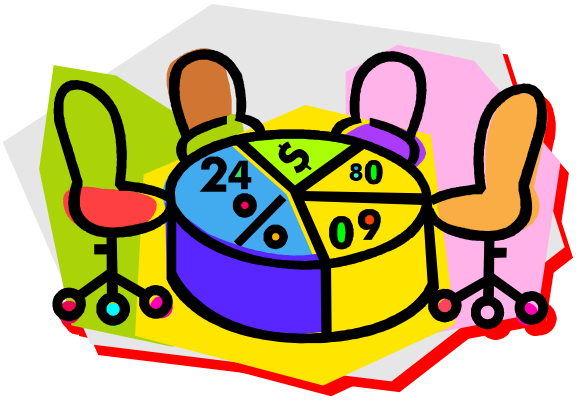
# Formative Assessment Probes and Interventions

## A Companion Tool for Math Trailblazers

### Grade 4

### 2010–2011



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## A Case for Formative Assessment

Delaware has adopted the definition of ***Response to Intervention*** as published by the National Association of State Directors of Special Education (2005):

> *"RTI is the practice of providing high-quality instruction and intervention matched to student need, monitoring progress frequently to make decisions about change in instruction or goals and applying child response data to important educational decisions. RTI should be applied to decisions in general, remedial and special education, creating a well-integrated system of instruction/intervention guided by child outcome data."*

Delaware's three-tiered RTI model is an efficient and effective model that emphasizes high-quality differentiated instruction based on meeting the needs of each student using data collected during formative assessment, diagnostic assessment, and summative assessment.

Successful implementation of ***Response to Intervention*** requires Delaware educators to:

1. Believe that you can effectively teach all students.
2. Use a research-based, scientifically validated core curriculum with fidelity.
3. Use assessment data to inform your instructional decisions.
4. Use a problem-solving method to make decisions within a multi-tier model of service delivery.
5. Use research-based, scientifically validated interventions matched to student need.

In response to these five principles, the Delaware mathematics community designed the ***Response to Intervention*** project. Formative assessment is the cornerstone of the Delaware ***RTI*** model based on the strongest possible research-based evidence connecting the use of diagnostic assessment and descriptive feedback to improved student achievement (Black, P., & Wiliam, D., 1998; and Crooks, 1988). We adopted the Chief Council of State School Officers' definition and five critical attributes of formative assessment listed below.

> Formative assessment is a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes.

Effective use of formative assessment requires attention to the following attributes:

- *Learning Progressions***:** Learning progressions contain clearly articulated sub-goals of the ultimate learning goal.
- *Learning Goals:* Learning goals and criteria for success are clearly identified and communicated to students.
- *Descriptive Feedback:* Students receive evidence-based feedback linked to the intended instructional outcomes and criteria for success.
- *Self- and Peer-Assessment:* Substantial student self- and peer-assessment are important for providing students an opportunity to think meta-cognitively about their learning.

- *Collaboration:* Practitioners establish and support a classroom culture in which teachers and students are partners in learning.

(Developed and approved by the CCSSO Formative Assessment Advisory Group and Formative Assessment for Teachers and Students (FAST) SCASS, October 2006)

These formative assessment principles have been used to build curriculum-based templates that guide teachers through the process of identifying learning gaps and implementing interventions. The work is remedial in nature and designed to be integrated with acceleration strategies. We invite you to join this stage of development.

## Formative Assessment: Delaware's Vision of Response to Intervention

As much as possible, as we wrote this document, we tried to imagine what this kind of implementation of Response to Intervention would look like in classrooms. We tried to imagine formative assessment in all of its messiness, as well as its potential, in order to create a document that would provide practical support to teachers as well as vision.

Delaware's RTI program is designed to occur within the classroom setting, using the curriculum materials that districts are already using. Teachers are supposed to analyze student work on pages that are part of the lessons themselves, and then use class time to intervene with students in need of Tier 1 intervention. While this may sound simple in conception, it is actually very complex and calls upon teachers to be experts in management, assessment, planning, and teaching.

### Classroom Management and Classroom Culture

Tier 1 of RTI may require teachers to carefully observe students as they work, ask "in the moment" questions, and carry out interventions with individual, pairs, or small groups of students during class time. For this to be accomplished they need to establish a classroom culture which will allow this. Students need to learn to:

- Work productively by themselves and with others.
- Get, share, and return the materials they need.
- Ask and receive help from other students when the teacher is unavailable.
- Find something else productive to do when they are finished or stuck.

For help establishing such a culture, we recommend these resources:

- "Building the MTB Classroom," Section 10 of the *Math Trailblazers Implementation Guide 3rd Edition.* This section focuses on establishing a flexible and challenging learning environment for all students. Strategies include addressing classroom culture, effectively grouping students, managing manipulatives, and pacing instruction.
- *The First Six Weeks of School* by Paula Denton and Roxann Kriete.
- Getting Metacognition Out of the Closet (http://investigations.terc.edu/library/implementing/qa-1ed/metacognition.cfm)

### Assessment

In order to gain as much information about students as possible, teachers need to assess their students in many ways. Teachers need to:

1. Observe students as they work and take notes in an organized way.
2. Carefully analyze student work on the formative assessment checkpoints after class.
3. Think about student understanding and misunderstanding not just in terms of "getting it" or "not getting it," but in specific language that describes what the student does or does not understand. In analyzing student work as part of this project, we found that most student mistakes or difficulties fell into one of these categories:

a. Lack of conceptual understanding;
b. Inefficient strategy;
c. Misusing or not using an important mathematical tool or representation;
d. Difficulty with verbal or written communication;
e. No connection to prior knowledge.

Recommended resources:

- "Assessment," Section 8 of the *Math Trailblazers Implementation Guide 3rd Edition*. This section details the philosophy, goals, and components as well as a description of how to do observational assessment.
- The "ongoing assessment" section of each lesson in the curriculum.
- "Math Facts and Whole Number Operations," Section 6 of the T*Math Trailblazers Implementation Guide 3rd Edition*.
- How Do I Organize My Observations? (http://investigations.terc.edu/library/implementing/qa-1ed/organize_observations.cfm)

**Planning**

In order to create lessons and interventions that help all students learn, teachers need to carefully plan their lessons. While management of materials and students is extremely important, **it is also extremely important that teachers plan the mathematical focus of their lessons**. For every activity, class discussion, or intervention, teachers need to figure out ahead of time what mathematical skills, concepts, or strategies they are trying to develop in their students. Much of this information can be gathered from

- Reading the "Background Information" section in each unit of the *Math Trailblazers Implementation Guide 3rd Edition*.
- Reading the lessons carefully.
- Doing the math ahead of time and anticipating student responses.
- Reading the "Lesson Overview" and the "At-A Glance" in each lesson of *Math Trailblazers Implementation Guide 3rd Edition*.

Class discussions are often not planned carefully ahead of time, and the students who are most affected by this lack of planning tend to be struggling students. Class discussions that are carefully focused, follow clear trains of thought, and draw clear connections among different strategies or ideas will extend the learning of all students in important ways.

Recommended resources for planning class discussions are:

- How Can I Help Special Needs Students Feel Included in Math Discussions? (http://investigations.terc.edu/library/implementing/qa-1ed/special_needs_class_disc.cfm)
- "Inclusive Math Communities" (http://investigations.terc.edu/library/implementing/qa-1ed/inclusive_communities.cfm)

- "Meeting Individual Needs," Section 11 of the *Math Trailblazers Teacher Implementation Guide 3rd Edition*. This section specifically focuses on the needs of the English Language Learner, special education students, and talented and gifted students.
- "Language," Section 12 of the *Math Trailblazers Teacher Implementation Guide 3rd Edition*. This section focuses on the use of language and gives strategies for working with reading comprehension, writing, and discourse in a math classroom.

Likewise, when teachers are intervening with students, they need to carefully think about the primary goal(s) for their intervention. Not all interventions are designed for the same purpose. Some are designed to help the student who is lagging behind in conceptual understanding of a topic. Others are written to improve computation, communication, representation, and connections.

- **Conceptual Leap** – This type of intervention is designed to prompt students to develop or clarify a particular concept or generalization. For example, a second-grade teacher might design an intervention to prompt a student to develop a generalization about breaking 2-digit numbers into 10s and 1s. Or, a teacher may need to diagnose where the child is in a specific learning trajectory and use an intervention based on the child's concept knowledge.
- **Computation Efficiency** – This type of intervention is designed to help students understand and use more efficient computation strategies. For example, a third-grade teacher might design an intervention to prompt students who are drawing pictures to solve multiplication problems, to begin using skip counting instead. A primary teacher might look at a student who is counting all and design an intervention for counting on.
- **Use of Tools or Models** – This type of intervention is designed to help students learn to use a particular tool or model to solve problems. Students in need of this intervention may not understand how a model relates to the mathematics. A fourth-grade teacher might design an intervention to help students understand and begin to use an array model to solve multiplication problems with 2-digit numbers. A primary teacher may design an intervention to help students make sense of a tens frame.
- **Communication** – This category includes both interventions designed to help students read and understand what a problem is asking and interventions designed to help students clearly communicate their thinking both verbally and in writing. It also includes carefully structuring a lesson so that vocabulary that is unknown (like campfire) does not get in the way of solving a problem. A fifth-grade teacher might design an intervention to help students read and make sense of multi-step word problems. Another fifth-grade teacher might design an intervention to help students record their work on multi-step word problems in a way that was clear and concise.
- **Prior Knowledge or Experiences** – This category of intervention is designed to prompt students to make connections among work they did earlier in the year or in a previous year and a current mathematical concept or task. For example, a fourth-grade teacher might design an intervention in which students divided a paper brownie into fractional parts and labeled the parts as they had in third grade. The purpose of this intervention would be to remind students of what they did know about fractions in order to prepare them for the fourth-grade fraction tasks.

## Teaching

In thinking about the teaching of interventions, we asked ourselves two basic questions:

- When would interventions happen?
- What would interventions look like?

Teachers who have a 15-minute class time for doing Daily Practice Problems (DPPs) have utilized this time to bring a small group together. Other teachers have started a lesson with the whole class and find time to pull a small group together while the rest of the class is working in small groups. Some schools are finding daily "intervention" time and sharing that time between English language arts and mathematics. While some students are receiving Tier 2 intervention outside of class, others can receive Tier 1 intervention or extension activities.

What the teaching of interventions should look like was a broader question for us. Most of the interventions we wrote are written as small-group activities, facilitated by the teacher, and can be implemented as such. However, we are aware that teachers may not always be available to lead small-group lessons, and sometimes the number of students needing an intervention might be more than a "small group." We also recognize that teachers may also not find it beneficial to gather the same students for small-group lessons too frequently. We do not want these students to become stigmatized in the eyes of their peers or themselves, and we do not want these students to become overly dependent upon adult help. We imagined that most interventions could be accomplished in one of four forms, with different forms being preferable under different circumstances. The four forms are:

- **Small-group lessons with the teacher** – This form is suggested most frequently in our intervention probes. In this form of intervention, the teacher meets target group of students for 10 to 25 minutes during the regular math class to provide interventions.
- **Strategic partnering of students** – In this form of intervention, a teacher carefully chooses particular students to collaborate as partners based on the belief that in working together one or both children will help the other child develop a particular skill or understanding. For example, a teacher might choose to have a student who had difficulty using a number line to solve subtraction problems work with a student who is able to use a number line to solve subtraction problems with the goal that the student who is having difficulty using the number line will learn from the other child. We recommend that teachers who use strategic partnering tell students in clear, but tactful language, what they hope the students will each contribute to and gain from the partnership. Strategic partnering could be especially helpful for students who lack vocabulary or background knowledge.
- **Whole-class interventions** – In this form of intervention, a teacher may modify a task for the whole class or plan a whole-class discussion in a particular way, based upon information learned from analyzing student work. For example, a teacher who realized that many students in her class have difficulty with the initial tasks in the fourth-grade fraction unit might revisit a third-grade fraction lesson with her whole class in order to activate the prior knowledge of all of the students in the class. Another teacher whose class was not clearly recording their work might take a lesson for addition strategies and focus on how each strategy could be recorded clearly.

- **Modification of tasks for particular students** – In this form of intervention, a teacher modifies a task or substitutes a different task for particular students based upon the learning goals for those students. For example, a third-grade teacher may have three students in her class who are just learning to add numbers by place. The teacher might give those three students the same word problems that she gives the other students in her class but changes the numbers in the word problems to make them 2-digit numbers instead of 3-digit numbers. That same teacher could also have the three students play a second-grade game during a Tier 1 math time as a way to improve their number sense and understanding of place value. This modification or substitution of a task is a good way to foster independence and perseverance among students who often ask for help. It can also be an effective homework strategy. By modifying a task or substituting a different task, a teacher can provide homework that can be done independently by the child and is focused on a targeted skill or concept.

We recommend the following resources for further ideas on implementing the types of interventions suggested above:

- "Meeting Individual Needs," Section 11 of the *Math Trailblazers Teacher Implementation Guide 3rd Edition*. This section specifically focuses on the needs of the English language learner, special education students, and talented and gifted students.
- "Strategies for Special Needs Students" (http://investigations.terc.edu/library/implementing/qa-1ed/special_needs_strategies.cfm)
- "How do I enrich math class to challenge gifted students?" (http://investigations.terc.edu/library/implementing/qa-led/enrich_for_gift_students.cfm )
- "Teaching ALL Math Trailblazers Students" a differentiation opportunity in each unit.

## Progress Monitoring

**Selected Daily Practice Problems (DPPs) from the *Math Trailblazers* curriculum will serve as Progress Monitors for your Tier 1 students**. Good teaching in the regular classroom along with a system to track students periodically will help all students meet the standards in mathematics. Look over the packet of DPPs for your grade level along with the spreadsheet that matches your grade level DPPs with the big ideas tested in the Universal Screening Tool.

**Formative assessment is used to monitor student achievement.**

- It is <u>not graded</u> but instead used to provide information about how each student in your classroom is doing based on the standards. Imagine giving each of these DPPs and walking around the room as students are working on them. The best part is that all students are doing the DPPs as a matter of fact each day, so asking students to do DPPs as you watch them work is not a new idea and children will not feel as though they are being graded.
- The teacher has a rare opportunity to watch students do math and …
  - *ask them questions to clarify the problem,*
  - *stop them to redirect their thinking,* and
  - *probe the students who need additional help.*
- Then, by looking at the student work, the teacher can …
  - *make a plan to have some students work in small groups based on your observations,*
  - *make a plan for whole-group instruction,*
  - *look to future lessons and use this information to teach what the students need.*
- The teacher directions are in a separate packet from the student DPPs but include a picture of the DPP formative assessment as well as three components.
  - *Suggestions for Implementation*: This section has been included and derived from the work that classroom teachers in the pilot study compiled. Since there are no teacher directions for administering DPPs in the curriculum, we asked teachers what worked best and included their findings here.
  - *Interventions*: Again, the teachers in the pilot tested some interventions and found some to be very workable. The teachers decided whether the intervention was to be large group, small group, individual, or sometimes whole class.
  - *Looking Ahead to Assess Again*: Since the DPPs do spiral, the authors looked ahead in the packet and made it very clear when this same topic was assessed in the Probes. There are many other opportunities to assess these skills during the curriculum as well, but this document takes you to the next DPP probe.
- The Universal Screening Tool includes a spreadsheet that groups questions into categories based on Numeric Reasoning. We have also grouped these DPPs into the same categories and have included a spreadsheet that shows how the student progresses. Instead of looking at one DPP at a time, the teachers decided to look at many chances to keep track of student progress by topic.

- In each grade level, there are certain manipulatives that are standard for your grade level. In almost every case, students should be given access to manipulatives when solving these problems. A rule of thumb for using manipulatives: Make available to each student what is usually given during the learning of the skill.

Please direct your questions to


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The work on this document began in August 2008 when elementary teachers from around the state gathered with a common purpose. They were charged with interpreting the Response to Intervention (RtI) regulation to determine the path forward for implementation in mathematics. This group of teachers identified the big ideas in the elementary grades, analyzed their curriculum materials for specific formative assessment possibilities, and began the intervention writing process. Their contributions continued throughout the school year during several professional development sessions in which they provided student work to analyze and learned intervention techniques such as enhancing their questioning strategies and error analysis. The Delaware Department of Education would like to thank these dedicated professionals for the many hours of hard work that went into the initial planning and development of this document.

**A Special Thank You to the Teachers and Specialists of –**

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Lake Forest School District
Laurel School District
Milford School District
Red Clay Consolidated School District
Seaford School District
Smyrna School District

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**Our Gratitude Extends to the Teachers and Specialists of –**

Central Elementary School, Seaford
Welch Elementary School, Caesar Rodney
West Park Place Elementary School, Christina

The leadership team devoted a considerable amount of time and effort to the development of this document. Their invaluable contributions include countless hours of thoughtful planning, providing outstanding professional development, and providing support to teachers and schools with piloting this RtI project. The dedication and commitment to this vast and significant work demonstrates their passion and support for mathematics education!

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Sincerely,

Crystal Lancour & Diana Roscoe, Delaware Department of Education

# 4th Grade DPP Probes

All Daily Practice Problems (DPPs) are reprinted with permission from the *Math Trailblazers 3rd Edition* (copyright 2008 by Kendall/Hunt Publishing Company), specifically the *Unit Resource Guides (URGs).*

| **DPP I – Extra Subtraction Practice** | **Unit 2** |
|---|---|

Use mental math to solve the following problems.

A. 120 – 90 =  B. 120 – 70 =  C. 160 – 80 =

D. 150 – 70 =  E. 170 – 80 =  F. 180 – 90 =

Explain your strategy for either Question D or E.

| |
|---|
| |

.....

| **DPP K – More Subtraction Practice** | **Unit 2** |
|---|---|

Use mental math to solve the following problems.

A. 101 – 97 =  B. 412 – 15 =  C. 565 – 556 =

D. 306 – 299 =  E. 153 – 49 =  F. 215 – 110 =

G. Explain your strategy for solving Question B.

| |
|---|
| |

| **DPP W – Timing Reading** | **Unit 2** |
|---|---|

It takes Jackie 20 minutes to read a chapter in her book. How many chapters can she read in 2 hours, if all the chapters are about the same length? Show your work.

| **DPP F – Story Solving** | **Unit 3** |
|---|---|

1. Write a story to show 5 x 7. Draw a picture to go with your story. Write a number sentence on your picture.

2. Write a story and a number sentence to show 35 ÷ 7.

3. What are the other two facts in this fact family?

| **DPP J – How Can We Compare Two Numbers?** | **Unit 3** |
|---|---|

Use mental math to solve the following problems.

A. 4003 - 3997= _____ B. 4007 - 3995 = _____

C. 4001 - 3800 = _____ D. 4000 - 500 = _____

E. 4000 - 501 = _____ F. 4000 - 499 = _____

Explain your strategies for Questions A or E.

| **DPP C – Using Twos** | **Unit 4** |
|---|---|

A. 2 x 9 = B. 3 x 200 =

C. 2 x 1000 = D. 8 x 2 =

E. 5 x 20 = F. 20 x 2 =

G. 40 x 2 = H. 6 x 2 =

I. 2 x 7 = J. 0 x 2 =

| **DPP E – Finding Prime Factors—Using Factor Trees** | **Unit 5** |
|---|---|

Write 90 as the product of prime numbers.

| |
|---|
| |

.....

| **DPP T – Missing Factors** | **Unit 5** |
|---|---|

The letters m and n stand for missing numbers. Find the missing numbers in each of the following number puzzles.

A. $2 \times m = 4$

B. $m \times 8 = 24$

C. $6 \times m = 36$

D. $10 \times m = 100$

E. $64 \div m = 8$

F. $81 \div 9 = m$

G. $4 \times n = 16$

H. $m \times n = 11$

I. $m^2 = 25$

Write 3 more missing number puzzles. Then, solve them.

| **DPP D – The Long and the Short** | **Unit 6** |
|---|---|

Write the following numbers in words:

A. 421

B. 8,536

C. 58,972

D. 20,380

Write the following words as numbers:

E. six thousand nineteen

F. two thousand, three hundred forty-one

G. two hundred one thousand, five hundred two

.....

| **DPP E – Median and Mean** | **Unit 6** |
|---|---|

Ming experimented with 3 kinds of balls to find out which one bounced highest. He dropped each type of ball five times from 1 meter. The bounce heights for the tennis ball were:

52 cm, 47 cm, 55 cm, 52 cm, and 50 cm.

1. Find the median bounce height. Show your work.

2. Use a calculator to find the mean bounce height to the nearest cm. Show your work.

| **DPP I – Ana's Purchase** | **Unit 6** |
|---|---|

1. Ana bought a gallon of milk for $2.49, a box of crackers for $1.56, and a magazine for $2.95. Will $10.00 be enough to pay the bill? Show your work.

2. About how much change will Ana get back? Show your work.

.....

| **DPP P – Which Is the Closest?** | **Unit 6** |
|---|---|

1. Which number is closest to 4056? Show your work.

   450 4000 4100 5000

2. Which number is closest to 62,096? Show your work.

   7000 60,000 65,000 70,000

3. Which is a reasonable estimate for the height of a 5-story apartment building?

   30 feet 50 feet 300 feet 500 feet

| **DPP I – Divisible by 2, 3, or 6?** | **Unit 7** |
|---|---|

A. Circle the numbers that are divisible by 2. How do you know?

762 1025 8031 7532

B. Circle the numbers that are multiples of 3. How do you know?

762 1025 8031 7532

C. Which numbers have 6 as a factor? How do you know?

762 1025 8031 7532

.....

| **DPP O – Multiplying by 10** | **Unit 7** |
|---|---|

A. 7 x 80 = B. 6 x 400 = C. 8000 x 6 =

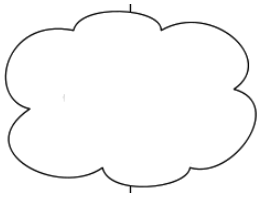
D. 700 x 4 = E. n x 60 = 420 F. 800 x n = 3200

G. 10 x 700 = H. 0 x 600 =

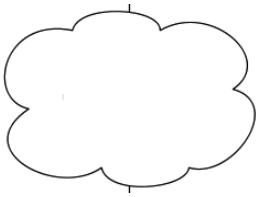
| **DPP B – More Multiplication** | **Unit 8** |
|---|---|

Solve the following problems. First, estimate and put your estimate in the cloud next to the problem. Then, solve each problem using pencil and paper or mental math to find the answers.

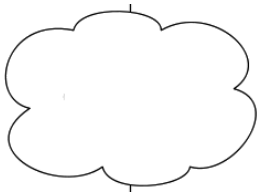
A. 26 x 8 =



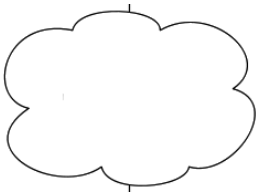
B. 47 x 6 =



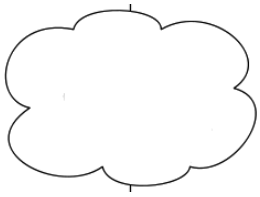
C. 87 x 7 =



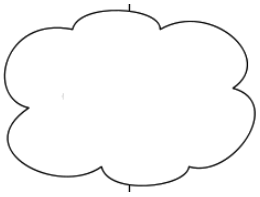
D. 93 x 5=



E. 63 x 6 =



F. 45 x 8 =

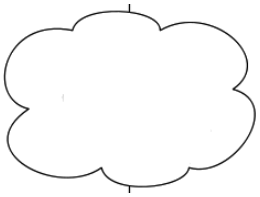


G. Explain how you solved Question 1A.

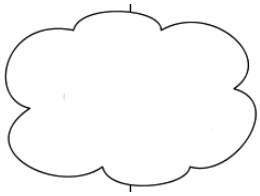
| **DPP N – Arithmetic Review** | **Unit 8** |
|---|---|

Solve the following problems. First, estimate and put your estimate in the cloud next to the problem. Then, solve each problem using pencil and paper or mental math to find the answers.

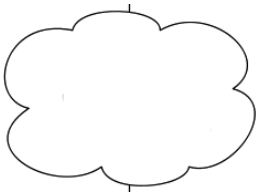
A. 3005 + 61 + 458 =



B. 17 + 608 + 3 + 1060=



C. 917 - 145 =



D. Explain how you made your estimate for Question B.

| **DPP Q – Sharing Money** | **Unit 8** |
|---|---|

Five children found $4.00 in the hall at school. The principal said they could share it, if no one claimed it. How much would each child get? Show your work.

| **DPP E – Division Stories** | **Unit 9** |
|---|---|

The following two problems can be solved using division.

1. Mrs. Randall gave each of her children $4 to spend on games at the neighborhood carnival. If Mrs. Randall gave out $20 in all, how many children does she have? Show your work.

2. One package of bus tokens contains 10 tokens. Keenya's mother needs 40 tokens to get to and from work for one month. How many packages does Keenya's mother need? Show your work.

| **DPP K – More Fact Practice** | **Unit 9** |
|---|---|

Find n to make each number sentence true.

A. 8 x 5 = n

B. n x 7 = 70

C. n ÷ 4 = 5

D. 80 ÷ n = 10

E. 10 x n = 50

F. 30 ÷ 5 = n

G. 9 x 10 = n

H. 15 ÷ n = 5

I. n x 8 = 80

J. 10 ÷ 10 = n

Explain how you solved either Question C or J.

| **DPP M – Multiplying with Zeros** | **Unit 10** |
|---|---|

A. 80 x 20 = B. 40 x 3 = C. 3000 x 40 = D. 20 x 500 =

E. 50 x 30 = F. 600 x 2 = G. 0 x 20 = H. 10 x 30 =

.....

| **DPP U – Smallest, Largest, and In Between** | **Unit 10** |
|---|---|

For the following base-ten shorthand, the flat is one whole. Write the number beside each base-ten shorthand.

1.  _____

2.  _____

3. 

4.  _____

A. Which is the smallest?

B. Which is the largest?

C. Which are equal?

D. Put the numbers you wrote in order from smallest to largest.

| **DPP X – Dividing It Up** | **Unit 10** |
|---|---|

26 ÷ 8 = Write a story for 26 ÷ 8. Then, draw a picture. Include any remainder in your picture. Solve the problem.

| |
|---|
| |

……

| **DPP D – Decimals** | **Unit 11** |
|---|---|

Write each of the following decimals using base-ten shorthand. The flat is one whole.

A. 2.02 B. 0.6 C. 2.6

D. 0.06 E. 2.16

F. Write the decimals in order from smallest to largest.

| |
|---|
| |

| **DPP E – Missing Number** | **Unit 11** |
|---|---|

A. n x 300 = 900 B. 7 x n = 490 C. 80 x n = 640

D. 6 x n = 36,000 E. n x 40 = 160 F. 900 x n = 8100

.....

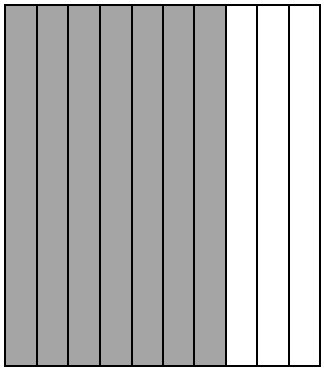
| **DPP S – Measuring Volume** | **Unit 11** |
|---|---|

Romesh filled and empties a 250-ml cylinder into a large container 6 times. After the sixth time, the container was full. What is the volume of the container in milliliters? Show your work.

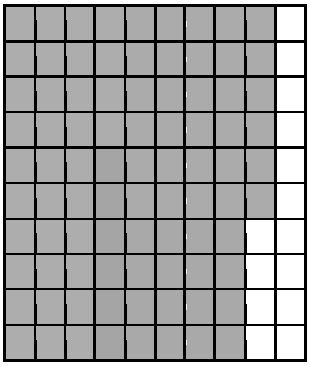
| **DPP I – Decimals and Pictures** | **Unit 12** |
|---|---|

Write a decimal for the shaded part of each picture. The flat is one whole.

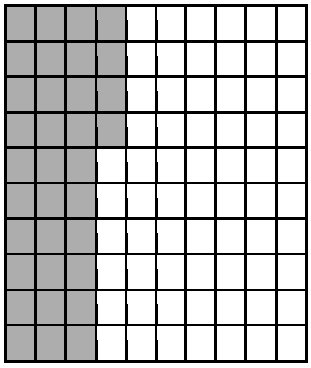
1. _____



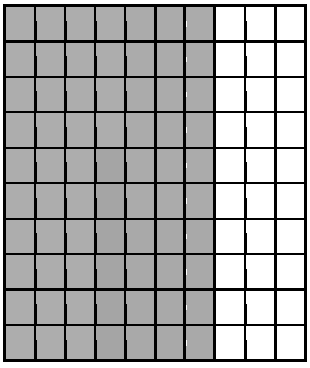
3. _____



2. _____



4. _____



| **DPP P – Division Stories** | **Unit 12** |
|---|---|

Write a division story for 28 ÷ 9. Then, draw a picture. Include any remainder in your picture. Solve the problem.

| **DPP S – Words to Numbers** | **Unit 12** |
|---|---|

1. Write the following words as numbers.

A. two-thirds B. six-tenths C. five-eighths D. one-twelfth

2. Write the following numbers as words.

A. $\frac{3}{4}$ B. $\frac{7}{9}$ C. $\frac{1}{2}$ D. $\frac{2}{5}$

| **DPP E – Trumpet Practice** | **Unit 13** |
|---|---|

Ana's trumpet teacher told her to practice $\frac{3}{4}$ hour every day. Since 1 hour = 60 minutes, Ana used this number sentence to find the number of minutes she must practice each day:

$$\frac{3}{4} = \frac{?}{60}$$

1. Compete Ana's number sentence to find the number of minutes in $\frac{3}{4}$ hour.

2. Use Ana's method or your own strategies to find the following:

   A. How many minutes are in $\frac{1}{4}$ hour?

   B. How many minutes are in $\frac{1}{2}$ hour?

   C. How many minutes are in $\frac{1}{3}$ hour?

   D. How many minutes are in $\frac{1}{6}$ hour?

   E. How many minutes are in $1\frac{1}{4}$ hours?

   F. How many minutes are in 3.5 hours?

| **DPP M – TV Survey** | **Unit 13** |
|---|---|

Use paper and pencil to solve the following problems.

1. According to a recent study, the average American eighth grader spends about 4 hours a day watching TV. How many hours in a year will an eighth grader spend watching TV? Show your work.

2. A school year is approximately 180 days long. How many hours is a student in school if a school day is 6 hours long? Show your work.

| **DPP Y – Order Fractions** | **Unit 13** |
|---|---|

1. Which is greater $\frac{1}{12}$ or $\frac{1}{10}$? How did you decide?

2. Which is greater $\frac{3}{5}$ or $\frac{3}{8}$ ? How did you decide?

3. Which is greater $1\frac{1}{2}$ or $\frac{5}{4}$? How did you decide?

| **DPP F – Lots of Money** | **Unit 14** |
|---|---|

Pretend you are at the store without your calculator or paper and pencil. See how many of the answers you can figure out in your head. Tell what strategies you used.

**How much change will you receive from a $10 bill if you spend the following amounts?**

A. $5.60

B. $7.45

C. $1.97

D. $4.25

| **DPP J – Adding and Subtracting** | **Unit 14** |
|---|---|

Use mental math to solve the following problems.

A. 1700 + 400 =

B. 4300 - 400 =

C. 1450 + 350 =

D. 1658 - 500 =

E. 5033 + 9100=

F. 2099 + 301 =

G. Explain your mental math strategy for Questions C and D.

| **DPP L – Related Multiplication and Division Sentences** | **Unit 14** |
|---|---|

Find a number for n in each number sentence that makes the statement true.

A. 4 x n = 2400

B. 2400 ÷ 4 = n

C. n x 9 = 360

D. 360 ÷ 9 = n

E. 7 x n = 63,000

F. 63,000 ÷ 7 = n

G. 6 x n = 54

H. 54 ÷ 6 = n

I. 9 x n = 4500

J. 4500 ÷ 9 = n

K. 8 x n = 560

L. 560 ÷ 8 = n

M. 3 x n = 0

N. 0 ÷ 3 = n

| **DPP G – Fractions** | **Unit 16** |
|---|---|

Which is larger? Explain how you decided on yours answers.

A. $\frac{1}{12}$ or $\frac{1}{10}$

B. $\frac{3}{2}$ or $1\frac{1}{4}$?

C. $\frac{6}{12}$ or 0.5?

D. $\frac{5}{8}$ or 0.4?

# 4th Grade Math Trailblazers
# Directions and Interventions

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| DPP I – Extra Subtraction Practice | Unit 2 |
|---|---|

Use mental math to solve the following problems.

A. 120 – 90 = B. 120 – 70 = C. 160 – 80 =

D. 150 – 70 = E. 170 – 80 = F. 180 – 90 =

Explain your strategy for either Question D or E.

## Implementation Suggestions

- This assessment will inform teachers to what extent students are thinking in base ten. Ask students to think about the distance between 90 and 120 to be able to mentally solve the problems. Allow students to use a model: 12 tens – 9 tens is 3 tens. Do not shortcut their thinking by covering 0s. This "trick" only allows students to NOT think in tens.
- Students who easily think in tens will invent the rule, "If I know 12 - 9 then I can do 12 tens - 9 tens or 12 hundreds - 9 hundreds."
- Some students need practice in mental math. Start with small numbers like 12 - 9 to make sure that they are using mental strategies instead of paper and pencil.

## Interventions

- Return to building models. Students in 4$^{th}$ grade should know that 12 tens is the same as 120. Perhaps start with base ten model for 120 and trade. Name the number 3 ways:
  - 1 hundred 2 tens
  - 12 tens
  - 120 ones
- Then students will start building the relationship between tens and hundreds.
- Play digits game: URG 2, Lesson 4.

## Follow-Up DPP

- The next DPP that assesses this skill is Unit 2 – DPP K.

| DPP K – More Subtraction Practice | Unit 2 |
|---|---|

Use mental math to solve the following problems.

A. 101 – 97 =  B. 412 – 15 =  C. 565 – 556 =

D. 306 – 299 =  E. 153 – 49 =  F. 215 – 110 =

G. Explain your strategy for solving Question B.

## Implementation Suggestions

- This assessment also asks students to think about the distance between numbers on a number line. Students who can only use the algorithm to solve these problems need some intervention.
- Provide hundreds charts or number lines.

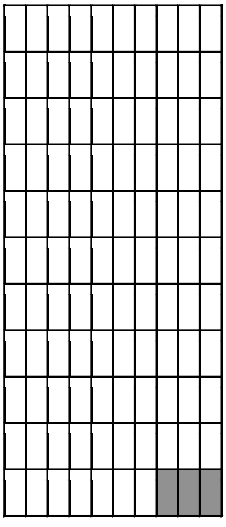
  0_____100_____200_____300_____400_____500_____600

  Example:

  A, "The distance between 97 and 101 is 4. It takes 3 hops to get to 100, and I need one more."

  B, "The distance is 397 because if I hop back 15, I will go three hops in front of 400." OR, "To subtract 15 first I take away 12 and I'm at 400. Then I must take away three more."

## Interventions



- Another way to start this process is for students to estimate the difference.
- For B, the difference is close to 400 because the estimate is 400 - 0 =. Since the answer is close to 400 could we count back to subtract? Model the problem with base ten. 4 flats 12 bits - 15 bits. Taking away the 12 bits is easy and then count back on the flat touching the last 3 bits. Imagine breaking off these 3 bits. What would be left?
- The teacher shows the mental calculation strategy with the model.
- The teacher could also present the number line model as shown above.
- This intervention is done with students who only can use the algorithm model. The hope is for all students to begin inventing mental strategies.

## Follow-Up DPP

- Unit 3 – DPP J will check on understanding of mental math.

| DPP W – Timing Reading | Unit 2 |
|---|---|

It takes Jackie 20 minutes to read a chapter in her book. How many chapters can she read in 2 hours, if all the chapters are about the same length? Show your work.

## Implementation Suggestions

- This assessment gives students an opportunity to solve a problem about time. Read the DPP together and let students talk in table groups about what fact they need to know to solve this problem.
- Answer the question, "How many minutes are there in an hour?", and then give students "time" to solve the problem.
- Allow students to use model clocks. Make note of which students do not have fluency in telling time or using a clock.
- There are many strategies that work:
  - Counting by 20s to get to 120 or counting by 20s to get to 60 and then doubling.
  - Dividing 120 by 20 or dividing 60 by 20 and then doubling.
  - Adding 20s to get to 60 and then doubling or adding 20s to get to 120.
  - Drawing a clock with 20-minute segments or drawing two clocks.

## Interventions

- Give other scenarios about things that take 15 minutes or 30 minutes or 10 minutes. How many could happen in an hour?
- This discussion could happen as students have pictures of clocks and students subdivide the clock face into groups of 15 minutes. The student needs to be directed to "see" the 15-minute segments as ¼ of an hour. There are four, 15-minute segments in an hour.
- Students might also benefit from using a table of values to solve problems.

| Number of chapters | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Time in minutes | 20 | 40 | 60 | 80 | 100 | 120 | 140 |

- This acts as a way to move from counting by 20s to multiplying by 20s or multiplying by 15 instead of counting by 15s.

| Number of chapters | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Time in minutes | 15 | 30 | 45 | 60 | 75 | 90 | 105 |

## Follow-Up DPP

- Unit 13 – DPP E, is the next DPP that asks students to solve problems with fractional parts of clocks.

| DPP F – Story Solving | Unit 3 |
|---|---|

1. Write a story to show 5 x 7. Draw a picture to go with your story. Write a number sentence on your picture.
2. Write a story and a number sentence to show 35 ÷ 7.
3. What are the other two facts in this fact family?

## Implementation Suggestions

- This assessment is one that is repeated from 3rd grade. Drawing a picture of 5 x 7 should look like 5 groups of 7. Since students are very familiar that 5 x 7 is the same answer as 7 x 5, they may solve this fact with either number sentence. Make sure the picture and the number sentence show "groups of." It is important that the picture and story problem match. Review the concept that a story problem asks a question and does not give the answer.
- Division story problems are new to 4th grade. This picture and story should show sharing 35 things in 7 groups OR 35 things shared in groups of 7. Both are acceptable.
- "There are 35 students who will make 7 teams. How many students will be on each team?" OR "There are 35 students who will be placed in teams of 7. How many teams will there be?" These examples are defined as partitive and measurement division. One form of division is fair share, dealing out one at a time to 7 groups, and one is modeled as taking 7 at a time.
- Redirect any student who starts drawing 35 groups of 7 for question 2.
- Redirect any student who draws good pictures and does not write the story. You may provide lined paper as a way to make writing easier for some students.

## Interventions

- Students might need to practice with triangle flashcards to write all four equations for a fact family.
- Provide square-inch tiles or cm sq. paper to model the array 5 x 7. Use the array to show:
  - 5 rows of 7
  - 7 columns of 5
  - 35 is made of 5 rows
  - 35 is made of 7 columns

## Follow-Up DPP

- Unit 10 – DPP X, checks on writing a division story problem.

| DPP J – How Can We Compare Two Numbers? | Unit 3 |
|---|---|

Use mental math to solve the following problems.

A. 4003 - 3997= _____ B. 4007 - 3995 = _____ C. 4001 - 3800 = _____

D. 4000 - 500 = _____ E. 4000 - 501 = _____ F. 4000 - 499 = _____

Explain your strategies for Questions A or E.

## Implementation Suggestions

- This assessment also asks students to think about the distance between numbers on a number line. Students who can only use the algorithm to solve these problems need some intervention.
- Provide hundreds charts or number lines.
- 3000_____3500_____4000_____4500

## Interventions

- Check the notes you took about students completing Unit 2 DPP K. Are those students more easily using a number line? Have they tried something other than an algorithm?

## Follow-Up DPP

- The next DPP that assesses this skill is Unit 6 – DPP P.

| DPP C – Using Twos | Unit 4 |
|---|---|

A. 2 X 9 =
B. 3 X 200 =
C. 2 X 1000 =
D. 8 X 2 =
E. 5 X 20 =
F. 20 X 2 =
G. 40 X 2 =
H. 6 X 2 =
I. 2 X 7 =
J. 0 X 2 =

## Implementation Suggestions

- Students may show understanding of doubling to complete this DPP. Observe to note what strategies students use to solve these problems and with how much ease they complete this assignment.
- Strategies may include:
  - Rethinking all of these questions as n x 2. Even though B says 3 x 200 the strategy remains. This student thinks 3 x 200 is the same as 3 x 2 x 100 and does the 300 x 2 as his fact.
  - Knowing that 2 x 40 is the same as 2 x 4 x 10 or 2 x 4 tens.
  - Repeated addition or skip counting. B is thought as 200 + 200 + 200.
  - Writes the problem and solves with multiplication algorithm.
- Not all of these are efficient. Students who are showing their calculations (either + or x) to solve these problems need intervention.

## Interventions

- Move all students to some mental math strategy that involves skip counting by 10s or groups of 10s or doubling.

## Follow-Up DPP

- The next opportunity to assess this is Unit 7 – DPP I.

| DPP E – Finding Prime Factors—Using Factor Trees | Unit 5 |
|---|---|


## Implementation Suggestions

- This is an opportunity to give students independent practice in solving a factor tree. Teachers may tell students who cannot get started to use a factor tree. On the math word wall should be an example of a factor tree as well as a definition of prime numbers.
- Some teachers will ask students to first look at the word wall and try to decide what this question is asking. Allowing students to self-correct is giving them an opportunity to solve problems. Make a note of the students who had difficulty remembering how to get started.
- Also observe students who do get started but then stop before the number is completely factored in primes (9 x 5 x 2). Redirect these students to make sure that all of their factors are prime.
- Observe to see if some students use 1 as a prime factor. Redirect these students to make sure that all of their factors are prime.

## Interventions

- More practice making factor trees correctly.
- One of the benefits from knowing prime factors will occur much later. Students who know the prime factors of a number also can find all of the factors of a number.
- (2 x 3 x 3 x 5) are prime factors of 90. I can combine these in many ways to get the factors of 90.
- 2 x (3 x 3 x 5) 2 x 45
- (2 x 3) x (3 x 5) 6 x 15
- (2 x 3 x 3) x 5 18 x 5
- 3 x (2 x 3 x 5) 3 x 30
- (3 x 3) x (2 x 5) 9 x 10
- Perhaps by solving for prime factors and then multiplying those primes back into factors, the students will see the benefit and the connection between finding factors and finding prime factors.
- I can also find common factors and the greatest common factor of any two numbers by starting with the prime factors of both numbers.
- Prime factors of 18 are 2 x 3 x 3
- Prime factors of 16 are 2 x 2 x 2 x 2
- The only common factor is 2.

- Prime factors of 18 are 2 x 3 x 3
- Prime factors of 12 are 2 x 2 x 3
- The common factors are 2, 3, and 6. The GCF is 6.
- (This is another way to show that if a number is divisible by 2 and 3, it is divisible by 6.)

---

## Follow-Up DPP

| DPP T – Missing Factors | Unit 5 |
|---|---|

The letters m and n stand for missing numbers. Find the missing numbers in each of the following number puzzles.

| | | |
|---|---|---|
| A. 2 X m = 4 | B. m X 8 = 24 | C. 6 X m = 36 |
| D. 10 X m = 100 | E. 64 ÷ m = 8 | F. 81 ÷ 9 = m |
| G. 4 X n = 16 | H. m X n = 11 | I. $m^2 = 25$ |

Write 3 more missing number puzzles. Then, solve them.

## Implementation Suggestions

- This assessment allows the teacher to discover which students are using multiplication to solve for division. The students are asked to use inverse operations to solve for unknowns. Make sure students know that in each case the number could be different. The letter m does not always equal the same number.
- Notice that questions H and I are checking for understanding of prime and square numbers. A student may solve all questions except questions H and I and show good understanding of solving for missing numbers and using inverse operations.

## Interventions

- The final question, allowing students to write some missing number puzzles, might be above the standard for some fourth graders. Do not hold a student to this degree of knowledge.
- However, students who cannot solve for an unknown may need to practice fact families.
- Return to sample E and ask the student to write fact families that have the numbers 8 and 64.

## Follow-Up DPP

- Unit 9 – DPP K for the next example of finding missing factor.

| DPP D – The Long and the Short | Unit 6 |
|---|---|

Write the following numbers in words:

A. 421
B. 8536
C. 58,972
D. 20,380

Write the following words as numbers:

E. six thousand nineteen
F. two thousand, three hundred forty-one
G. two hundred one thousand, five hundred two

## Implementation Suggestions

- This assessment tests whether a student can read and write large numbers. Being able to read and/or write a number is a way to understand the value of each digit in a number. Students who can say four hundred twenty one for 421 can most likely tell that the 2 stands for 20.
- The teacher may ask individual students to read numbers aloud as he/she circulates around the room. Correct students who say four hundred AND twenty one but do not consider it needing intervention. Look at the written numbers and allow for misspellings. Consider having the words hundred and thousand on the word wall to help students self-correct their spelling.

## Interventions

- Provide a place value chart so students can put the numerals in the chart and then read it.
- Use the comma as a clue to group the numbers and say them together.

## Follow-Up DPP

| DPP E – Median and Mean | Unit 6 |
|---|---|

Ming experimented with 3 kinds of balls to find out which one bounced highest. He dropped each type of ball five times from 1 meter.
The bounce heights for the tennis ball were:
52 cm, 47 cm, 55 cm, 52 cm, and 50 cm.

1. Find the median bounce height. Show your work.

2. Use a calculator to find the mean bounce height to the nearest cm. Show your work.

## Implementation Suggestions

- This assessment will give the teacher information about which students can successfully find BOTH median and mean with a set of data. Teachers have found that completing a Frayer Map for both median and mean and posting it on the word wall gives students sufficient practice with both forms of central tendency.
- Since students have only used cubes to even out towers for smaller numbers to find mean, this assessment does ask for the student to use a calculator. Hopefully the lessons have been sufficient for students to learn how to read a number on the calculator and find the nearest whole number. Observe to see which students are not proficient at this skill.
- As an aside, there are fourth grade students who can use the cube model to solve for mean even with larger numbers. One student solved this problem by thinking of towers of cubes and NOT building the 50, just building the towers larger than 50. Imagine his thinking:
  - 52 is really 2 cubes
  - 55 is 5 cubes
  - 52 is 2 cubes
  - and 50 is 0 cubes
- That is 9 total cubes, but I need to take away 3 cubes because I need to build the 47 cubes into a 50 tower.
- So that leaves me with 6 cubes for 5 towers. Each tower would get one cube. The answer is 51.
- He/she is processing with positive and negative integers, and he/she does it very quickly.
- 2 + ( -3) + 5 + 2 = 6
- 6 divided by 5 is 1

## Interventions

- Review the difference between median and mean.
- These might be placed on the word wall.
- Median is the MIDDLE number in a group of numbers that are sequenced.

- Mean is really fair share.  It is as though every bounce is the same.
- Practice reading numbers on the calculator and determining the closer whole number. There are many decimal lessons later in fourth grade.  This process is really just deciding if the decimal is less than half or bigger than half.

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## Follow-Up DPP

| DPP I – Ana's Purchase | Unit 6 |
|---|---|

1. Ana bought a gallon of milk for $2.49, a box of crackers for $1.56, and a magazine for $2.95. Will $10.00 be enough to pay the bill? Show your work.

2. About how much change will Ana get back? Show your work.

## Implementation Suggestions

- This assessment has already been scaffolded for the student. Question 1 asks for a total, and question 2 asks for the difference. There are many problems within the URG that are multi step problems. This gives the teacher an opportunity to discern if students do know to use the correct operation to solve each part of the problem. Teachers may also see a student making errors in regrouping for addition and/or subtraction.
- Question 2 can be solved with either subtraction or adding on.
- $10.00 - $6.98 = $3.02
- $6.98 + $0.02 + $3.00 = $10.00

## Interventions

- Practice with computation strategies.

## Follow-Up DPP

- Unit 14 – DPP F, checks on finding differences to $10.

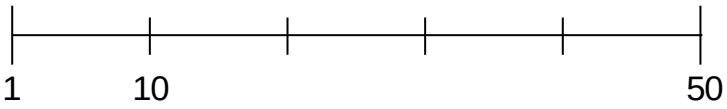
| DPP P – Which Is the Closest? | Unit 6 |
|---|---|

1. Which number is closest to 4056? Show your work.
   450 4000 4100 5000
2. Which number is closest to 62,096? Show your work.
   7000 60,000 65,000 70,000
3. Which is a reasonable estimate for the height of a 5-story apartment building?
   30 feet 50 feet 300 feet 500 feet

## Implementation Suggestions

- Read the directions and make sure that students show how they made their choices.
- You might provide an example.

  Which number is closest to 4?




- The discussion might include eliminating the 50 right away. Cross it out. Then looking at the other choices and by finding the difference, 10 is 6 away, 1 is 3 away which makes 1 closest to 4. This might also be modeled as a number line.
- If a student asks what a 5-story building is explain that it has 5 floors and probably has an elevator.

## Interventions

- Students who successfully complete 1 and 2 need no intervention. Others will need to return to work on the number line.

## Follow-Up DPP

- Unit 14 – DPP J, checks on understanding of number line and size of number.

| DPP I – Divisible by 2, 3, or 6? | Unit 7 |
|---|---|

A. Circle the numbers that are divisible by 2. How do you know?

762 1025 8031 7532

B. Circle the numbers which are multiples of 3. How do you know?

762 1025 8031 7532

C. Which numbers have 6 as a factor? How do you know?

762 1025 8031 7532

## Implementation Suggestions

- The words divisible, factor, and multiple are all on the word wall. Discuss how these three words are alike.
- These three sentences are good examples to start this discussion.
  - 25 is divisible by 5.
  - 5 is a factor of 25.
  - 25 is a multiple of 5.
- Allow students to work independently on this assessment. The explanations they give for their choices will give the teacher information about the students' ability to use the rules for divisibility. Here are some misconceptions that might surface:
  - If a number ends in 2, it is divisible by 2.
  - If a number ends in 3, it is divisible by 3.

## Interventions

- Use the divisibility charts made in lesson 2 and allow students to work on this assessment and other examples as well. Students should make the connection that, if a number is divisible by 2 AND 3, it is also divisible by 6.
- Likewise, if a number is divisible by 2 and 5, it is also divisible by 10.
- If a number is divisible by 9 and 5, it is also divisible by 45.
- Sometimes by making the rule more general, it makes more sense to some students.
- If a number is divisible by a and b then it is divisible by a times b.

## Follow-Up DPP

| DPP O – Multiplying by 10 | Unit 7 |
|---|---|

A. 7 X 80 =  B. 6 X 400 =  C. 8000 x 6 =

D. 700 X 4 =  E. n X 60 = 420  F. 800 X n = 3200

G. 10 X 700 =  H. 0 X 600 =

## Implementation Suggestions

- This assessment checks on understanding of multiplying and dividing by multiples of tens and hundreds. Students can use many strategies to solve these problems. Observe and ask students how they are using mental math to solve the problems.
- Example – Question A:
  - 7 x 80 might be solved by thinking 7 x 8 tens is 56 tens or 560.
  - 7 x 80 might be solved by (7 x 8 x 10) is (56 x 10) or 560.
- Students who have good understanding of powers of ten will probably be more proficient at E and F. Students who are merely counting 0s will be lost in understanding the magnitude of the answers for E and F.
  - Question E – How can I make 42 tens? 7 x 6 tens will give me 42 tens.
  - Question F – How can I make 32 hundreds? 8 hundreds x 4 will give me 32 hundreds.

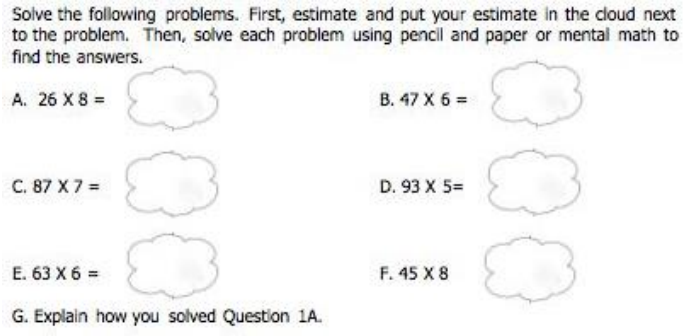
## Interventions

- There may be students who need interventions when multiplying by 10s and 100s. Rewriting the problems as 7 x 8 tens and modeling the problem with groups of skinnies might be necessary.
- For students who only miss Questions E and F, diagnose the error. Is it not understanding the inverse operation or is it guessing a number like 70 when 7 is correct? Plan the intervention to review missing variables or magnitude of numbers.

## Follow-Up DPP

- The next DPP to check on multiplying by 10s and 100s is Unit 10 – DPP M.

| DPP B – More Multiplication | Unit 8 |
|---|---|



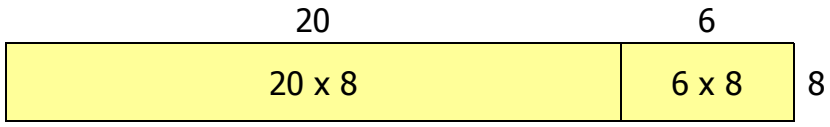

## Implementation Suggestions

- Begin this DPP by giving a word problem example.
- A basketball player played 9 games. He averaged 27 points per game. About how many points did he score? Draw a cloud on the board and ask students to think of a way we could estimate this problem and get an answer close to the actual answer. Allow groups time to discuss and then take ideas. In the cloud write 27 x 10 = 270. This basketball player got ABOUT 270 points. Is the estimate high or low? Then have students practice partial products and solve for 27 x 9 = 243. Yes, our estimate was high but close to the actual answer.
- Students are then directed to find an estimate for each multiplication problem, and then solve by using partial products. Usually by putting the estimate in a scenario, there is a better understanding of using approximation.
- Estimates might include: (accept any good use of friendly numbers)
  - 25 x 8 = 200 (208)
  - 50 x 6 = 300 (282)
  - 90 x 7 = 630 (609)
  - 90 x 5 = 450 (465)
  - 60 x 6 = 360 (378)
  - 50 x 8 = 400 (360)

## Interventions

- Two different small groups might be formed: those who cannot choose a good problem to use as an estimate or solve that estimate and those who cannot compute the actual answer with few errors.
- For the first group, practice using friendly numbers as factors. One of the problems with some students choosing friendly numbers is NOT really having the ability to use mental math for multiplication. They have no numbers that they can multiply with ease. Much work needs to be done with multiplying by multiples of 10s and 100s.

- For the second group, use an array model to show the partial products.

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## Follow-Up DPP

| DPP N – Arithmetic Review | Unit 8 |
|---|---|

Solve the following problems. First, estimate and put your estimate in the cloud next to the problem. Then, solve each problem using pencil and paper or mental math to find the answers.

A. 3005 + 61 + 458 =

B. 17 + 608 + 3 + 1060=

C. 917 - 145 =

D. Explain how you made your estimate for Question B.

## Implementation Suggestions

- Ask students to follow the directions and solve for an estimate first and then use a different strategy to solve for the exact answer. Warning: you may have students who can find the exact with mental math. Do not penalize these students. Accept their cloud answer but ask them to write their thinking below the cloud. Here are examples of students adding mentally left to right.
  - 3000 + 400 + 110 gives 3510 +14 (5 + 1 + 8) gives 3524
  - 1000 + 600 + 70 (60 + 10) + 18 (7 + 8 + 3) gives 1688.\
- Cloud answers might include:
  - 3560
  - 1620
  - between 700 and 800

## Interventions

- Students who add with the algorithm accurately need no intervention. Look carefully at the estimates students use to decide if they are using some sense making. Have students talk together about how they made their estimate for B.

## Follow-Up DPP

| DPP Q – Sharing Money | Unit 8 |
|---|---|

Five children found $4.00 in the hall at school. The principal said they could share it, if no one claimed it. How much would each child get? Show your work.

## Implementation Suggestions

- Read the problem aloud to the class. Remind students that they can use pictures or number sentences to show their work.
- Students may clearly draw pictures and show how they might share 16 quarters or 40 dimes among five students.
- When students start with quarters, they need to trade the final quarter for 5 nickels to fair share the $4 into 5 groups of $0.80. If they start with 40 dimes, each student gets 8 dimes.
- There may be students who can divide 400 by 5 to get 80 for each student.
- 5 x ___ = 400

## Interventions

- Small-group instruction might highlight how to model fair share. Keep the number of groups intact and make sure that each group gets the same amount. Keep fair sharing until there is not enough to give to each group.

## Follow-Up DPP

- The next division DPP is Unit 9 – DPP E.

| DPP E – Division Stories | Unit 9 |
|---|---|

The following two problems can be solved using division.

1. Mrs Randall gave each of her children $4 to spend on games at the neighborhood carnival. If Mrs. Randall gave out $20 in all, how many children does she have? Show your work.

2. One package of bus tokens contains 10 tokens. Keenya's mother needs 40 tokens to get to and from work for one month. How many packages does Keenya's mother need? Show your work.

## Implementation Suggestions

- This assessment can be solved with inverse operations, using multiplication instead of division. In each case, the problem gives the whole and asks the student to find one of the factors. They can both be solved by missing factors—4 x n = 20 and 10 x n= 40.
- In both problems, we know the number in each group and want to find the number of groups. They can also be solved with division.

## Interventions

- Have students write problems using fact families. With practice, students can get comfortable moving from division to multiplication. By writing problems that can be solved with either multiplication or division, the power of inverse operation becomes apparent.
- Have students write a word problem for the problem 5 x 8 = 40. Then, have students write a problem for 40 divided by 8 = 5. Finally, ask students how are these problems alike and how are they different.

## Follow-Up DPP

| DPP K – More Fact Practice | Unit 9 |
|---|---|

Find n to make each number sentence true.

| | | |
|---|---|---|
| A. 8 X 5 = n | B. n X 7 = 70 | C. n ÷ 4 = 5 |
| D. 80 ÷ n = 10 | E. 10 X n = 50 | F. 30 ÷ 5 = n |
| G. 9 X 10 = n | H. 15 ÷ n = 5 | I. n X 8 = 80 |
| J 10 ÷ 10 = n | | |

Explain how you solved either Question C or J.

## Implementation Suggestions

- Students move from thinking with multiplication to division to solve equations with missing factors. Another task that students can do with this assessment is to write the other facts that match each equation. The more students see a connection between multiplication and division, the better their fluency with facts will be.

## Interventions

- Students who can explain how they think and solve for missing factors will not need interventions. However, students who have difficulty saying, "To solve for n / 4 = 5 means that I multiply 4 x 5 to get the product, then the product / factor gives the other factor," need to practice explaining inverse thinking.

## Follow-Up DPP

| DPP M – Multiplying with Zeros | Unit 10 |
|---|---|

A. 80 X 20 =  B. 40 X 3 =  C. 3000 X 40 =  D. 20 X 500 =

E. 50 X 30 =  F. 600 X 2 =  G. 0 X 20 =  H. 10 X 30 =

## Implementation Suggestions

- Check for understanding of multiplication by powers of 10. Ask students questions about what strategies they are using to solve the problems.

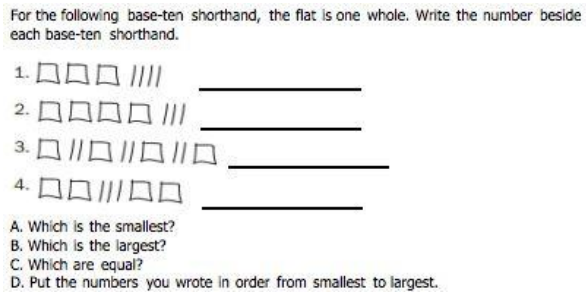
## Interventions

- There should be very few students who are NOT able to master this skill. Some students may need to use a calculator to solve these problems.

## Follow-Up DPP

- Unit 11 – DPP E gives another opportunity to check for understanding.

| DPP U – Smallest, Largest, and In Between | Unit 10 |
|---|---|




## Implementation Suggestions

- The teacher will get a lot of information about students' understanding of decimal notation as well as comparing decimal numbers. Provide base ten pieces so students can build the decimals if necessary. These decimal numbers only use tenths. Notice that the wholes and the tenths are not placed together. Some students might need to redraw the numbers before counting the wholes and tenths.
- Students can be redirected to do part D first and then answer A, B, and C. Some students may find this easier.

## Interventions

- To review this skill, ask students to build decimals when given the decimal number, example 4.6, 6.3, 8.2, 2.8, etc. Then, have students put the numbers in order.
- Another game to play is to say a decimal number and then ask students to build a number that is one-tenth more OR one-tenth less OR one whole more OR one whole less.

## Follow-Up DPP

- Unit 11 – DPP D will check this same concept.

| DPP X – Dividing It Up | Unit 10 |
|---|---|

26 ÷ 8 = Write a story for 26 ÷ 8. Then, draw a picture. Include any remainder in your picture. Solve the problem.

## Implementation Suggestions

- This assignment should provide information about the variety of ways students think about division. Since this problem has a remainder, the story has to fit dividing something with a few left over. Both the quotient and the remainder should be labeled in the solution.
- Partitive or measurement division would both work. Writing a story about 3 x 8 + 2 does not meet the requirement.

## Interventions

- Allow the students to use manipulatives to solve the problem (or a new problem). Then, use the model to write the story and draw the picture and write the solution with labels.
- Sometimes just reading good examples and listing the parts of the stories or pictures that make sense will give a student an idea to write a different story.

## Follow-Up DPP

- Unit 12 – DPP P is the next example of writing and drawing a story problem.

| DPP D – Decimals | Unit 11 |
|---|---|

Write each of the following decimals using base-ten shorthand. The flat is one whole.

A. 2.02 B. 0.6 C. 2.6

D. 0.06 E. 2.16

F. Write the decimals in order from smallest to largest.

## Implementation Suggestions

- This assessment gives the student another opportunity to show his knowledge of decimal notation, both numbers and base ten. As students are working, have each student read the decimals aloud to you. Perhaps point to just two or three so that each child says B, D, and E and shows knowledge of tenths and hundredths.

## Interventions

- There may be a small group of students who are not proficient. Prepare a set of cards with decimal numbers and another set with base ten and perhaps another with the cm sq model. Play fish or a matching game.

## Follow-Up DPP

- The next opportunity to check on this understanding is Unit 12 – DPP I.

| DPP E – Missing Number | Unit 11 |
|---|---|

A. n X 300 = 900

B. 7 X n = 490

C. 80 X n = 640

D. 6 X n = 36, 000

E. n x 40 = 160

F. 900 X n = 8100

## Implementation Suggestions

- This is another opportunity to check for understanding of multiples of 10s with missing factors. If students are held back by not knowing multiplication facts, allow them to use the multiplication tables and try again.

## Interventions

- Teach a system of guess and check.
  - n x 300 = 900
  - Try 3 x 300 = 900
  - Try 30 x 300 = 9000

  - 7 x n = 490
  - Try 7 x 7 = 49
  - Try 7 x 70 = 490

## Follow-Up DPP

- The next DPP that assesses this skill is Unit 15 – DPP L.

| DPP S – Measuring Volume | Unit 11 |
|---|---|

Romesh filled and empties a 250-ml cylinder into a large container 6 times. After the sixth time, the container was full. What is the volume of the container in milliliters? Show your work.

## Implementation Suggestions

- This story problem asks students to fill a container with 6 cylinders of 250 ml.
- Students at this time of fourth grade should be using multiplication to solve 6 x 250.
- Students who use repeated addition should be asked to solve it with a different operation.
- Remind students to draw a picture and write a number sentence are good strategies to solve problems.

## Interventions

- Although Visiting Egypt is not part of the prompts, it could be used as a group of different problems to give students as an intervention.

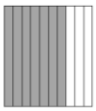
## Follow-Up DPP

- Unit 13 – DPP M is the next DPP that asks students to solve word problems with multiplication.

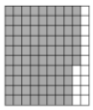
| DPP I – Decimals and Pictures | Unit 12 |
|---|---|

Write a decimal for the shaded part of each picture. The flat is one whole.

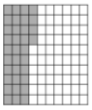
1. _____



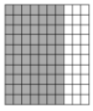
3. _____



2. _____



4. _____



## Implementation Suggestions

- Students can use manipulatives to solve these problems. As a way to differentiate, ask students to write a number sentence that follows this model.
  - Shaded + Unshaded = 1 whole

## Interventions

- Give students cm sq paper with flats drawn on the page. Give the students a decimal and have the student shade the flat.

## Follow-Up DPP

| DPP P – Division Stories | Unit 12 |
|---|---|

Write a division story for 28 ÷ 9. Then, draw a picture. Include any remainder in your picture. Solve the problem.

## Implementation Suggestions

- This assignment should provide information about the variety of ways students think about division. Since this problem has a remainder, the story has to fit dividing something with a few left over. Both the quotient and the remainder should be labeled in the solution.
- Partitive or measurement division would both work. Writing a story about 3 x 9 +1 does not meet the requirement.

## Interventions

- Allow the students to use manipulatives to solve the problem (or a new problem). Then use the model to write the story and draw the picture and write the solution with labels.
- Sometimes just reading good examples and listing the parts of the stories or pictures that make sense will give a student an idea to write a different story.

## Follow-Up DPP

| DPP S – Words to Numbers | Unit 12 |
|---|---|

1. Write the following words as numbers.

A. two-thirds B. six-tenths C. five-eighths D. one-twelfth

2. Write the following numbers as words.

A. $\frac{3}{4}$ B. $\frac{7}{9}$ C. $\frac{1}{2}$ D. $\frac{2}{5}$

---

## Implementation Suggestions

- This assessment informs the teacher to what extent a student can read and write fractions. Notice that in question 1, students read the words and write the fractions, and then in question 2, students reverse that skill. ¾ is written as "three-fourths" NOT "3 out of 4." Often students say "3 over 4" or "3 slash 4" instead of showing understanding of the name of the fraction's denominator is said in ordinal numbers, and the name of the fraction's numerator is a counting number. Teachers need to ask individual students to read fractions to make sure that all students are saying the fractions correctly.

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## Interventions

- Allow more practice and perseverance to continue having students change the way they read fractions.
- Example: Show a fraction picture and students say and write the fraction representation. Show a fraction representation and ask students to draw the fraction picture. This can be a partner activity.

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## Follow-Up DPP

| DPP E – Trumpet Practice | Unit 13 |
|---|---|

Ana's trumpet teacher told her to practice $\frac{3}{4}$ hour every day. Since 1 hour = 60 minutes, Ana used this number sentence to find the number of minutes she must practice each day:

$$\frac{3}{4} = \frac{?}{60}$$

1. Compete Ana's number sentence to find the number of minutes in $\frac{3}{4}$ hour.

2. Use Ana's method or your own strategies to find the following:

   A. How many minutes are in $\frac{1}{4}$ hour?

   B. How many minutes are in $\frac{1}{2}$ hour?

   C. How many minutes are in $\frac{1}{3}$ hour?

   D. How many minutes are in $\frac{1}{6}$ hour?

   E. How many minutes are in $1\frac{1}{4}$ hours?

   F. How many minutes are in 3.5 hours?

## Implementation Suggestions

- In Unit 2 – DPP W, students reviewed how to use a clock as a way to understand fractions. Many strategies were accepted. This assessment shows how to use equivalent fractions to find fractional parts of 60.
  - ¼ = ?/60
  - ½ = ?/60
- It is quite acceptable for students to use other strategies to find fractional parts of this whole (60 minutes).
- Allow use of clock faces, table of values, division.

## Interventions

- Observe students and suggest strategies to students who are not solving accurately.
- Often students are not ready for this "efficient" strategy, but with some picture or data table, the student can find success.

## Follow-Up DPP

| DPP M – TV Survey | Unit 13 |
|---|---|

Use paper and pencil to solve the following problems.

1. According to a recent study, the average American eighth-grader spends about 4 hours a day watching TV. How many hours in a year will an eighth-grader spend watching TV? Show your work.

2. A school year is approximately 180 days long. How many hours is a student in school, if a school day is 6 hour long? Show your work.

## Implementation Suggestions

- Students at this time of fourth grade should be using multiplication to solve 4 x 365 and 6 x 180.
- Students who use repeated addition should be asked to solve the problems with a different operation.
- Remind students to draw a picture and write a number sentence are good strategies to solve problems.

## Interventions

- There are many opportunities for students to learn to use multiplication to solve problems. Take a student's work using repeated addition and show how adding 365 + 365 + 365 + 365 is really adding 4 (300s), 4 (60s) and 4 (5s) just like the partial product algorithm. Repeat this side-by-side problem solving and allow students to think first in addition and then write it like partial products.

## Follow-Up DPP

| DPP Y – Order Fractions | Unit 13 |
|---|---|

1. Which is greater $\frac{1}{12}$ or $\frac{1}{10}$? How did you decide?
2. Which is greater $\frac{3}{5}$ or $\frac{3}{8}$? How did you decide?
3. Which is greater $1\frac{1}{2}$ or $\frac{5}{4}$? How did you decide?

## Implementation Suggestions

- Students in fourth grade have compared fractions using fraction strips and pattern pieces. Allow students to use the fraction chart URG 12, page 61. The teacher may make the decision which students need the chart to complete the assessment and which students might want to use it to check their answers. Remind students that they need to draw or write how the student decided which fraction is greater.
- Notice that question 1 asks students to compare unit fractions, question 2 is comparing like numerators, and question 3 asks students to think of mixed numbers versus improper fractions and then compare unit fractions.

## Interventions

- Return to the fraction strips to compare fractions.
- Review comparing unit fractions.
- Review comparing like numerators.

## Follow-Up DPP

- Next DPP checking on this skill is Unit 16 – DPP G.

| DPP F – Lots of Money | Unit 14 |
|---|---|

Pretend you are at the store without your calculator or paper and pencil. See how many of the answers you can figure out in your head. Tell what strategies you used.

**How much change will you receive from a $10.00 bill if you spend the following amounts?**

A. $5.60

B. $7.45

C. $1.97

D. $4.25

## Implementation Suggestions

- The first DPP to check on understanding of change for $10 is Unit 6 – DPP I. Check on the students who had difficulty and ask them to talk about a strategy that will work for this assessment. Each time allow for adding on or subtraction.

## Interventions

- Use coins and bills to count out change.
- Play a game called, “How much to 100 or 1000?” This skill of adding on to 100 will increase a student’s ability to think about what two addends make 100 or 1000.

## Follow-Up DPP

| DPP J – Adding and Subtracting | Unit 14 |
|---|---|

Use mental math to solve the following problems.

A. 1700 + 400 = B. 4300 - 400 = C. 1450 + 350 =

D. 1658 - 500 = E. 5033 + 9100= F. 2099 + 301 =

G. Explain your mental math strategy for Questions C and D.

## Implementation Suggestions

- This is the last assessment for the teacher to check on the student's ability to use mental math in addition and subtraction. Students who are still using an algorithm to solve these problems need intervention.

## Interventions

- Give the student any number in the thousands (5436) and ask, "What is 100 more? What is 100 less? What is 1000 more? What is 1000 less? What is 500 more? What is 60 more?"
- Give each student a white board and allow the students to look at the number 5436. Ask one question at a time and have the student solve the problem mentally, writing only the answer. Then have students talk about the strategy they use to tell what is 60 more?

## Follow-Up DPP

| DPP L – Related Multiplication and Division Sentences | Unit 14 |
|---|---|

Find a number for n in each number sentence that makes the statement true.

| | | |
|---|---|---|
| A. 4 X n = 2400 | B. 2400 ÷ 4 = n | C. n X 9 = 360 |
| D. 360 ÷ 9 = n | E. 7 X n = 63,000 | F. 63,000 ÷ 7 = n |
| G. 6 X n = 54 | H. 54 ÷ 6 = n | I. 9 X n = 4500 |
| J. 4500 ÷ 9 = n | K. 8 X n = 560 | L. 560 ÷ 8 = n |
| M. 3 X n = 0 | N. 0 ÷ 3 = n | |

## Implementation Suggestions

- This is the last assessment for the teacher to check on the student's ability to use mental math to solve problems with multiplication and division with missing factors. Allow students who need use of the multiplication tables full use. These students still need to understand the power of ten and inverse operation to solve these problems even with the access to the facts.

## Interventions

- Refer to early mental math interventions.

## Follow-Up DPP

| DPP G – Fractions | Unit 16 |
|---|---|

Which is larger? Explain how you decided on yours answers.

A. $\frac{1}{12}$ or $\frac{1}{10}$ ?

B. $\frac{3}{2}$ or $1\frac{1}{4}$ ?

C. $\frac{6}{12}$ or 0.5 ?

D. $\frac{5}{8}$ or 0.4?

## Implementation Suggestions

- This assessment mirrors DPP 13Y. Students are asked to compare unit fractions as well as comparing mixed numbers and improper fractions. Questions C and D compare fractions to decimals.
- This assessment asks students to explain the way the student decided.
- For C, the fraction and decimal are equal. A student might convert both numbers into fractions or convert both to decimals. For D, the fraction is greater than ½, and the decimal is smaller than ½.

## Interventions

- Students can use the chart from Unit 12, page 61 and make up problems for their partner. One student can write 2 fractions, and then his/her partner has to choose the larger and explain.

## Follow-Up DPP