication sentence can also be used in a related division sentence.

APPROXIMATE TIME: 1 Day

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, pages 62-63)

"Students can benefit from a few activities with models and no context. The purpose of such activities is to focus the meaning of the operation and the associated symbolism.

Make sure to draw students' attention to the dimension of the rectangles (length and width). You want students to make the connection that the factors in the multiplication expression

they have written indicate the number of rows and columns (the dimensions) in a rectangle that consist of the given number of squares."

COMMON MISCONCEPTION

Students may have difficulty seeing multiplication and division as inverse operations. In order to develop an understanding of this relationship, students need to have ample opportunities to explore these two operations simultaneously.

ESSENTIAL QUESTIONS

- How are multiplication and division related?
- How can the same array represent both multiplication and division?

MATERIALS

- Grid paper
- Colored pencils or markers
- "Array-nging Our Fact Families" recording sheet

GROUPING

Individual/Partner Task

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION (SMP 1, 2, 3, 4, 5, 6, 7, and 8)

PART 1

Give students 12 blocks that represent the total area of an array. Have them arrange the blocks in an array and identify the dimensions of their array, noting different arrays are possible for 12. Then ask if there is a way they can make a division sentence with the dividend represented by the total area of the array. For example, a student may make a 4 x 3 array. The dividend (area of 12) can be divided by 4 or 3, both factors of 12. Both dimensions are utilized, one as the divisor and the other as the quotient.

PART 2

Task Directions

Students will follow the directions below from the "Array-nging Our Fact Families" recording sheet.

1. Draw the following arrays:
   6 by 3
   4 by 8
   2 by 7
2. Use the example to complete the following for each array:
   - Label the dimensions and total area.
   - Write a multiplication sentence and tell the factors and the product.
   - Write a division sentence and indicate the divisor, dividend, and quotient.
3. Select one of your arrays and write two story problems that can be modeled with the array, one for multiplication and one for division.

FORMATIVE ASSESSMENT QUESTIONS

- How can you describe your array?
- How does the array show both multiplication and division?
- What does the word "by" mean (i.e. 6 by 3)?
- What is the difference between a factor and a product? With what operation would you use these words?
- Explain the meaning of the divisor, dividend, and quotient in a division sentence?

DIFFERENTIATION

Extension

- Have students build an array of their choice and have a partner describe the dimensions and area of the array and all related vocabulary relating to both multiplication and division.
- Have students build arrays for multiplication and division that involve larger numbers. Limit the dimensions to a three-digit number times a one-digit number.

Intervention

- If students are not ready to transition to grid paper without the use of the base-ten blocks, allow the use of these manipulatives to guide student work.

TECHNOLOGY CONNECTION

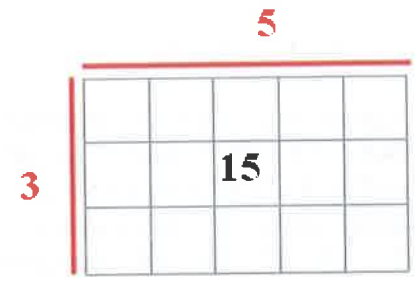
http://www.eduplace.com/math/mw/background/3/08/te_3_08_overview.html Provides background information on the relationship between multiplication and division.

Name _____ Date _____

Array-nging Our Fact Families

This 3 by 5 array has a total of 15 square units.
3 x 5 = 15.

3 and 5 are **factors**.
15 is the **product**.



Fifteen divided by three equals five.
15 ÷ 3 = 5
15 is the **dividend.**
3 is the **divisor.**
5 is the **quotient.**

1. Draw the following arrays listed in the table below.
2. Following the example above, complete the following for each array:
   - ☑ Label the dimensions and total area.
   - ☑ Write a multiplication sentence and label the factors and the product.
   - ☑ Write a division sentence and label the divisor, dividend, and quotient.

6 by 3	4 by 8	2 by 7

3. Select one of your arrays. On the back of this paper, write two story problems that can be modeled with the array, one for multiplication and one for division.

SCAFFOLDING TASK: Finding Factors

APPROXIMATE TIME: 3-4 Days

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.5. Apply properties of operations as strategies to multiply and divide.
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
Use arrays, area models, and manipulatives to develop understanding of properties.

MCC.3.OA.6. Understand division as an unknown-factor problem.
For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
Conversations should also include connections between division and subtraction.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

STANDARDS OF MATHEMATICAL PRACTICE (SMP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

BACKGROUND KNOWLEDGE/ COMMON MISCONCEPTIONS
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 64)

"Students look for patterns in the factors they find for numbers, such as the number of factors, the type of factors, the shape of the resulting array, and so on. Rather than always assigning numbers that have several factors, this activity suggests including numbers that have only a few factors so that differences between numbers become more distinct."

Make sure to draw student's attention to the dimensions of the rectangles (length and width). You want students to make the connections that the factors in the multiplication expression they have written indicate the number of rows and columns (the dimensions) in a rectangle that consist of the given number of squares. Your class will undoubtedly want to decide if a rectangle that is 3 by 8 should be counted differently from one that is 8 by 3. Leave the decision to the class but take advantage of the opportunity to discuss how 3 rows of 8 are the same as 8 rows of 3. Note that if sets rather than arrays are made, 3 sets of 8 look very different from 8 sets of 3.

ESSENTIAL QUESTIONS

- How can multiplication products be displayed on a multiplication chart?
- What strategies can be used to find factors or products?
- How does the order of the digits in a multiplication problem affect the product?

MATERIALS

Part I:

- Square tiles
- Finding Factors with Arrays Student recording Sheet

Part II:

- Counters
- Blank Multiplication chart
- Right angle guide (on colored card stock)
- Colored card stock cut out or Paper dessert plates or coffee filters or index cards
- Finding Factors Using Equal Sets Student Recording Sheet (will also use for Multiplication Chart Mastery Task)

GROUPING

Small Group/Partner

NUMBER TALK

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition Catherine Fosnot has developed "strings" of numbers that could be included in a

number talk to further develop mental math skills. (See Minilessons for Early Multiplication and Division by Willem Uittenbogaard and Catherine Twomey Fosnot (2007).

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION (SMP 1, 2 3, 4, 5, 6, 7, and 8)

Part I:
Begin the lesson by writing 6 x 4 = 24 on the board/overhead/smart board. Ask three volunteers to label the two factors and the one product. Discuss and review the terms from the task before to ensure retention. You may do a few together then ask the students to write their favorite multiplication sentence (expression) on their desk/wipe off board/ paper and label. Circulate and check for comprehension.

Next, write a large 24 on the board. Ask the class to think if there is one factor, or more than one factor for the number 24. Is there more than 1? 2? Ask for three predictions for how many factors there are for the number 24. After taking the predictions, use square tiles (square magnets, square image on the smart board) to have students model their thinking. Have three sets of 24 prepared and ask three volunteers (or have a set for each student/pair of students) and ask them to create an array using the 24 tiles. Show the student where and how to label the length and width. Share the results. Discuss commutative property. Decide, as a class, if 4 x 6 and 6 x 4 are the same. If they did not find all the factors, prompt to see if they can find more ways. There are four arrays for 24: 1 x 24 or 24 x 1, 2 x 12 or 12 x 2, 3 x 8 or 8 x 3 and 4 x 6 or 6 x 4. Students can draw and label the different arrays on the board. Identify the factors and the product. Students can then record the different arrays for the product 24 on their Finding Factors with Arrays Student Response Sheet.

Part II:
Divide the students into pairs or small groups of mixed abilities. Prepare stations (you may need multiple) with bags of 12, 18, 30 and 36 square tiles. Allow the students to create arrays for each product. They may draw and label the arrays on their Finding Factors Student Recording Sheet. Circulate and encourage with questions such as: Is there another way to create an array with your product? How many factors do you think your product will have? Is there a difference between these two arrays? Can you explain why these arrays are/aren't different?

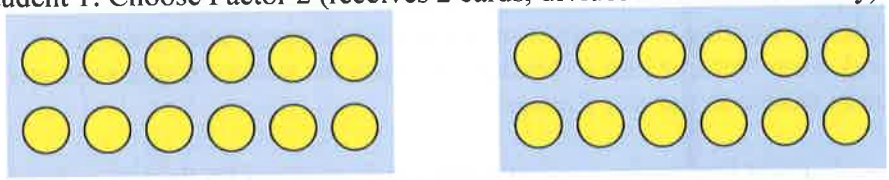
Conclude with sharing the arrays and discuss the student's findings. End the lesson with reviewing factors and products.

Part III:
Find Factors using Equal Sets Student Recording Sheet to build the multiplication chart.

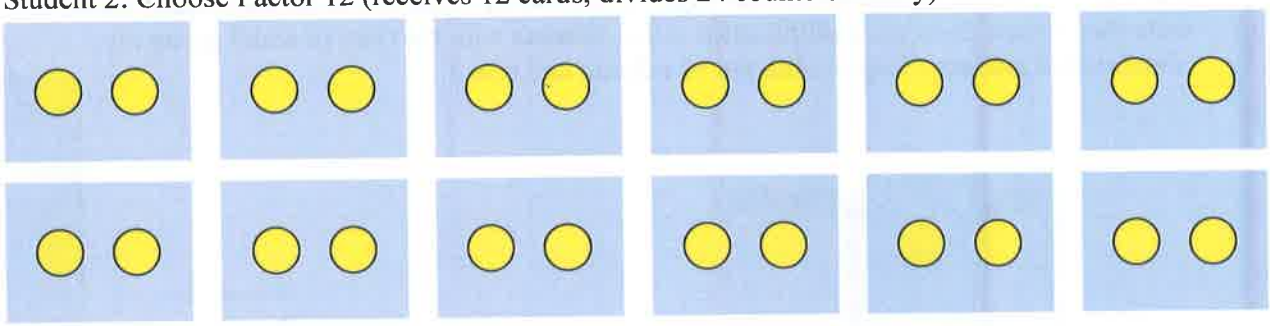
Students will continue to build on the concept of factors and products using counters and index cards/coffee filters/ paper dessert plates or the cut out from the right angle guide. The teacher will model with the number 24. Using two sets of 24 counters (one for each student or you can choose to use the smart board) the teacher will ask two volunteers to bring their Student

Response Sheet from yesterday. These students will decide on one multiplication fact whose product is 24. For example, the students choose 2 x 12 = 24 or 12 x 2 = 24. Each student selects a factor and receives that amount of cards (index, cut out from right angle guide, coffee filter, paper dessert plate) and 24 counters. Ask each student to evenly divide the counters onto each card. Ask each student to use math language to create a division sentence and multiplication sentence for their model.

Student 1: Choose Factor 2 (receives 2 cards, divides 24 counters evenly)



Student 2: Choose Factor 12 (receives 12 cards, divides 24 counters evenly)



As a class discuss who has 2 groups of 12 and who has 12 groups of 2. Provide time to discuss if this is the same or different and why. Discuss the commutative property and the relationship that multiplication has with division. Repeat with new volunteers and the remaining factors for 24. Create and record the multiplication expressions and corresponding division sentences for the factors of 24. Discuss what the division sentences have in common. (The product from the multiplication expression is first in the division sentence.)

Divide the students into pairs or small groups of mixed abilities. Prepare stations (you may need multiple) with bags of 12, 18, 30 and 36 counters and up to 36 index cards (coffee filters, cut outs, paper dessert plates). Allow the small groups or partners to go to each station to model the different factors using the counters and index cards (or what you have chosen to be the designated grouping). They may draw and label grouping on their Finding Factors Student Recording Sheet from the previous task (if the students are working the same groups as Part 1 or new sheets for today's grouping). Circulate and encourage with questions like:

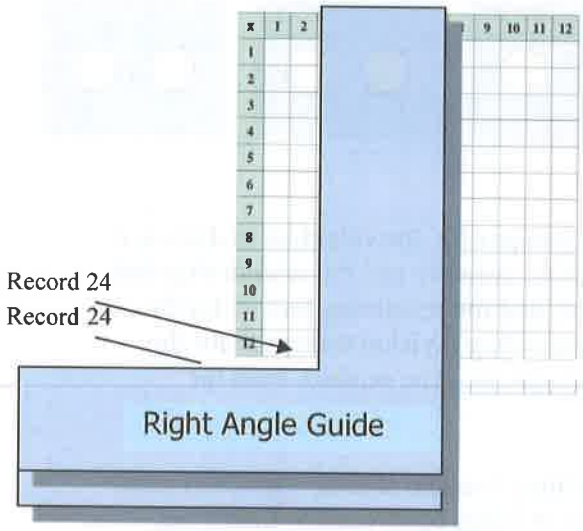
- *Is there another way to create a multiplication sentence with your product?*
- *How many factors do you think your product will have?*

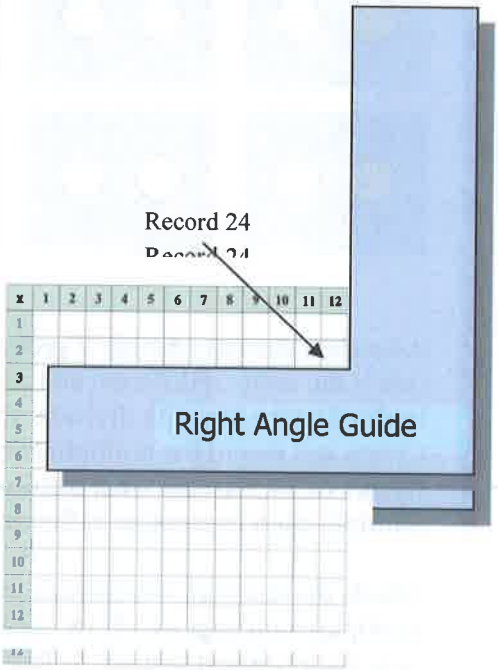
- *Is there a difference between these two facts?*
- *Can you explain why or why not these facts are different?*
- *What does the division sentence look like?*
- *What do you notice about how you arranged the division sentence?*

Part IV
Building the Multiplication Chart using the factors from the numbers 12, 18, 24, 30 and 36.

Today the students will begin recording the products on the multiplication chart. The teacher will demonstrate using a whole class setting or may choose to use a small group setting while students are finishing work from Part I or Part II. Students will use the student response sheets from both days or they may choose to use one strategy.

Starting with the 2 the teacher will line up the vertical Right Angle Guide with the two columns. Ask the class what was the other factor that is multiplied with 2 to get the product 24. Find the row with 12 and line up the horizontal Right Angle Guide. This will show the student where to write the product 24 in the multiplication chart. Discuss with the class or small group the commutative property. Repeat with the 12 column and row 2.





Continue to complete the factors of 24 as a class or with the small group. Circulate and check for comprehension. The students may work in pairs or individually to complete the facts for 12, 18, 30 and 36.

FORMATIVE ASSESSMENT QUESTIONS

- How can multiplication products be displayed on a multiplication chart?
- What strategies can be used to find factors or products?
- How does the order of the digits in a multiplication problem affect the product?
- How does a division sentence use the factors and product from a multiplication expression?

DIFFERENTIATION

Extension

- Provide the student with larger numbers to determine factors
- Provide the student with odd numbers, lead to discussion of remainders (although the understanding and interpreting remainders is not necessary in Third Grade, they naturally occur in most contexts and would be a great lead into MCC.4.OA.

Intervention

- Provide the student with one of the two factors
- Provide the student with two factors to determine the product using one strategy
- Focus on one strategy in both lessons
- Some students will need to fill every box within the multiplication chart.
- For a visual, see the Square of Pythagoras- http://moosehuntress.blogspot.com/2011/03/square-of-pythagoras-first-presentation.html

NAME _____ DATE _____

Multiplication Chart

x	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

NAME _____ DATE _____

Finding Factors with Arrays
Student Recording Sheet

Directions:
Record as many arrays you can for each number given.
Label each array with factor, factor and product.

Product 12

Product 18